

Can Stablecoins be Stable?

Adrien d'Avernas* Vincent Maurin[†] Quentin Vandeweyer[‡]

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Abstract

This paper provides a framework to analyze the stability of stablecoins – cryptocurrencies pegging their price to a currency. We study the problem of a monopolist platform earning seigniorage revenues from issuing stablecoins and characterize equilibrium stablecoin issuance-redemption and pegging dynamics, allowing for various degrees of commitment over the system's key policy decisions. Even with full commitment, purely algorithmic (uncollateralized) platforms are vulnerable to large demand shocks and always feature a zero price equilibrium. Collateral is helpful to stabilize a platform but costly to equity token holders. Decentralization acts as a substitute for commitment by delegating issuance to atomistic agents.

Keywords: Stablecoins, Cryptocurrencies, Target Leverage, Dynamic Games, Coase Conjecture

*Stockholm School of Economics

[†]Stockholm School of Economics

[‡]University of Chicago: Booth School of Business

1 Introduction

A stablecoin is a cryptocurrency designed to maintain a stable value vis-à-vis an official currency. It aims to avoid a fundamental drawback of conventional cryptocurrency: being too volatile to be used as a means of payment or store of value. Stablecoins therefore allegedly combine the benefits of the blockchain technology with the stability of established currencies and have gained in popularity in the last couple of years, with combined market capitalization growing from \$3 billion in 2019 to \$181 billion in April 2022.¹ Confronted with the rapid growth of stablecoin platforms along with multiple crashes (e.g., the Terra-Luna platform in May 2022), legislators have become increasingly concerned about the financial stability risks posed by stablecoins and have introduced new regulatory initiatives to balance the perceived risks and benefits associated with this new technology.²

Stablecoin protocols rely on a wide variety of pegging mechanisms to fulfill their promise of price stability: algorithmic supply adjustments (e.g., Terra), collateralization with dynamic liquidation (e.g., Frax), and decentralization of the issuance process (e.g., DAI). To this date, however, the academic literature provides little guidance about the stability of these tools and their optimal design. This paper aims to fill this gap by developing a general model of stablecoins to analyze the performance of various pegging mechanisms and assess their riskiness.

We propose a framework to study the dynamic problem of a stablecoin platform that caters to a time-varying demand from users. Users, who value price-stability, enjoy liquidity benefits from owning stablecoins when its price is pegged with respect to some unit of account. The platform acts as a monopolistic issuer and extracts profits from these liquidity benefits when able to maintain the peg. The existence of seigniorage revenues make a stablecoin platform akin to a (private or central) bank. Like a bank who may overprint money, a stablecoin platform has a tendency to overissue stablecoins, which ultimately undermines the peg. A bank's ability to maintain parity thus relies to a large extent on credibility and trust. The main technological proposition of stablecoins in this regard is the possibility to supersede trust by effectively committing to specific key policies such as issuance and redemption, interest rates and fees, and collateral liquidation rules via smart

¹<https://www.statista.com/statistics/1255835/stablecoin-market-capitalization/>

²For instance, the US Congress is working on a STABLE (Stablecoin Tethering and Bank Licensing Enforcement) Act while in the UK, the Treasury has launched the “UK regulatory approach to cryptoassets and stablecoins: Consultation and call for evidence”.

contracts.

Our objective is to characterize the price of stablecoins, the value of the platform’s equity tokens, and provide conditions under which the peg holds and under which it doesn’t. To maintain a stable price, the platform reacts to a negative demand shock by repurchasing (issuing) stablecoins to reduce (increase) its supply. In an expansion phase, the platform generates revenues by minting new stablecoins. In a contraction phase, the platform finances the stablecoin buybacks by issuing additional equity shares and diluting legacy holders. An equilibrium in our model has two components. On the one hand, the monopolistic platform chooses dynamic issuance-repurchase, interest rate, and collateralization policies. On the other hand, users price the stablecoin competitively given the liquidity benefits they derive from owning stablecoins and the interest paid by the platform. The unique state variable is the demand ratio between current stablecoin demand and supply by the platform—the inverse of which represents the platform’s implicit leverage.

We first study stablecoin protocols that can fully commit to all its policies through *credible* smart contracts. This analysis provides an upper bound for the value of *algorithmic stablecoin* protocols that rely on programmable adjustments of stablecoin supply. Our first result is that even under full commitment, uncollateralized platforms always admit an equilibrium in which stablecoins and equity tokens are worth zero. This equilibrium always arises because both stablecoin dividends, that is, liquidity benefits and interest payments depend themselves on the value of stablecoins. As with any fiat currency, this self-referential feature implies a zero value fixed point.

Under full commitment, we then also show that a second equilibrium exists in which the peg is *locally* stable but vulnerable to large negative demand shocks. In this equilibrium, the system generates seigniorage revenues, and equity tokens may have positive value. Thanks to this positive value, the platform is able to reduce stablecoin supply in the face of small negative demand shocks and maintain a constant optimal demand ratio. After a large negative demand shock, however, the value of equity tokens may be too low so that the platform cannot finance the necessary stablecoin repurchase and cannot cut back supply enough to maintain the peg even with full dilution. The peg is then broken as a too high stablecoin supply implies that the market clears at a price below par. Although the peg is lost and equity tokens are worth zero, the stablecoin price may still be positive and fluctuates exogenously with demand as users hope for resurrection. At some point, stablecoin demand may recover enough so that governance token holders can recapitalize

the platform to repurchase the quantity of stablecoin necessary to re-establish the peg. This mechanism is consistent how the collapse of the Terra-Luna platform unfolded. Following a large negative demand shock, the protocol reacted by minting exponentially increasing quantities of equity tokens Luna to withdraw stablecoins Terra but failed to restore the peg because the price of Luna converged to zero at a point for which the supply of Terra was still excessive. In Appendix A, we provide a descriptive analysis of the May 2022 stress for the five largest stablecoin platforms, including Terra-Luna.

We then investigate the stability properties of a stablecoin scheme under a weaker form of commitment. In practice, it might be preferable retain some ability to make discretionary changes in key parts of its algorithm to preserve adaptability to new market developments and technical issues. More precisely, we relax our initial assumption that all policies can fully be programmed via smart contracts and assume that the platform can commit to an interest rate rule while retaining discretion over its issuance-redemption policy. For a constant interest rate rule, we show that the leverage ratchet effect of [Admati, DeMarzo, Hellwig, and Pfleiderer \(2018\)](#) and [DeMarzo and He \(2021\)](#) applies and it is never optimal for the system to reduce its leverage so that designing a platform that maintains a peg is not possible. We find, however, that an equilibrium with local stability still exists if the interest rate payment decreases with the demand ratio. Such a rule penalizes overissuance and is sufficient to incentivize the platform to implement repurchase despite these interests being paid in units of stablecoin.

We also consider how locking an external asset as collateral affects the system’s stability. This design is common in practice, with many stablecoins such as DAI or Frax relying on external crypto-currency holdings as part of their pegging mechanism. In our model, adding collateral is helpful to stability because it provides an extra buffer against large jump shocks. Ex-ante, however, holding this asset as collateral can be costly if it generates a return below the economy’s discount rate. It is therefore often optimal to design a platform that is not fully collateralized and, hence, remains vulnerable to large shocks.

Finally, we examine the stability of a stablecoin platform that decentralizes the issuance and redemption of its stablecoin. This feature is present in DAI: a stablecoin that anyone with access to the Ethereum platform can mint freely. We find that this decentralization can act as an effective substitute for a commitment technology on stablecoin redemption and issuance. In this setting, investors acting as arbitrageurs prevents the price from moving away from the peg by creating more (redeeming) stablecoins in reaction to a positive

(negative) demand shock. Hence, decentralization allows the system to locally maintain the peg as in the full-commitment setting because the decisions affecting that system’s leverage have been externalized to agents that—unlike equity token holders—are not hurt by a reduction of leverage.

Related literature Our paper contributes to an interdisciplinary literature on stablecoins. From the computer sciences literature, [Klages-Mundt and Minca \(2019, 2020\)](#) develop models featuring endogenous stablecoin price and an exogenous collateral and find deleveraging spirals and liquidation in a system with imperfectly elastic stablecoin demand. [Gudgeon, Perez, Harz, Livshits, and Gervais \(2020\)](#) simulate a stress-test scenario for a DeFi protocol and find that excessive outstanding debt and drying up of liquidity can lead the lending protocol to become undercollateralized. Our paper also relates to a descriptive literature on stablecoins ([Arner, Auer, and Frost, 2020](#); [Berentsen and Schär, 2019](#); [Bullmann, Klemm, and Pinna, 2019](#); [ECB, 2019](#); [Eichengreen, 2019](#); [G30, 2020](#)). In closely-related contemporaneous work, [Li and Mayer \(2022\)](#) study the peg dynamics of stablecoin platforms under the assumption that stablecoins generate network externalities and the systems’ reserves are subject to stochastic shocks. Our paper differs by considering various commitment technologies and demand shocks that affect the system’s seigniorage and equity token value, allowing the study of fully algorithmic stablecoins.

In studying the stabilization mechanisms across stablecoin types and the failure of governance incentives to recapitalize undercollateralized systems, our paper is connected to the corporate finance literature examining firm shareholders’ attitudes towards leverage. In [Black and Scholes \(1973\)](#) and [Myers \(1977\)](#) firm shareholders do not have incentives to voluntarily reduce leverage as this always implies a transfer of wealth to existing creditors. [Admati, DeMarzo, Hellwig, and Pfleiderer \(2018\)](#) generalize these findings to multiple asset classes of debt and with agency frictions and document a *leverage ratchet effect*, whereby shareholders never have any incentives to delever. [DeMarzo and He \(2021\)](#) also show delevering resistance effects in a dynamic setting, although in their model leverage mean-reverts to a target because of asset growth and debt maturity. Our paper contributes to this literature by considering cases in which the firm (stablecoin platform in our setting) can partially commit or decentralize the buybacks and coupon payment decisions through a smart contract algorithm. These features can be seen as an extreme form of debt-covenants as studied in [Smith and Warner \(1979\)](#), [Bolton and Scharfstein \(1990\)](#), [Aghion and Bolton](#)

Platform	
Collateral	Stablecoins
Future Seigniorage	
	Equity Tokens

Figure 1: Sketch of a Centralized Platform Balance Sheet

(1992), and [Donaldson, Piacentino, and Gromb \(2020\)](#) in a continuous state-space.

More broadly, our paper contributes to the literature applying finance theory to model digital platforms and token valuations. While not mainly focusing on stablecoins, [Cong, Li, and Wang \(2020a\)](#) develop a continuous-time model of token-based platform economy with network effects and endogenous token price and also document conflicts of interests between platform owners and users, resulting in an under-investment outcome. [Cong, Li, and Wang \(2020b\)](#) build a dynamic asset pricing model with network effects and intertemporal linkages in endogenous token price and user adoption, and analyze the Markov equilibrium with platform productivity as the state variable.

2 General Environment

In this section, we describe our model of stablecoins. The central premise of our analysis is that users enjoy utility benefits from holding stablecoins issued by the platform, as they would do for money or bank deposits. Our model also embeds users' preferences for stable means of payment. As a result, the stablecoin platform can generate seigniorage revenues if (but only if) it can maintain a peg between the stablecoin price and some target unit of account. To fix ideas, we provide in [figure 1](#) a sketch of a balance sheet for a generic centralized platform. We describe the formal building blocks of the model below.

2.1 Stablecoin Demand

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space that satisfies the usual conditions. All agents are risk neutral with an exogenous discount rate of $r > 0$.³ Time is continuous with $t \in [0, \infty)$.

We consider a platform that issues stablecoins. Stablecoins are a liability of the platform that trade at (endogenous) price p_t expressed in the unit of account. The outstanding stock of these stablecoins at date t is C_t . Stablecoins have value because users enjoy direct utility from holding them: at date t , holding stablecoins generates utility flow $U(A_t, p_t C_t)$ per unit with A_t an exogenous driver of stablecoin utility value. To fix ideas, one could interpret variable A_t as the value of some cryptoassets, which proxies for users' demand for alternative means of payment. The utility derived from holding stablecoins can be thought as a liquidity benefit users derive because stablecoins are a form of money.⁴ We denote the marginal utility benefit from holding an additional unit of stablecoin, or its *convenience yield*, as $\ell(A_t, p_t C_t)p_t \equiv \partial U(A_t, p_t C_t) / \partial C_t$ and make some restrictions on its properties.

Assumption 1. *The convenience yield on stablecoins $\ell(A, pC)$ is (i) positive and continuously differentiable in both arguments; (ii) strictly increasing in A ; (iii) bounded with $0 \leq \ell(A, pC) \leq r$; (iv) homogeneous of degree 0; and (v) equal to 0 if the stablecoin price p is not pegged to 1. Finally, (vi) the product of the convenience yield with the total value of stablecoins $\ell(A, pC)pC$ is single-peaked with $\lim_{x \rightarrow \infty} \ell(A, x)x = 0$.*

Property (i) rules out negative marginal utility from stablecoin holdings and ensures differentiability. Property (ii) states formally that the value of stablecoins increases with demand driver A_t . Property (iii) and (iv) are technical assumptions ensuring respectively that the stablecoin price is well-defined and that the problem ultimately economizes on one state variable. Property (v) states that stablecoin owners enjoy a liquidity benefit only if it is pegged to the unit of account. This assumption is meant to capture in a tractable way that users value the stablecoin as means of payment to the extent that its issuer can maintain its parity with the unit of account.⁵ The peg at 1 is chosen for convenience and

³Alternatively, we can interpret the model as written under a fixed risk-neutral measure that is independent of the stablecoin platform policies.

⁴Our reduced-form specification can be microfounded assuming that stablecoins are essential to carry some transactions.

⁵Without this “extreme-peg” assumption, a stablecoin could have value even though there is no active management of the supply of stablecoins to stabilize its price, as a standard cryptocurrency. This assumption allows us to precisely characterize the optimal equilibrium policies in Lemma 2 and Proposition 3 below.

because it corresponds to market practice but our results do not depend upon it; only the real value of stablecoin holdings pC matters. Finally, Property (vi) ensures that the optimal amount of stablecoins is interior. An example of a class of functions that satisfy Assumption 1 is $\ell(A, C) = \kappa \exp((C/A)^{-\alpha} - (C/A)^{-\beta})$ for $\beta > \alpha > 0$ and $\kappa = r / \max_a \{\ell(a, 1)\}$. In this example, when C is low, the convenience yield is increasing in C as more stablecoins are in circulation but eventually declines as the stock becomes too large.

The cryptoasset value A_t that drives stablecoins' demand has the following law of motion

$$dA_t = \mu A_t dt + \sigma A_t dZ_t + A_{t-}(S_t - 1)dN_t, \quad (1)$$

where dZ_t is the increment of a standard Brownian motion and dN_t is a Poisson process with constant intensity $\lambda > 0$ adapted to \mathcal{F} . The size of a downward jump, $-\ln(S)$ is exponentially distributed with parameter $\xi > 0$ and the expected jump size is $\mathbb{E}[S - 1] = -1/(\xi + 1)$. The Poisson process generates large negative shocks to stablecoin demand that can be thought of as news about the usefulness of the stablecoin or speculative attacks. Overall, the expected growth rate of stablecoin demand is given by $\mu - \lambda/(\xi + 1)$, which we assume is strictly lower than the discount rate r . We use notation A_{t-} to denote the cryptoasset's value before a jump.⁶

Finally, there exists a safe asset that the platform can hold as collateral to back the issuance of stablecoins. This collateral trades in a competitive market at price p_t^k with

$$dp_t^k = \mu^k p_t^k dt. \quad (2)$$

This specification implies that collateral delivers a (safe) return μ^k with $\mu^k < r$. The difference between the discount rate and the return on collateral, $r - \mu^k$ can be interpreted as a convenience yield enjoyed by collateral asset owners. As we will see, this feature generates a cost from holding collateral for the stablecoin platform.⁷ Examples of this asset include cash, government securities, or bank deposits denominated in the target currency.

⁶ A_{t-} denotes the left limit $A_{t-} = \lim_{h \rightarrow 0} A_{t-h}$. Also note that $A_{t-}dt = A_t dt$ as the set $\{T_k\}_{k \geq 1}$ of jump times has zero measure of length.

⁷ Our assumption of a safe collateral asset comes with some loss of generality because some stablecoin platforms are implicitly or explicitly backed by cryptoassets. In this case, the collateral price would likely be correlated with demand process A_t . It is intuitive, however, that such correlation would reduce the usefulness of collateral as a hedge against demand fluctuations. From a technical standpoint, introducing correlation significantly complicates the analysis.

2.2 Platform Operation

We will analyze both a centralized and a decentralized platform. For clarity, we postpone the description of a decentralized platform to Section 5.

Definition 1 (Centralized Platform Policies). *A sequence of policies for the platform is an issuance-repurchase policy $\{d\mathcal{G}_t\}_{t \geq 0}$, a interest rate policy $\{\delta_t\}_{t \geq 0}$ paid in stablecoins, with $\delta_t > 0$, a collateral purchase policy $\{d\mathcal{M}_t\}_{t \geq 0}$, and a stochastic default time τ .*

The main policy of the platform is the issuance(-repurchase) policy $\{d\mathcal{G}_t\}_{t \geq 0}$. A positive (negative) value of $d\mathcal{G}_t$ corresponds to an issuance (repurchase) of stablecoins at price p_t at date t . The platform can also pay a coupon as interest to stablecoin owners. As in practice, this interest is paid in stablecoins, not in the unit of account. A platform may hold collateral with $d\mathcal{M}_t$ denoting the change in collateral value held by the platform at date t .

There exists a useful analogy between the stablecoin platform and a central bank. When it issues stablecoins ($d\mathcal{G}_t > 0$), the platform receives a payment $p_t d\mathcal{G}_t$ from users in the unit of account. Similarly, when it credits the account of a depository institution with reserves, the central bank receives an asset in exchange. The stablecoin's coupon policy whereby every stablecoin user is credited with $\delta_t \geq 0$ units of free stablecoins per unit owned is akin to an interest payment on reserves. Finally, collateral holdings of the platform correspond to central bank's holdings of foreign reserves.

We focus on a constant collateralization rule for the platform.

Assumption 2. *The platform maintains a fixed collateralization ratio φ , that is,*

$$K_t = \varphi C_t. \tag{3}$$

Assumption 2 states the platform must maintain a constant ratio $\varphi \in [0, 1]$ between the value of its collateral and the par value of stablecoins. The case $\varphi = 0$ ($\varphi = 1$) corresponds to a so-called algorithmic stablecoin (narrow bank). Assumption 2 helps simplify our analysis in that it eliminates collateral as a state variable and is in line with actual stablecoin designs such as DAI. Although this collateralization rate φ is fixed, the platform still has to choose a value for it at the initiation date 0 and does so to maximize its equity value.⁸

⁸As locking collateral in the platform is costly ($\mu^k < r$), we can show that it is never optimal to hold a

Assumption 3. *The equilibrium policies must satisfy the no-Ponzi-game condition:*

$$\lim_{T \rightarrow \infty} \mathbb{E}_t[e^{-rT} p_T C_T] = 0 \quad \forall t \geq 0.$$

Finally, Assumption 3 restricts our analysis to equilibrium policies that do not generate a Ponzi scheme. That is, the platform can not sustain the value of stablecoins simply by issuing new stablecoins. In other words, the value of a platform must rely on the creation of liquidity benefits for the owners of stablecoins, and not on the overaccumulation of debt.⁹

Laws of Motion The platform's policies imply the following laws of motion for the stock of stablecoin outstanding, C_t , and the value of its collateral, denoted K_t :

$$dC_t = \delta_t C_t dt + d\mathcal{G}_t, \tag{4}$$

$$dK_t = \mu^k K_t dt + d\mathcal{M}_t. \tag{5}$$

Equation (4) is the law of motion for stablecoins. The first term on the right-hand side captures the contribution of the coupon policy δ_t to stablecoin issuance. It must be treated separately from the active issuance component $d\mathcal{G}_t$ because the coupon policy increases the stablecoin stock without compensation for the platform. Equation (5) is the law of motion for the collateral value. The first term on the right-hand side corresponds to passive changes in collateral value. The second term corresponds instead to active changes in value due to purchases or sales. The collateral policy $d\mathcal{M}_t$ is fully determined by the issuance policy $d\mathcal{G}_t$ and the coupon policy δ_t at date t because $dK_t = \varphi dC_t$ under Assumption 2.¹⁰

Jump Notation There are both Brownian shocks and jumps to the value of cryptoassets in our model. We therefore also allow the platform's policies to feature jumps. A jump represents a discrete, instantaneous change in a variable. We denote the value of a variable X just before and after the jump by X_{t-} and X_t , respectively.

buffer of collateral above the minimum requirement and, in our Markov equilibria in Section 4 and Section 5, $K_t = \varphi C_t$ at all times.

⁹Although our analysis excludes Ponzi games in a technical sense, we will see that stablecoin equilibria may have features that are casually associated with Ponzi schemes such as requiring a positive growth rate of demand and financing interest paid to holders today by diluting future seigniorage revenues.

¹⁰Law of motion (5) can alternatively be written $dK_t = S_t^k dp_t^k + p_t^k dS_t^k$ with S_t^k the quantity of collateral held by the platform. The term $d\mathcal{M}_t$ in (5) corresponds to $p_t^k dS_t^k$.

2.3 Stablecoin Pricing and Platform's Objective

Stablecoin Pricing Users price the stablecoin competitively taking as given the platform policies. They enjoy two income streams from holding stablecoins: the direct utility benefits when the price is pegged and coupon payments when the stablecoin platform pays interest, with respective value $\ell_t p_t$ and $\delta_t p_t$ per unit of stablecoin. Should the platform default, an instantaneously liquidation procedure applies in which stablecoin owners are treated as pari-passu creditors. They receive any platform's collateral up to the parity value of stablecoins. At date t , the competitive stablecoin price given the platform's continuation policies is thus

$$p_t = \mathbb{E}_t \left[\int_t^\tau e^{-r(s-t)} (\ell_s + \delta_s) p_s ds + e^{-r(\tau-t)} \varphi \right]. \quad (6)$$

Users compute expected future cash flows by forming rational expectations over the platform's policies from date t onward. Upon liquidation of the platform, users receive φ per unit of stablecoins as the collateralization ratio is weakly lower than the par value of stablecoins ($\varphi \leq 1$).¹¹

Platform's Objective The platform starts with no stablecoin outstanding at date 0, that is, $C_0 = 0$ and maximizes its value E_0 , which is the sum of the issuance benefits net of the collateral purchases.

$$E_0 = \max_{\varphi, \tau, \{\delta_t, d\mathcal{G}_t\}_{t \geq 0}} \mathbb{E}_0 \left[\int_0^\tau e^{-rt} (p_t d\mathcal{G}_t - d\mathcal{M}_t) \right], \quad (7)$$

where the price p_t is given by equation (6). When the platform is liquidated, as $\varphi \leq 1$, all its collateral is used to partially repay stablecoin users. Hence, the platform receives zero payoff upon liquidation. As a monopolistic issuer, the platform has price impact. Hence, it pays the post-issuance (post-repurchase) price when it issues (repurchases) stablecoins. As in [Admati et al. \(2018\)](#) and [DeMarzo and He \(2021\)](#), this feature is important because it weakens the platform's incentives to buyback blocks of debt.

As we will see, a platform's ability to implement at date $t > 0$ a policy chosen at date 0 depends on its commitment power. A key technological proposition of stablecoins is that rules and procedures can be programmed in advance through algorithms, so-called

¹¹Later, as the policies are Markov and thus only functions of the current values of the payoff relevant states variables (A_t, C_t) , we substitute $\mathbb{E}_t[\dots]$ by $\mathbb{E}[\dots | A_t, C_t]$.

smart contracts. In many cases, however, platforms retain some flexibility over parts of the algorithm for technical maintenance, future adaptability, or to decrease vulnerability to hacking. To reflect these concerns and to capture heterogeneity in smart contracts’ credibility and transparency, we will analyze optimal policies under varying degrees of commitment.

2.4 Discussion of the Environment

Platform Fees We assume that the platform coupon policy is always positive $\delta_t \geq 0$ —that is, the platform never levies fees on stablecoin users. Doing so simplifies the analysis as only buy-backs can be used to reduce stablecoin supply.¹² This assumption also corresponds to the practice of the main stablecoin platforms: Terra notoriously subsidized platform usage by paying an annual interest rate of 20%; DAI’s interest rate is typically fluctuating between 1% and 7%; and Tether does not pay any interest nor levies fees.

Peg vs. Redemption Rights In our model, the platform does not provide redemption rights to investors. Instead, investors must trade in a competitive market to exchange their stablecoins for the unit of account and the platform administer the peg through supply adjustments. To the extent that the platform maintains the peg, however, investors are effectively guaranteed a fixed exchange rate between stablecoins and the unit of account and not defending the peg is observationally equivalent to stop redeeming stablecoins at par.

Platform Competition We focus on the analysis of an economy featuring a single stablecoin platform. In practice, several stablecoin platforms compete to cater to users’ demand for alternative means of payment. Although we refrain from modeling competition and entry of platforms for parsimony, one can interpret the platform’s convenience yield as investors’ residual demand for a platform’s stablecoins after accounting for supply from other platforms. All that is needed is that the platform enjoys some market power, which would arise naturally with payment network effects as in [Cong, Li, and Wang \(2020a\)](#).

¹²In theory, allowing for time-varying negative interest would give the platform an additional tool to reduce the supply of stablecoin by effectively taxing or diluting legacy stablecoin holders. Doing so would allow maintaining a nominal peg, but stablecoin holders would face the same loss as in a de-pegging. The real value of the stablecoin would not be different than in our analysis.

Equity Tokens As in most traditional corporate finance settings, it is only the total value of equity (or market capitalization) that matters for equilibrium so that we do not need to separately keep track of the nominal quantity of equity token outstanding and its price per unit. Consequently, we also abstract from the exact mechanism that is used by the platform to buyback stablecoins by diluting governance token holders, although our model is fully consistent with the common practice of issuing additional equity tokens to finance buybacks as described in Appendix A in which we review the most popular stablecoin platforms.

3 Credible Smart Contracts

In this section, we analyze the problem of a stablecoin platform that can commit to all future policies. A platform with full commitment can be viewed as a stablecoin platform with credible smart contracts governing all policies including issuance and repurchase of stablecoins. The analysis under full commitment provides minimal necessary conditions for a stablecoin platform to have positive value and to be able to maintain the peg.

For this analysis, the only constraint on the platform’s policy choices at date 0 is that its equity cannot become negative at some future date t , that is, limited liability applies.¹³ To clearly highlight the role of this constraint, we first consider a benchmark with unlimited liability in Section 3.1 and then reintroduce limited liability in Section 3.2.

3.1 Unlimited Liability Benchmark

We first assume the platform’s equity value may become negative. In this analysis, there is no default so we set $\tau = \infty$. The platform chooses a stablecoin issuance-redemption policy $\{d\mathcal{G}_t\}_{t \geq 0}$, an interest policy $\{\delta_t\}_{t \geq 0}$, and a collateralization rate φ to maximize the value

¹³The term “limited liability” typically refers to the legal protection provided to shareholders that the company’s liability does not extend beyond the company’s assets. In this work, we use this term to refer to the fact that, in the anonymity of the blockchain, it is impossible to credibly commit to recapitalizing a platform beyond what is effectively pledged or escrowed in a smart contract (which could include the forced dilution of the equity).

of the platform at date 0 given by

$$E_0 = \max_{\varphi, \{\delta_t, d\mathcal{G}_t\}_{t \geq 0}} \mathbb{E} \left[\int_0^\infty e^{-rt} (p_t d\mathcal{G}_t - d\mathcal{M}_t) \middle| A_0, C_{0-} = 0 \right] \quad (8)$$

subject to (6), (4), and (3).

The platform's payoff is the net present value of issuance proceeds net of collateral purchase costs. Equation (4) is the law of motion for stablecoins implied by the issuance policy and the initial condition $C_{0-} = 0$. Equation (6) is the competitive pricing function for stablecoins at any date t , given policies chosen by the platform for dates $s \geq t$.

Our first result is that even under full commitment, there exists an equilibrium with zero stablecoin price and zero platform value if the platform does not hold collateral.

Proposition 1. *For an uncollateralized platform with $\varphi = 0$, there always exists a zero-price equilibrium in which $p_t = 0$, for all $t \geq 0$.*

The zero-price equilibrium arises because there is no anchor between the stablecoin and the unit of account for an uncollateralized platform. In particular, the coupon is paid in stablecoins, not in the unit of account. To see why a zero-price equilibrium exists, suppose the price is indeed 0. Then both components of the stablecoin dividend in the pricing function (6) are equal to 0. Stablecoin owners enjoy no liquidity benefit because an asset that has a price of 0 cannot provide liquidity benefits and the real coupon $p\delta$ is also worth 0 even if the platform promises a very large nominal coupon payment δ . Finally, without collateral, the price is not supported by an external asset. Hence, the stablecoin price is equal to zero and the platform has no value because it never captures any stablecoin issuance benefits.

Proposition 1 shows that unbacked stablecoins, like any fiat money, are fragile: stablecoins may be worth zero even when issuance and repurchase are fully programmable. Having shown this result, we now consider equilibria with positive stablecoin value if any. Under full commitment and with unlimited liability, there exists an equilibrium in which the stablecoin has value and the platform enjoys seigniorage revenues.

Proposition 2. *With full commitment and unlimited liability, the equilibrium with positive*

stablecoin price features a target demand ratio $A_t/C_t = A_t/C_{ul}^*(A_t) = a_{ul}^*$ for all t with

$$C_{ul}^*(A) = \arg \max_C \{ \ell(A, C)C \}. \quad (9)$$

The interest rate policy at demand ratio a_{ul}^* is $\delta_{ul}^* = r - \ell(a_{ul}^*)$ to peg the stablecoin price to 1 and is not determined otherwise. The platform sets collateralization ratio $\varphi_{ul}^* = 0$.

As we show formally in the Appendix, the platform value is the present value of liquidity benefits enjoyed by investors net of the collateral holding costs,

$$E_0 = \mathbb{E} \left[\int_0^\infty e^{-rt} (\ell_t C_t + (\mu^k - r) \varphi C_t) dt \middle| A_0, C_{0-} = 0 \right], \quad (10)$$

with $\ell(A, C)C$ the instantaneous total seigniorage revenues. This equivalence is intuitive because the platform captures all gains from trade. Maximizing the platform value E_0 with unlimited liability thus becomes a static optimization problem to the extent the platform can maintain the peg. In this case, the optimal collateralization rate is $\varphi_{ul}^* = 0$ because holding collateral is costly. Given current demand A , an interior optimum $C_{ul}^*(A)$ for stablecoin supply exists under Assumption 1. Homogeneity of the liquidity benefit, $\ell(A, C)$ further implies that $C_{ul}^*(A)$ is linear in A , and we call a_{ul}^* the *target demand ratio*.

The need to maintain the peg, $p_t = 1$ in order to capture liquidity benefits determines the platform's coupon policy. In equilibrium, the demand ratio a_t is constant, so we only need to specify $\delta_{ul}^* \equiv \delta(a_{ul}^*)$. It is easy to verify that the peg holds when δ_{ul}^* is given as in Proposition 2 because, for all t , we then have

$$p_t = \frac{\ell(a_{ul}^*) + \delta_{ul}^*}{r} = 1. \quad (11)$$

Proposition 2 implies that the platform issues (buys back) stablecoins when demand A_t increases (decreases) in order to implement its target demand ratio. This policy reflects supply adjustments practice by algorithmic stablecoin platforms. In our benchmark, the platform is always able to perform these adjustments and, as a result, always maintains the peg. However, as we show next, the mere introduction of limited liability jeopardizes the platform's ability to always maintain the peg even under full commitment.

3.2 Limited Liability

The full commitment policy with unlimited liability requires the platform to conduct large stablecoin repurchases when the underlying cryptoasset value drops in order to restore an optimal demand ratio. For a large drop, however, the repurchase cost might exceed the post-repurchase platform value. In practice, the platform would then be unable to finance the entirety of repurchase necessary to maintain the peg by issuing new equity tokens even if it fully dilutes legacy equity holders.

From this point, we assume that policies must satisfy limited liability. That is, the platform's equity value must be positive at all times.¹⁴ In other words, no smart contract may impose actions such that the platform's continuation value is negative. The value of the platform under limited liability constraint at date t , $E_t \geq 0$ can be derived at each point in time through the same steps as Proposition 2:

$$E_t = \mathbb{E} \left[\int_t^\tau e^{-r(s-t)} \left(\ell(A_s, C_s) C_s + (\mu^k - r) \varphi C_s \right) ds \middle| A_t, C_{t-} = 0 \right] - (p_t - \varphi) C_{t-} \geq 0. \quad (12)$$

The first term in (12) is the total platform value from date t onward as in (10) at date 0. The second term, $(p_t - \varphi) C_{t-}$, captures the net value of outstanding debt. Note that this term is zero at date 0 as $C_{0-} = 0$, so that it does not appear in (10). The equity value of the platform at time t is thus equal to the value of a new platform starting with zero stablecoins net of the cost of repurchasing all outstanding stablecoins. The effective repurchase cost per unit is given by $p_t - \varphi$ because buying back one stablecoin frees up collateral value φ . Equation (12) therefore shows that collateral can help relax the limited liability constraint, an intuition we formalize below.¹⁵

First, we argue that for a large enough negative demand shock, the optimal policy with unlimited liability in Proposition 2 violates constraint (12). After a negative demand shock,

¹⁴The limited liability assumption has a similar interpretation as for limited liability companies: a claimant to the company (platform) cannot expect to recover more than the value of assets belonging explicitly to the company (platform), thereby protecting the private wealth of the shareholders. For regular companies, this feature derives from a limited liability contractual arrangement, whereas for stablecoin platforms, it is the consequence of the platform policy and asset accessibility. Only escrowed collateral can effectively be accessed at liquidation.

¹⁵The equity value decomposition in (12) does not imply that the platform must repurchase all stablecoins before issuing new ones. It simply breaks down any policy into two steps which happen simultaneously at the same price: (i) repurchase all outstanding stablecoins C_{t-} , and (ii) issue new stablecoins to the new level, C_t .

the platform should repurchase a large stock of stablecoins to implement target a^* . If the shock is large enough, however, this cost can exceed the present value of future convenience yields. In this case, the platform's equity value would then become negative if the platform were to implement the policy in Proposition 2. At that point, the platform is unable to finance the stablecoin buybacks necessary to maintain the peg.

To analyze the platform's problem under full commitment and limited liability—problem (8) with additional constraint (12)—we focus on a set of policies defined below. To characterize these policies, it is useful to define the demand ratio $a_t \equiv A_t/C_{t-}$.

Definition 2. A target Markov policy (TMP) is given by demand ratio thresholds $\{\underline{a}, \bar{a}, a^*\}$ with $\underline{a} \leq \bar{a} \leq a^*$, an interest rate policy $\delta_t = \delta(A_t, C_{t-}) = \delta(a_t)$ and an issuance policy

$$d\mathcal{G}_t = \begin{cases} G(A_t, C_{t-})dt & \text{if } \underline{a} \leq a_t < \bar{a}, \\ \frac{A_t}{a^*} - C_{t-} & \text{if } a_t \geq \bar{a}, \end{cases} \quad (13)$$

where the issuance policy over $[\underline{a}, \bar{a}]$ is said to be smooth (of order dt). The platform enters liquidation when the demand ratio is below \underline{a} .

We call policies considered in Definition 2 Markov because they depend on the history of shocks and actions only via state variables A_t and C_{t-} . This memoryless property considerably simplifies our analysis in the presence of constraint (12).¹⁶ The optimal policy in the unlimited liability benchmark is a TMP with $\underline{a} = \bar{a} = 0$. Definition 2 generalizes this policy in two ways. First, a TMP may feature a *smooth region* $[\underline{a}, \bar{a}]$ in which the platform abandons the target and switches to an issuance policy of order dt , that is, it makes smooth adjustments to the stablecoin stock.¹⁷ Second, there may also be a liquidation region below demand ratio threshold \underline{a} . As discussed above, relaxing the strict commitment to the target demand ratio may prove necessary to satisfy limited liability constraint (12). Target Markov policies may come with some loss of generality under full commitment and limited liability. When we relax commitment to stablecoin issuance in Section 4, however, we show that the optimal policy belongs to this class.

¹⁶The general problem is not standard because limited-liability constraints (12) are forward-looking, which means equity value E_t is not the solution to a standard Hamilton-Jacobi-Bellman (HJB) equation. Techniques developed by Marcet and Marimon (2019) do not apply to our problem; the additional complexity comes from the term $(p_t - \varphi)C_{t-}$ on the right-hand side of (12) as a state variable C_{t-} multiplies forward-looking variable p_t , which depends on all future policy choices. Our focus on Markov policies ensure the equity value and the stablecoin price solve HJB equations.

¹⁷See the definition provided by DeMarzo and He (2021, p. 1205).

The platform's equity value and the stablecoin price inherit the Markov property of the platform's policies, denoted now by $E(A, C)$ and $p(A, C)$, thereafter omitting the time index. Due to the homogeneity of the problem, the ultimate state variable for our problem is the demand ratio $a = A/C$ so we also define $e(a) \equiv E(A, C)/C$ and $p(a) \equiv p(A, C)$ where $e(a)$ is the platform's equity value per stablecoin outstanding. Using this notation, the platform's objective (8) can be rewritten as

$$E_0 = E(A, C^*(A_0)) + (p(a^*) - \varphi)C^*(A_0) = A_0 \frac{e(a^*) + p(a^*) - \varphi}{a^*}, \quad (14)$$

with $C^*(A) = A/a^*$. The platform's objective is comprised of the sum of date-0 issuance gains, $(p(a^*) - \varphi)C^*(A_0)$, and the post-issuance equity value.

To solve for the optimal policy, equation (14) shows that we need to characterize the equilibrium equity value and the price at the target ratio a^* . In our model, this ultimately requires solving for these functions over the whole state space. To do so, we guess and verify that the equilibrium price satisfies $p(a) = 1$ for $a \geq \bar{a}$ and $p(a) < 1$ for $a \in [\underline{a}, \bar{a}]$, which implies investors enjoy liquidity benefits only in the target region. We first show that the platform designs the policy in the smooth region $[\underline{a}, \bar{a}]$ so that the limited liability constraint binds when the peg is lost.

Lemma 1. *In the smooth region $[\underline{a}, \bar{a}]$, an optimal TMP under full commitment and limited liability satisfies $\delta(a) = 0$ and*

$$g(a) \equiv \frac{G(a, C_-)}{C_-} = -\frac{\mu^k \varphi}{p(a) - \varphi}, \quad (15)$$

that is, the platform does not pay coupon when the peg is lost and it uses all collateral proceeds to repurchase stablecoins. Under repurchase policy (15), $e(a) = 0$ for all $a \in [0, \bar{a}]$ and the price solves the following equation when $a \in [\underline{a}, \bar{a}]$

$$(r + \lambda)p(a) = (\mu - g(a))ap'(a) + \frac{\sigma^2}{2}a^2p''(a) + \lambda\mathbb{E}[p(Sa)], \quad (16)$$

subject to the two boundary conditions $p(\underline{a}) = \varphi$ and $p(\bar{a}) = 1$.

The intuition for this result is as follows. As shown by (10), the platform has value to the extent it captures investors' liquidity benefits. As a result, the platform seeks to minimize the time it spends in the smooth region $[\underline{a}, \bar{a}]$ where the peg is lost ($p(a) < 1$) and

investors enjoy no such benefit. To do so and increase the growth rate of $a_t = A_t/C_t$ when $a_t \in [\underline{a}, \bar{a}]$, stablecoin issuance is minimized in this region. This policy involves paying no interest to investors and using all the returns on collateral to buyback stablecoins. This condition yields equation (15) because each stablecoin is backed by collateral value φ that grows at rate μ^k . The net repurchase cost of a stablecoin is $p - \varphi$ because buying back a stablecoin frees up collateral value φ . Hence, equation (15) defines the maximum rate at which the platform can repurchase stablecoins while satisfying limited liability constraint $e(a) \geq 0$.

The second part of the Lemma confirms that the platform's equity value is zero when the peg is lost. Intuitively, the platform would otherwise still have slack in the limited liability constraint to buy back stablecoins and defend the peg. Finally, Lemma 1 characterizes the stablecoin price dynamics in the smooth region $[\underline{a}, \bar{a}]$ when the peg is lost. Its evolution is governed by HJB equation (16). Optimal repurchase policy (15) enters this equation because it governs the rate at which the demand ratio a_t increases in region $[0, \bar{a}]$.

Thanks to Lemma 1, we can characterize the equilibrium equity value under a TMP up to the level of the interest rate $\delta(a^*)$ paid at the target ratio.

Lemma 2. *Under a TMP that satisfies Lemma 1, the platform's equity value under full commitment and limited liability is characterized by*

$$e(a) = \begin{cases} 0 & \text{if } a \leq \bar{a}, \\ [e(a^*) + (p(a^*) - \varphi)] \frac{a}{a^*} - (p(a^*) - \varphi) & \text{if } a \geq \bar{a}, \end{cases} \quad (17)$$

$$(r + \lambda - \mu)e(a^*) = \mu^k \varphi + \mu(p(a^*) - \varphi) - \delta(a^*)p(a^*) + \lambda \mathbb{E}[e(Sa^*)], \quad (18)$$

where $\delta(a^*)$ is the interest rate maintaining the peg at parity $p(a^*) = 1$, defined by

$$\delta(a^*) = r - \ell(a^*) + \lambda(1 - \mathbb{E}[p(Sa^*)]). \quad (19)$$

The characterization of the equity value when $a \leq \bar{a}$ follows directly from Lemma 1 and the fact that $e(a) = 0$ when the platform is in default ($a \leq \underline{a}$). Consider now the target region $[\bar{a}, \infty)$, in which the platform issues or repurchases stablecoins to maintain a

constant demand ratio a^* . By definition, the equity value is then given by

$$E(A, C) = E(A, C^*(A)) + (p(a^*) - \varphi)(C^*(A) - C). \quad (20)$$

That is, the equity value away from the target demand ratio is equal to the equity value at the target plus the net issuance proceeds (repurchase costs) when issuing (buying back) stablecoins to reach the target. Dividing both sides of (20) by the amount of stablecoins C , we obtain (17). Equation (18) is the HJB equation for equity value at the target ratio a^* . When at the target, equity holders receive interest on collateral $\mu^k \varphi$, issue new stablecoins at (expected) rate μ , gain $p(a^*) - \varphi$, and pay interest $\delta(a^*)$. The last term on the right-hand-side of (18) corresponds to a large (Poisson) negative shock to demand A .

The equity value at the target depends on the interest rate $\delta(a^*)$ the platform must pay to maintain the peg given by (19). In the absence of Poisson shocks ($\lambda = 0$), equation (19) is the same as in Proposition 2 with unlimited liability. Under limited liability, however, large negative demand shocks may force the platform to abandon the peg, in which case the stablecoin price falls below 1. This effect requires the platform to pay a larger interest at the target ratio in order to compensate for this expected devaluation, an effect captured by the last term of equation (19).

Our last preliminary result is that default is not optimal under full commitment.

Lemma 3. *Under full commitment and limited liability, the platform never defaults: $\underline{a} = 0$.*

The platform never defaults because doing so cannot increase the platform's value at date 0. The result is most intuitive for an uncollateralized platform ($\varphi = 0$). If the platform defaults below some threshold $\underline{a} > 0$, the stablecoin price then falls permanently to 0. As the demand ratio can fall below \underline{a} following a large enough negative shock, default below \underline{a} reduces investors' willingness to pay for the stablecoin at date 0. To maintain the peg, the platform would need to pay a large interest $\delta(a^*)$ at the target as suggested by (19). This feature in turn depresses the platform's equity value as can be seen from (18). If instead the platform keeps on operating when $a \leq \underline{a}$, it can wait for positive shocks to recover and regain the peg. This lowers the required interest rate $\delta(a^*)$ relative to the case in which the platform defaults below \underline{a} , which increases equity value at the target a^* .

The argument for Lemma 3 is more involved when the platform holds collateral ($\varphi > 0$) as default allows the platform to transfer collateral to users. Thanks to Lemma 1, however, we can show that the equilibrium price without default satisfies $p(a) \geq \varphi$ for all $a > 0$,

which implies that such transfer does not increase the platform's ex-ante value. Indeed, if $p(a) \leq \varphi$, it would be optimal for the platform to repurchase a stock of stablecoins at no additional cost than selling the corresponding collateral. Thus, there is a reflecting boundary at \tilde{a} such that $p(\tilde{a}) = \varphi$ (i.e., $g(\tilde{a}) = -\infty$). Overall, while the platform may lose the peg, it never defaults under full commitment.

3.3 Optimal Platform Design

We now use our preliminary results to characterize the optimal policy as the solution to an optimization problem over the target ratio a^* and the lower bound \bar{a} . The last necessary step to rewrite objective function (14) as a function of these parameters only is to derive the interest rate at the target ratio $\delta(a^*)$ as a function of (\bar{a}, a^*) . This requires solving for the equilibrium price for $a \in (0, \bar{a})$. Unfortunately, we cannot provide a general analytical solution for the general case because of the feedback loop in equation (16) for the price via the issuance rate $g(a)$ given by (15).

Nonetheless, two special cases of interest allow for an explicit characterization of the platform's optimal policy: the uncollateralized case ($\varphi = 0$) and the fully-collateralized case ($\varphi = 1$). We make use of these two extreme cases to characterize the effect of collateralization on platform stability and provide numerical results for the general case. We first present results assuming an equilibrium exists and then state conditions for existence.

Purely-algorithmic Platforms Consider first an uncollateralized platform with $\varphi = 0$. In this case, we obtain an analytic solution for the price thanks to equation (16) because Lemma 1 shows that $g(a) = 0$ for $a \in (0, \bar{a}]$ when $\varphi = 0$. We then obtain the following characterization of the optimal policy.

Proposition 3 (Purely-Algorithmic Platform Equilibrium). *If an equilibrium with positive stablecoin value exists for an uncollateralized platform ($\varphi = 0$), then:*

1. *The region $[0, \bar{a}]$ in which the peg is lost is non-empty and the equilibrium stablecoin price is given by $p(a) = \left(\frac{a}{\bar{a}}\right)^{-\gamma}$, for $a \leq \bar{a}$ where $\gamma < -1$ is the unique negative root of*

$$r + \lambda = -\mu\gamma + \frac{\sigma^2}{2}(1 + \gamma)\gamma + \frac{\lambda\xi}{\xi - \gamma}. \quad (21)$$

2. The optimal policy is characterized by Lemma 1, 2, and 3, and (\bar{a}, a^*) that solve

$$\frac{e(a^*) + p(a^*)}{a^*} = \max_{\bar{a}, a^*} \left\{ \frac{\ell(a^*)/a^*}{r + \frac{\lambda}{\xi+1} - \mu + \left(\frac{\lambda\xi}{\xi+1} - \frac{\lambda\xi}{\xi-\gamma} \right) \left(\frac{a^*}{\bar{a}} \right)^{-(\xi+1)}} \right\} \quad (22)$$

$$\text{subject to} \quad e(\bar{a}) = [e(a^*) + p(a^*)] \frac{\bar{a}}{a^*} - 1 = 0. \quad (23)$$

An uncollateralized platform loses the peg after a large enough negative demand shock, even if it can commit to an issuance policy. When demand drops, the platform would like to repurchase a sufficient amount of stablecoins to maintain the peg. For a large enough drop, however, the net present value of liquidity benefits is so low that the value of equity is zero. In that case, repurchases cannot be financed through equity dilution and the peg is lost. The intuition behind this result is similar to [Del Negro and Sims \(2015\)](#) and [Reis \(2015\)](#) showing that an insolvent central bank without fiscal support is unable to control inflation as the only way it can keep paying interest on reserves is to violate a no-Ponzi game condition.

This dynamics can be observed in the crash of the two algorithmic stablecoins Terra and NuBits for which the market capitalization of their governance tokens fell to zero at the time of losing the peg (see [Appendix A](#)). In the region $[0, \bar{a}]$ in which the peg is lost, the stablecoin price remains strictly positive although investors enjoy no liquidity benefit since $p(a) < 1$. The stablecoin value is then driven entirely by the probability that the demand ratio a_t exogenously reaches the peg threshold \bar{a} due to a series of positive demand shocks. The speed of this process depends on the value of the root γ .

The second part of [Proposition 3](#) characterizes the optimal policy choice of an uncollateralized platform under limited liability. Given $e(a) = 0$ for all $a \leq \bar{a}$ and $e(a)$ is linear and increasing for $a \in [\bar{a}, \infty)$, limited liability holds for all a if $e(\bar{a}) = 0$, which is constraint (23). As in the case with unlimited liability, the platform maximizes the present value of liquidity benefits. With limited liability, however, the platform's effective discount rate increases and depends on its policy choices, as shown by (22), because the platform may lose the peg. Given threshold $\bar{a} > 0$, a lower value of a^* increases the probability of losing the peg, which raises the platform's discount rate. Setting $\bar{a} = 0$ to kill this effect is not possible because this would violate limited liability constraint (23), that is, $e(\bar{a}) > 0$.

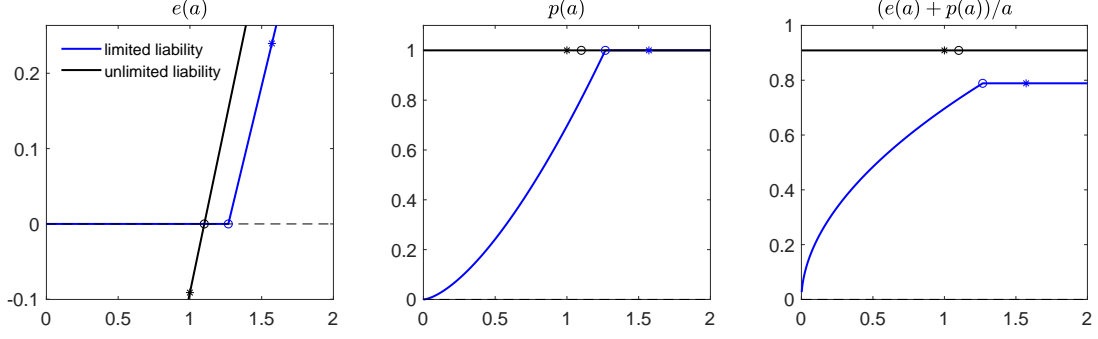


Figure 2: Full-commitment Solution with limited liability (blue) and unlimited liability (black) without collateral ($\varphi = 0$). The set of parameters is given by $r = 0.06$, $\mu = 0.05$, $\sigma = 0.1$, $\ell(A, C) = r \exp(-C/A)$, $\xi = 6$, $\lambda = 0.10$. Star symbols “*” represent the target demand ratio a^* while dot symbols “o” indicate \bar{a} , the point at which $e(a)$ reaches zero.

This discount rate effect implies that the optimal target demand ratio a^* is higher than its counterpart with unlimited liability, a_{ul}^* : reducing stablecoin issuance from $C_{ul}^*(A)$ to $C^*(A) < C_{ul}^*(A)$ protects the platform against large negative demand shocks.

We provide a numerical illustration in Figure 3 contrasting the solutions under limited and unlimited liability. The left panel shows that limited liability protects equity holders, as their equity value is always positive after large negative shocks. From an ex-ante perspective however, the inability to conduct large repurchases lowers the total platform value as can be observed in the rightmost panel. The center panel illustrates that the stablecoin price loses the peg at \bar{a} under the limited liability case.

Fully-collateralized Platforms We now turn to the analysis of fully-collateralized stablecoin platforms. The feature that simplifies the analysis in this case is that limited liability constraint (12) never binds because stablecoin repurchases are financed entirely from collateral holdings.

Proposition 4 (Fully-Collateralized Platform Equilibrium). *If an equilibrium with positive stablecoin value exists for a fully collateralized platform, the following results apply*

1. *The peg is always maintained, that is, $\bar{a} = 0$.*

2. The optimal policy is given by Lemma 1, 2, and 3, $\bar{a} = 0$ and $a^* > a_{ul}^*$ that solves

$$\max_{a^*} \frac{e(a^*)}{a^*} = \frac{\ell(a^*) + \mu^k - r}{r - \mu + \lambda/(\xi + 1)} \frac{1}{a^*} \quad (24)$$

The key difference between a fully-collateralized platform (Proposition 4) and an uncollateralized one (Proposition 3) is that the peg is never lost in the former, that is, $\bar{a} = 0$. This result follows directly from the observation that limited liability constraint (12) may never bind when $p(a^*) = \varphi = 1$ because the net value of outstanding stablecoins is zero. With a fully-collateralized platform any stablecoin repurchase is fully financed by collateral holdings, which means no equity dilution is ever required to pay for a repurchase. Hence, the limited liability constraint does not affect the ability of the platform to perform these operations and maintain the peg under full collateralization.¹⁸

This observation implies that the optimization problem (24) is identical to that of a fully-collateralized platform under unlimited liability. An important difference, however, is that with unlimited liability, the platform would choose not to hold collateral because it is costly ($\mu^k < r$) and large stablecoin repurchases would be financed by letting equity become negative. With limited liability, the platform must hold collateral to maintain the peg at all times. The comparison between target ratios a^* and a_{ul}^* reflects this additional collateral cost. Under limited liability, the platform issues less stablecoins because the net liquidity benefit per stablecoin accounting for the collateral cost is lower than under unlimited liability as $\ell + \mu^k - r \leq \ell$ given our assumption that $\mu^k \leq r$.

3.4 Existence Conditions

Proposition 3 and 4 characterize a platform's optimal policy given that an equilibrium with positive stablecoin value exists. Now we provide existence conditions for both cases.

Proposition 5 (Existence Condition). *Given collateralization ratio $\varphi \in \{0, 1\}$ a sta-*

¹⁸The result holds in part because the collateral asset is uncorrelated with the demand process A_t . If it were (positively) correlated, the value of collateral holdings would decrease when demand drops. In this case, collateral available to finance a repurchase may fall short of the repurchase cost. Unless collateral is fully correlated with the demand process, however, the qualitative effect that collateral relaxes limited liability constraint (12) remains.

blecoin platform with positive value exists under full commitment only if

$$\max_a \ell(a) \geq \begin{cases} r - \mu + \frac{\lambda}{\xi+1} & \text{if } \varphi = 0, \\ r - \mu^k & \text{if } \varphi = 1. \end{cases} \quad (25)$$

We derive the existence conditions in Proposition 5 from imposing $e(a^*) \geq 0$. This condition implied by $e(\bar{a}) \geq 0$ and $a^* \geq \bar{a}$ is necessary and sufficient for an equilibrium with positive stablecoin value to exist under limited liability. We report this condition directly in the fully-collateralized case ($\varphi = 1$) while we only provide a necessary condition in the uncollateralized case ($\varphi = 0$) for simplicity. The proof contains a sufficient condition.

In the fully-collateralized case ($\varphi = 1$), the existence condition states that the liquidity benefit captured by the platform $\ell(a^*)$ must exceed the collateral holding cost $\mu^k - r$. As discussed above, the collateral holding cost can be interpreted as a liquidity benefit from holding the underlying asset, which the platform forgoes when using the asset as collateral. Condition (25) states that issuing stablecoins that are fully backed by another asset can only be profitable if the former commands larger liquidity benefits.

To form intuition for the condition in the uncollateralized case ($\varphi = 0$), set arbitrarily the lower bound of the target region $\bar{a} = 0$. We can then use expression (22) to approximate the platform's value at date 0 as follows:

$$[e(a^*) + p(a^*)] \frac{A_0}{a^*} \approx \frac{\ell(a^*)}{r + \frac{\lambda}{\xi+1} - \mu} \frac{A_0}{a^*}, \quad (26)$$

Given that $p(a^*) = 1$, condition (25) is then necessary for equity value $e(a^*)$ to be positive. If this condition did not hold, equity owners would need to buy back stablecoins on average to maintain the peg following the initial issuance. Condition (26) shows that the platform's equity is positive only if the growth rate of stablecoin demand, $\mu - \frac{\lambda}{\xi+1}$ and the liquidity benefits are high enough. In this case, equity owners expect a positive profit after the initial issuance. In other words, an uncollateralized platform exists only if demand for the stablecoin keeps growing over time.

Corollary 1. *An uncollateralized platform value can exist only if stablecoin demand grows, that is, if $\mu - \frac{\lambda}{\xi+1} \geq 0$.*

Corollary 1 follows from Proposition 5 and Assumption 1 stating that the marginal

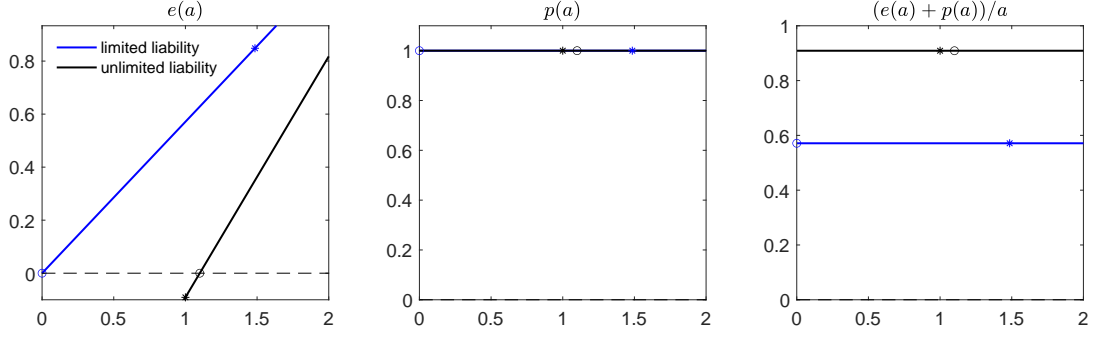


Figure 3: Full-Commitment Solution with limited liability (blue) and unlimited liability (black) fully collateralized ($\varphi = 1$). The set of parameters is given by $r = 0.06$, $\mu = 0.05$, $\mu^k = 0.055$, $\sigma = 0.1$, $\ell(A, C) = r \exp(-C/A)$, $\xi = 6$, $\lambda = 0.10$. Star symbols “*” represent the target demand ratio a^* while dot symbols “o” indicate \bar{a} , the point at which $e(a)$ reaches zero.

liquidity benefit is no larger than the discount rate ($\ell < r$). Stablecoin demand must grow over time for an uncollateralized stablecoin platform to have any value. Without this growth component, platform owners would prefer to default immediately after the initial stablecoin issuance because all gains from trade would have been realized already. In that sense, this result relates to existing argument on algorithmic stablecoins, which are portrayed as “Ponzi Schemes.” If the growth rate of the demand for the stablecoin were to unexpectedly and permanently fall to zero, the equity value of the platform would also fall to zero and it would loose its peg permanently.

3.5 Under-Collateralized Platforms: Numerical Solution

[Make reference to FRAX]

4 Non-Programmable Issuance

In this section, we analyze the centralized platform’s problem under a weaker form of commitment. We assume that stablecoin issuance and repurchase cannot be fully programmed via smart contracts at date 0. We maintain commitment with respect to the coupon policy $\{\delta_t\}_{t \geq 0}$ and the minimum collateralization rule φ chosen at date 0.¹⁹ The study of

¹⁹Limited liability still applies. If equity token holders find the interest policy or collateralization requirements too costly ex-post, they can liquidate the platform.

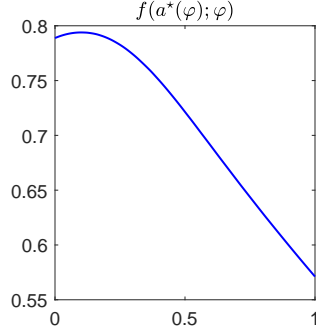


Figure 4: Total platform value $f(a^*; \varphi) = (e(a^*(\varphi); \varphi) + p(a^*(\varphi); \varphi) - \varphi) / a^*(\varphi)$ as a function of φ . The set of parameters is given by $r = 0.06$, $\mu = 0.05$, $\mu^k = 0.055$, $\sigma = 0.1$, $\ell(A, C) = r \exp(-C/A)$, $\xi = 6$, $\lambda = 0.10$.

this weak form of commitment is motivated by the many stablecoin protocols that retain discretion over repurchase and issuance of stablecoins.²⁰

In line with our previous analysis, we assume that the coupon policy δ is Markov in that it only depends on the demand ratio $a_t = A_t/C_t$. In what follows, we first refine our equilibrium concept under partial commitment and highlight the new implementation constraints that arise in this case. Finally, we characterize the optimal design a centralized platform that cannot commit to its issuance policy.

4.1 Equilibrium Concept under Partial Commitment

We refine our equilibrium concept under limited commitment by introducing the concept of a Markov Perfect Equilibrium (MPE), defined with respect to the state variables of our economy (A_t, C_t) . In a MPE, the platform's issuance policy and the stablecoin pricing function depend only on (A_t, C_t) , as opposed to the entire history of shocks.

Definition 3. *Given a coupon policy $\delta(A, C)$ homogeneous of degree 0 and a collateralization ratio $\varphi \in [0, 1]$, a Markov Perfect Equilibrium (MPE) is given by equity token value function $E(A, C)$, a stablecoin pricing function $p(A, C)$, an issuance policy $d\mathcal{G}(A, C)$, and an optimal default policy τ such that the issuance policy $d\mathcal{G}$ and default policy τ maximize*

²⁰As described below, some degree of commitment is necessary for an equilibrium with positive stablecoin value to exist. See Appendix D for a proof of this claim.

the platform's equity value in any state (A, C) . That is,

$$E(A, C) = \max_{\tau, d\mathcal{G}} \mathbb{E} \left[\int_t^\tau e^{-r(s-t)} (p_s d\mathcal{G}_s - \varphi d\mathcal{C}_s) \middle| A_t = A, C_t = C \right], \quad (27)$$

given the law of motion for stablecoins (4) and stablecoin pricing function

$$p(A, C) = \mathbb{E} \left[\int_t^\tau e^{-r(s-t)} (\ell_s + \delta_s) p_s ds + e^{-r(\tau-t)} \varphi \middle| A_t = A, C_t = C \right]. \quad (28)$$

Optimality criterion (27) states that the issuance policy must be sequentially optimal under limited commitment. In a Markov equilibrium, this means the policy must be optimal in any state (A, C) . In writing equation (27) for the equity value above, we simplified equation (6) by substituting for the collateral repurchase policy $d\mathcal{M}_t = \varphi d\mathcal{C}_t$ implied by collateralization rule (3) and letting the platform's value in default be 0 because the collateral value is always weakly inferior to the par value of stablecoins $K_t = \varphi C_t \leq C_t$.

Debt pricing equation (28) embeds the restriction imposed by the Markov property. The stablecoin price may depend only on current state variables (A, C) , not on the history of past issuance decisions. This restriction implies investors may not “punish” the platform for deviating. Suppose the platform announces policy $\{d\mathcal{G}_t\}_{t \geq 0}$ but deviates to $d\hat{\mathcal{G}}_\tau \neq d\mathcal{G}_\tau$ at some date $\tau > 0$. In a Markov equilibrium, the new price faced by the platform may change only because state variables (A, C) change after $d\hat{\mathcal{G}}_\tau$, not because $d\hat{\mathcal{G}}_\tau$ is a deviation. If instead we let the stablecoin price depend on the entire history of actions, investors could explicitly punish the platform following a deviation using so-called grim-trigger strategies. For instance, investors could refuse to buy stablecoins, thereby setting the price to 0. We use Markov Perfection to discipline out-of-equilibrium behavior but also for realism because disperse investors would find difficult to coordinate and implement grim-trigger strategies.²¹

The first step of the analysis is to characterize an equilibrium stablecoin issuance policy $d\mathcal{G}$ and default policy τ under partial commitment. Remember that under full commitment, we posited that $d\mathcal{G}$ belongs to the class of targeted Markov policies. Under limited commitment, however, sequential optimality puts stronger requirements on the equilibrium policy, which allow us to prove that it must belong to this class.

²¹See Malenko and Tsoy (2020) who consider such explicit punishments in a related dynamic leverage choice problem for firms.

Proposition 6 (Equilibrium Policy). *For an optimal coupon policy δ chosen at date 0, if an equilibrium exists, the equilibrium issuance policy $d\mathcal{G}$ under limited commitment belongs to the class of TMP introduced in Definition 2.*

The proof of Proposition 6 has several technical steps that we briefly outline below. We first establish that the equilibrium equity function is weakly convex and the stablecoin price is weakly increasing as a function of the demand ratio a . Following arguments from DeMarzo and He (2021), we then show that the equilibrium issuance policy is smooth (features jumps) on intervals for which the equity value is strictly convex (linear). The existence of a default threshold \underline{a} follows from the fact that the equity value is increasing in a . Next, we show that if the coupon policy $\delta(a)$ is chosen optimally at date 0, the issuance policy is smooth over the first part of the no-default region, $[\underline{a}, \bar{a}]$ for some $\bar{a} \geq \underline{a}$. For values of $a \in [\bar{a}, \infty)$, it features a jump to some target demand ratio a^* . These results imply that the equilibrium policy belongs to the class of TMP.

4.2 Commitment Constraints

The platform chooses the policy parameters of the TMP anticipating the equilibrium induced by its choice. Relative to the full commitment case, the platform faces additional constraints because the issuance and default decisions must now be sequentially optimal. The following proposition characterizes these constraints generated by lack of commitment.

Proposition 7. *Under limited commitment, a feasible TMP and the equity value function of the induced non-zero MPE must satisfy the following properties:*

1. *The platform's issuance rate in the smooth region $[\underline{a}, \bar{a}]$ is given by*

$$g(a) = \frac{a\delta'(a)p(a) + (\mu^k - r)\varphi}{ap'(a)} \quad (29)$$

and the platform's equity value is the same as if it issued no debt.

2. *The value of equity tokens must satisfy the following smooth-pasting conditions at the lower bound of the target region \bar{a} and at the default threshold \underline{a} if $\underline{a} > 0$, respectively:*

$$e'(\bar{a}) = \frac{e^* + 1 - \varphi}{a^*}, \quad (30)$$

$$e'(\underline{a}) = 0. \quad (31)$$

3. In the target region, for all $a \geq \bar{a}$, the interest rate must satisfy

$$\delta(a) - \delta(a^*) \frac{a}{a^*} \geq \left[(r + \lambda)(1 - \varphi) + \mu^k \varphi \right] \left(1 - \frac{a}{a^*} \right) + \lambda \left[\mathbb{E}[e(Sa)] - \mathbb{E}[e(Sa^*)] \frac{a}{a^*} \right]. \quad (32)$$

Condition 1 of Proposition 7 characterizes the optimal issuance policy in the region $[\underline{a}, \bar{a}]$ with smooth stablecoin issuance. For smooth issuance to be optimal, it must be that the returns from issuance are equal to 0. We show in the proof that this condition writes

$$p(a) - \varphi = e'(a)a - e(a). \quad (33)$$

Equation (33) states that the net marginal benefit of issuing stablecoin, $p(a) - \varphi$, is equal to the marginal loss of equity value from such issuance. The corollary that returns to issuance are zero in the smooth region is similar to the leverage ratchet effect of DeMarzo and He (2021) for a firm issuing debt. In their work, the leverage ratchet effect implies that the firm can never capture the tax advantage of debt, which is akin to the liquidity benefit in our model. While this result holds in the smooth region $[\underline{a}, \bar{a}]$, our optimal policy also feature a target region $[\bar{a}, \infty)$. We explain below that such region can be supported in our model thanks to a state-contingent interest policy. Moreover, as issuance returns are zero, equity value can be solved as if the platform issued no debt. Equilibrium issuance is determined, however, by the optimality condition for smooth debt issuance, equation (33).

The equilibrium issuance policy in the smooth region has two components captured by the two terms at the numerator of the right-hand side of (29). First, the platform tends to repurchase stablecoin to reduce the amount of costly collateral it must hold because $\mu^k < r$ by assumption. Second, the platform issues (repurchases) stablecoins if $\delta'(a) > 0$ ($\delta'(a) < 0$). In particular, if the interest rate decreases with a , $\delta'(a) < 0$, the interest rate policy induces the platform to repurchase stablecoins so as to increase its demand ratio.

Condition 2 gathers standard smooth-pasting conditions the equity value must satisfy in equilibrium. Equation (30) ensures that the platform optimally switches from discrete repurchases in the target region to a smooth issuance policy at threshold \underline{a} . This condition requires that the derivative of the equity value is continuous at threshold \bar{a} . Equation (31) ensures that the liquidation threshold is optimally chosen by the platform.

Finally, Condition 3 ensures that implementing demand ratio a^* is ex-post optimal with-

out commitment when the platform is in the target region $a \in [\bar{a}, \infty)$. This condition is thereby crucial to support an equilibrium with a target region. We obtain this condition by considering a “one-step” deviation whereby, starting from some demand ratio $a \neq a^*$, the platform would stay idle during an interval of period dt and then revert back to the conjectured equilibrium policy.²² To understand constraint (32), rewrite it as:

$$\begin{aligned} & \delta(a)C - \delta(a^*)C^*(A) + \underbrace{\lambda(\mathbb{E}[E(SA, C^*(A))] - \mathbb{E}[E(SA, C)])}_{\text{protection against large shocks}} \\ & \geq \underbrace{(r + \lambda)(1 - \varphi)(C - C^*(A))}_{\text{gains from postponing repurchase}} + \underbrace{\mu^k \varphi(C - C^*(A))}_{\text{collateral dividends}}. \end{aligned} \quad (34)$$

To fix ideas, assume that $C \geq C^*(A)$ so that the conjectured equilibrium policy is to repurchase $C - C^*(A)$ stablecoins from current stock C . The terms on the left-hand side of equation (34) represent the net advantages of adhering to the equilibrium policy relative to deviating. The first term, $\delta(a)C - \delta(a^*)C^*(A)$, represents the net interest savings from increasing the demand ratio from a to a^* . The second term measures the relative protection against negative (Poisson) demand shocks at target demand ratio a^* relative to ratio a . On the right-hand-side, the two terms capture respectively the benefit from delaying the repurchase of stablecoins and the relative gains from larger collateral dividends when deviating.

Constraint (34) shows that promising a high interest rate in the target region for $a \neq a^*$ can discipline the platform. Intuitively, the platform has more incentives to implement the target ratio a^* if it must deliver a large interest rate to users when it deviates. The notion that off-equilibrium punishments can sustain the implementation of the peg is intuitive. We want to stress that in our model, the cost of this punishment is endogenous. The intuition is clearest for an uncollateralized platform that faces no direct cost from issuance. Paying a high interest rate in stablecoins is still costly because the platform’s equity value is decreasing with the outstanding stock of stablecoins in equilibrium.²³

A direct corollary to Proposition 7 is that a non-zero MPE only exists if the platform’s design follows a state-contingent interest policy.

²²We show that this condition also rules out deviations such that the platform repurchases or issues debt smoothly during an interval dt .

²³Our analysis of the deviation leading to condition (34) in the proof of Proposition 7 shows the argument is similar for a collateralized platform. Although paying interest comes with a direct collateral cost, this cost is recouped by the platform when it reverts back to the equilibrium policy.

Corollary 2. *With a noncontingent interest policy, $\delta(a) = \delta^*$ for all a , there exists no non-zero MPE under limited commitment.*

This result states that the platform owners must be punished for deviating from a target Markov policy via a larger interest rate than at the target ratio a^* . Such a state-contingent interest policy is necessary to sustain an equilibrium under limited commitment even if the platform is fully collateralized. To see this, set $\varphi = 1$ and $\delta(a) = \delta(a^*)$ for all a so that condition (32) from Proposition 7 becomes

$$\forall a, \quad (\delta(a^*) - \mu^k) \left(1 - \frac{a}{a^*}\right) \geq 0 \quad \Rightarrow \quad \delta(a^*) = \mu^k, \quad (35)$$

where the term proportional to λ disappears because $E(A, C^*(A)) = E(A, C)$ for all C with a fully-collateralized platform (Lemma 2). Together with equation (19) to maintain the peg ($\delta(a^*) = r - \ell(a^*)$ when $\varphi = 1$), condition (35) implies that the platform's value is zero by Proposition 4 because the collateral cost $r - \mu^k$ then fully offsets the liquidity benefit $\ell(a^*)$ captured by the platform. Hence, a state-contingent interest policy is always necessary to sustain a non-zero MPE under limited commitment.

Corollary 2 shows that commitment issues remain even with full collateralization although the platform's net cost of repurchasing stablecoins is then zero. Condition (35) states that implementing target ratio a^* is optimal for the platform ex-post only if the interest it pays to users, $\delta(a^*)$ is equal to the interest earned on collateral, μ^k . If for instance $\delta(a^*) < \mu^k$, the platform would prefer to maintain a lower demand ratio, $a \leq a^*$ to earn the spread $\mu^k - \delta(a^*)$ on the extra supply of stablecoins relative to the target level $C^*(A)$.

4.3 Optimal Protocol under Limited Commitment

We now characterize the policy design problem of the platform under limited commitment. The objective of the platform is again to maximize its date-0 value, which is given by:

$$\frac{e(a^*) + 1 - \varphi}{a^*} = \frac{\ell(a^*) + (\mu^k - r)\varphi + \lambda \mathbb{E}[e(Sa^*) + p(Sa^*) - \varphi]}{r + \lambda - \mu} \frac{1}{a^*}, \quad (36)$$

To obtain (36), we used equations (18) and (19). The platform thus chooses the TMP that maximizes (36) subject to limited liability, $e(a) \geq 0$ as in the full commitment case. Without commitment to the issuance policy now, the platform also faces the implementation constraints we derived in Proposition 7. To express (36) and the constraints as a

function of policy parameters, we need to solve for the equilibrium value functions and the stablecoin price over the entire state space. In the target region $a \in [\underline{a}, \infty)$, we have $p(a) = 1$ by construction and $e(a)$ given by equation (17) as in the full commitment case. In the smooth region, we show in the proof that the dynamic equation for the equity value and the price is respectively

$$(r + \lambda)e(a) = (\mu^k - \delta(a))\varphi + \underline{\delta}(e(a) - ae'(a)) + \mu ae'(a) + \frac{\sigma^2}{2}a^2e''(a) + \lambda\mathbb{E}[e(Sa)], \quad (37)$$

$$(r + \lambda)p(a) = \delta(a)p(a) + (r - \mu^k)\varphi + (\mu - \delta(a))ap'(a) + \frac{\sigma^2}{2}a^2p''(a) + \lambda\mathbb{E}[p(Sa)]. \quad (38)$$

Analytical solutions for (37) and (38) cannot be obtained in the general case, which motivates us to impose some structure on the feasible set of TMPs.

Assumption 4. *The interest rate policy in the smooth region $[\underline{a}, \bar{a}]$ is given by $\delta(a) = \underline{\delta}$.*

Assumption 4 allows us to provide analytical solution for the MPE equity value and price in the smooth region. Our numerical analysis below suggests this assumption is innocuous because we find that the platform always chooses to set $\underline{\delta}$ as high as possible. This indicates that state-contingency in δ in the smooth region would not increase the platform value.

Thanks to Assumption 4, we thus guess and verify that the equity value and the stablecoin price have the following functional forms

$$e(a) = \begin{cases} 0 & \text{if } 0 \leq a < \underline{a}, \\ \underline{e} + \sum_{k=1}^3 c_k a^{-\gamma_k} & \text{if } \underline{a} \leq a < \bar{a}, \\ (e^* + 1 - \varphi)a/a^* - (1 - \varphi) & \text{if } a \geq \bar{a}, \end{cases} \quad (39)$$

$$p(a) = \begin{cases} \varphi & \text{if } 0 \leq a < \underline{a}, \\ \underline{p} + \sum_{k=1}^3 b_k a^{-\gamma_k} & \text{if } \underline{a} \leq a < \bar{a}, \\ 1 & \text{if } a \geq \bar{a}, \end{cases} \quad (40)$$

where γ_k s are roots of the characteristic equation

$$r + \lambda - \underline{\delta} = -(\mu - \underline{\delta})\gamma + \frac{\sigma^2}{2}(1 + \gamma)\gamma + \frac{\lambda\xi}{\xi - \gamma}, \quad (41)$$

and $\{\underline{p}, \underline{e}\}$ and $\{b_k, c_k\}_{k=1,2,3}$ are parameters.

As in the full commitment case, and even under Assumption 4, a general characterization

of the solution is difficult. We thus focus again on two extreme cases of interest: a purely-algorithmic protocol ($\varphi = 0$) and a fully-collateralized protocol ($\varphi = 1$).

Purely-algorithmic Protocol

We first consider a purely-algorithmic platform without collateral, that is, with $\varphi = 0$. In this case, the optimization problem of the platform can be characterized as follows.

Proposition 8. *Under limited commitment, a non-collateralized platform never defaults, that is, $\underline{a} = 0$. It chooses $\underline{\delta}, \bar{a}, a^*$ to maximize*

$$\frac{e(a^*) + 1}{a^*} = \frac{\ell(a^*)/a^*}{r + \frac{\lambda}{\xi+1} - \mu + \left(\frac{\lambda\xi}{\xi+1} - \frac{\lambda\xi}{\xi-\gamma} \right) \left(\frac{a^*}{\bar{a}} \right)^{-(\xi+1)}} \quad (42)$$

$$\text{subject to} \quad e(\bar{a}) \equiv [e(a^*) + 1] \frac{\bar{a}}{a^*} - 1 = -\frac{1}{1+\gamma} > 0. \quad (43)$$

where $\gamma < -1$ is the lowest negative root of (21)

The first result from Proposition 8 is that an uncollateralized platform never goes into liquidation. A collateralized platform may default ex-post because its interest payment to users (in stablecoins) must be backed by collateral purchases. Without collateral, however, the platform faces no such cost, which kills any value from default. Hence, similar to the commitment case, an uncollateralized platform never liquidates, $\underline{a} = 0$. This implies in particular that the platform's objective function (42) takes the same form as (22) with commitment. A key difference, however, is that under limited commitment, the platform faces constraint (43), which is stronger than its counterpart (23). Under commitment, (23) states that the platform's equity value should be 0 at the threshold \bar{a} to satisfy limited liability. Without commitment, (43) follows instead from smooth-pasting condition (30) at \bar{a} . Comparing (23) and (43) confirms the latter is more stringent because $\gamma > -1$.

We now characterize the optimal policy that solves the optimization problem presented in Proposition 8. The reduced-form variable γ depends only on the interest rate $\underline{\delta}$ and is decreasing. Hence choosing $\underline{\delta}$ is similar to choosing directly γ . This latter variable plays two opposite roles. First, as we show in the proof of Proposition 8, the total platform value when $a \leq \bar{a}$ increases with γ because this variable governs the speed at which the platform exits the smooth region. On the other hand, decreasing γ allows the platform to extend the target region $[\bar{a}, \infty)$ over which the price is pegged, as can be seen from constraint

(43). Overall, the second effect dominates because the platform's paramount objective is to maintain the peg to be able to capture liquidity benefits.

Proposition 9. *An optimal uncollateralized platform's policy under limited commitment features $\underline{\delta} = \infty$ and thus $\gamma = -\infty$ such that $e(a) = p(a) = 0$ for $a \in [0, \bar{a}]$. The optimal target ratio a^* is strictly higher than under full commitment.*

As mentioned above, the platform seeks to maximize the size of the target region over which the price is pegged. This requires setting $\underline{\delta} = \infty$ and thus $\gamma = \infty$, which decreases the lower bound \bar{a} of the target region. As noted below, the downside is that the platform value is zero in the smooth region $[0, \bar{a}]$. Maximizing the size of the target region is optimal, however, because stablecoin users only enjoy liquidity benefits in this region, which are ultimately captured by the platform. With $\underline{\delta} = \infty$, the stock of stablecoins jumps to $C = \infty$ whenever the platform enters the smooth region, which implies $e(a) = p(a) = 0$ for $a \in [0, \bar{a}]$. In other words, the platform designs the TMP such that it effectively shuts down once it loses the peg. Hence, there is no recovery once the peg is lost.

Proposition 9 also states that the target ratio is higher under limited commitment than under full commitment. A given negative shock to demand has worse consequences under limited commitment than under full commitment because it (optimally) triggers liquidation of the platform. Under full commitment instead, the platform can recover after a shock that forces the platform to abandon the peg. This effect implies that the platform is more conservative and issues less stablecoins for a given level of demand to accommodate larger negative shocks to demand. Hence, the platform choose a higher target ratio a^* under limited commitment than under full commitment.

[REMOVE?] Compare new existence condition with old one.

Fully-collateralized platform We now turn to the analysis of a fully-collateralized protocol. In this case, we show that the full-commitment outcome can be implemented thanks to a state-contingent interest policy.

Proposition 10. *Under limited commitment and with full collateralization, interest rate rule $\delta(a) = r - \ell(a)$ implements the full-commitment outcome.*

With full collateralization, the TMP has no smooth region. Hence, conditions 1 and 2 of Proposition 7 are moot. The commitment outcome can thus be sustained without commitment to the issuance policy if there exists an interest rate rule that satisfies condition

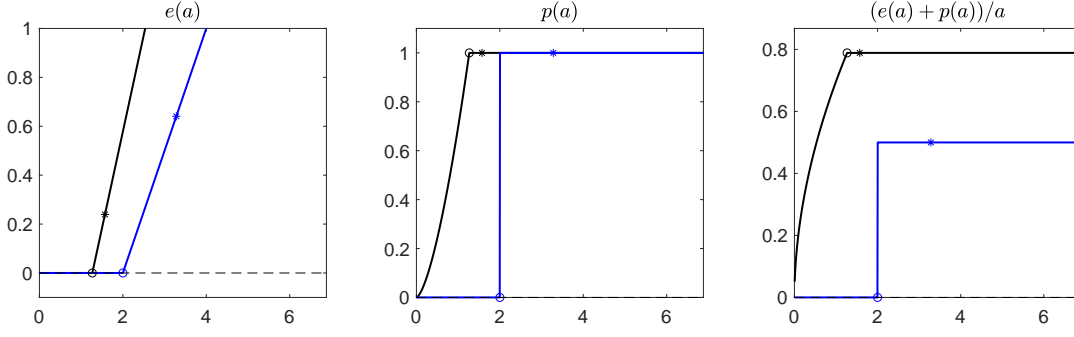


Figure 5: Solution with commitment and limited liability (black) and without commitment (blue). The set of parameters is given by $r = 0.06$, $\mu = 0.05$, $\sigma = 0.1$, $\ell(a) = r \exp(-C/A)$, $\xi = 6$, $\lambda = 0.10$.

(3). Plugging the rule of Proposition ?? in (35), we obtain the following condition:

$$\forall a, \quad \frac{\ell(a^*) + \mu^k - r}{a^*} \geq \frac{\ell(a) + \mu^k - r}{a}, \quad (44)$$

which holds by definition of a^* , the optimal demand ratio target chosen at date 0. The intuition for the result is that interest rate rule $\delta(a) = r - \ell(a)$ makes the platform internalize ex-post the cost of deviating from the ex-ante profit-maximizing demand ratio. Hence, lack of commitment to the issuance policy has

To conclude this section, we provide numerical illustrations for the optimal policy for an uncollateralized case. We also derive the loss in value from lack of commitment to the issuance policy for each collateralization level. In Figure ??, for each panel, the solution without commitment is compared to the solution where the only constraint is limited liability.

[TO BE CONTINUED]

5 Decentralized Protocols

In this section, we extend our baseline framework to accommodate *decentralized* stablecoin protocols. Such protocols—with Dai as its most prominent example—allow for the decentralized creation of new stablecoins by anyone with enough collateral. Individual investors who wish to issue stablecoins must lock some collateral asset in a smart contract generated by the protocol—a *vault*. Once the stablecoins are sold to outside investors, the vault rep-

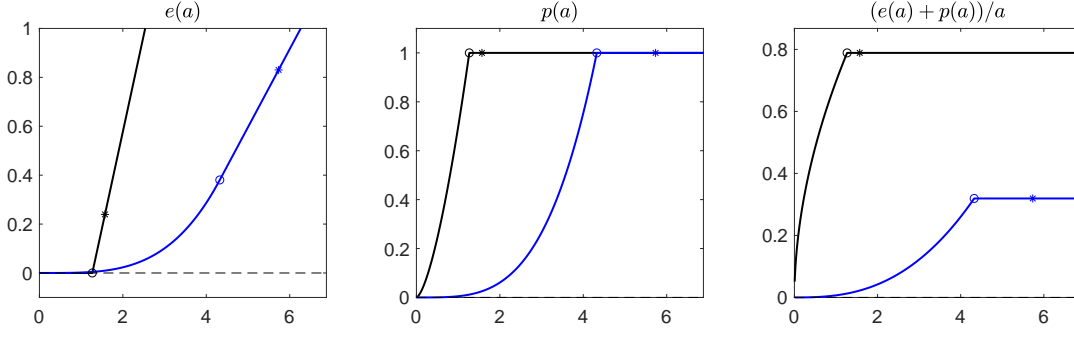


Figure 6: Solution with commitment and limited liability (black) and without commitment (blue) constrained such that $\delta(a) \leq \mu$. The set of parameters is given by $r = 0.06$, $\mu = 0.05$, $\sigma = 0.1$, $\ell(a) = r \exp(-C/A)$, $\xi = 6$, $\lambda = 0.10$.

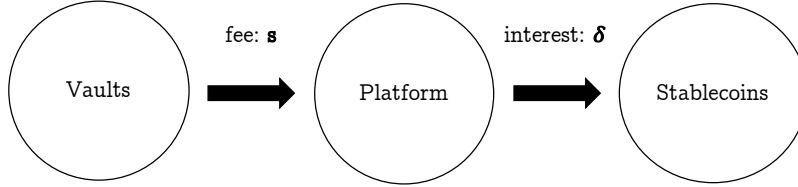


Figure 7: Flows in a Decentralized Platform

resents a leveraged position in the collateral asset for its owner. Vault owners can unlock their collateral assets by repurchasing and “burning” enough stablecoins to liquidate the vault.

Importantly, in decentralized stablecoins protocols all stablecoins are fungible, so that the identity of the specific vault that issued the coin is irrelevant. The stability of a decentralized protocol therefore hinges on the set of incentives provided by the governance token owners to the individual vault owners. In practice, governance token owners set the vaults’ collateralization ratio and charge vault owners a stability fee supposed to steer issuance and repurchase decisions by vault owners. As in the centralized case, governance token owners also set an interest rate paid to stablecoin users.

5.1 Decentralized Protocol

We first describe the policies available to individual vault owners and governance token owners. As mentioned above, any agent can open a vault and issue stablecoins subject to

the protocol rules. We index existing vaults by i and call C_t^i the amount of stablecoins outstanding for vault i at date t . Total stablecoin supply at date t is thus $C_t = \int_i C_t^i di$.

Governance token owners set a collateralization ratio φ at date 0, and choose a stability fee s_t charged to vault owners and an interest rate δ_t paid to stablecoin users at any date $t \geq 0$. They may also decide to shut down the system at some date τ . The stability fee is a new feature of a decentralized protocol: a vault owner with C_t^i stablecoins outstanding at date t must issue $s_t C_t^i$ coins which it transfers to the governance token owners. Of these $s_t C_t^i$ units of stablecoins it collects from vault owners, the platform transfers $\delta_t C_t^i$ as interest payment to users and sells the difference at market price p_t . These flows are summarized in Figure 7.

Taking as given the collateralization ratio φ , the price sequence $\{p_t\}_{t \geq 0}$ and the stability fee sequence $\{s_t\}_{t \geq 0}$, a vault owner i chooses its active supply $d\mathcal{G}_t^i$ and its default time τ^i . A vault owner with C_t^i stablecoins outstanding at date t thus solves:

$$V_t^i(C_t^i) = \max_{\tau^i, d\mathcal{G}_t^i} \mathbb{E}_t \left[\int_t^{\tau^i \wedge \tau} e^{-r(s-t)} (p_s d\mathcal{G}_s^i - d\mathcal{M}_s^i) + e^{-r(\tau^i \wedge \tau - t)} \max\{K_{\tau^i \wedge \tau}^i - C_{\tau^i \wedge \tau}^i, 0\} \right], \quad (45)$$

subject to

$$dC_t^i = s_t C_t^i dt + d\mathcal{G}_t^i, \quad (46)$$

$$d\mathcal{M}_t^i = \varphi dC_t^i - \mu^k \varphi C_t^i dt. \quad (47)$$

The vault owner enjoys the present value of active issuance gains $p_s d\mathcal{G}_s^i$ net of collateral purchases $d\mathcal{M}_s^i$ until it defaults or the platform shuts down. In that event, the vault owner receives the collateral value net of the par value of outstanding stablecoins, similar to the centralized case. Equation (46) is the law of motion for the vault owners' outstanding stablecoins and (47) is the law of motion for the vault's collateral value that ensures the collateralization ration $K_t^i = \varphi C_t^i$ is satisfied.

We now turn to the optimization problem of the governance token owners. At every date t , they choose whether to default, and if not, the stability fee s_t charged to vault owners

and the interest rate δ_t paid to stablecoin users so as to solve

$$E_t = \max_{\tau, \delta, s} \mathbb{E}_t \left[\int_t^\tau e^{-r(s-t)} \left((s_s - \delta_s) p_s C_s - \int_i \min\{C_s^i(1 - \varphi), 0\} \mathbb{1}\{\tau^i = s\} di \right) ds \right], \quad (48)$$

subject to $C_t = \int_i C_t^i di$ where C_t^i is determined by optimization problem (45) for individual vault owner i . The governance tokens' dividend is proportional to the difference between the stability fee s_t charged to vault owners and the interest δ_t paid to users. At every date t , governance token owners must also cover the collateral shortfall, equal to $\int_i \min\{C_s^i(1 - \varphi), 0\}$ of all liquidated vaults or shut down the system. Finally, stablecoins are priced competitively by users. We thus have

$$p_t = \mathbb{E} \left[\int_t^\tau e^{-r(s-t)} (\ell_s + \delta_s) p_s ds + e^{-r(\tau-t)} \min\{\varphi, 1\} \right]. \quad (49)$$

As in the centralized case, we analyze Markov equilibria of the decentralized model with states variables A_t and C_t . Besides aggregate state variables A_t and C_t , each vault owner i also considers its own stock of stablecoins C_t^i as an idiosyncratic state variable when solving problem (45). In the interest of space we do not provide a formal definition of a Markov equilibrium for the decentralized game as it resembles that of Section 4. We thus write $E(A, C)$ for the governance tokens' value and $p(A, C)$ for the stablecoin price as before, and $V(A, C, C^i)$ for vault owner i 's value function.

As a key difference relative to our analysis of a centralized platform, we fully relax our commitment assumption: governance token owners choose the interest rate policy (and the stability fee) sequentially at every date t , rather than once and for all at date 0. This assumption closely reflects market practice for decentralized protocols.

5.2 Arbitrage

First, we derive arbitrage relationships for equilibrium objects imposed by vault owners free entry and stablecoin optimization. A vault owner with stablecoins C^i can adjust its holdings to any \tilde{C}^i receiving net issuance benefits $p(A, C) - \varphi$ per unit from the adjustment. By definition of the value function, we thus have

$$V(A, C, C^i) \geq V(A, C, \tilde{C}^i) + (p(A, C) - \varphi)(\tilde{C}^i - C^i). \quad (50)$$

The same relationship must hold inverting C^i and \tilde{C}^i which implies that (50) must hold as an equality. The key element for this result is that the stablecoin price $p(A, C)$ is independent from each vault owner's individual stock of stablecoins with decentralized issuance. Imposing free entry for vault owners, that is, $V(A, C, 0) = 0$, equation (50) taken as an equality implies

$$V(A, C, C^i) = \varphi C^i - p(A, C) C^i. \quad (51)$$

Equation (51) states that the value of a vault is equal to the value of the collateral held minus the value of stablecoins outstanding. As vault owners are protected by limited liability, equation (51) further implies that an equilibrium price must satisfy $p \leq \varphi$. This inequality is also an arbitrage constraint: the equilibrium price of a stablecoin cannot exceed the value of collateral backing its issuance. Otherwise, investors would open vaults and issue an infinite amount of stablecoins.

Thus the absence of arbitrage opportunities implies that vault owners cannot capture the franchise value from the seigniorage revenues and the value of a vault must be equal to the value of collateral minus stablecoin debt.

As the continuation value of the vault itself then depends on the rate of return of collateral μ^k and the stability fee s , this no-arbitrage condition constrains the choice of the stability fee policy.

Equity holders then internalize that any deviation from the pair of stability fee $s(a)$ and interest payment $\delta(a)$ such that the arbitrage condition (51) is not satisfied triggers immediate changes in the supply of stablecoins. By adjusting the stability fee s , equityholders can incentivize arbitrageurs to adjust the supply to a desired target. Thus, by submitting a policy schedule, they can decide the equilibrium quantity of stablecoins. For example, equityholders can lower the stability fee to increase the value of a vault and incentivize the minting of stablecoins by vault owners. In Appendix B.13, we show that this problem can then be rewritten as

$$E_t = \max_C \mathbb{E}_t \left[\int_0^\tau e^{-r(s-t)} \left(\ell_s C_s + (\mu^k - r) \varphi C_s \right) ds \right]$$

subject to $\varphi - p(a) \geq 0$. In Proposition 11, we characterize the MPE of a decentralized platform.

Proposition 11 (Targeted MPE). *The non-zero MPE policies $s(a_t)$ and $\delta(a_t)$ are such that $d\mathcal{G}_t = \int d\mathcal{G}_t^i di$ is given by*

$$d\mathcal{G}(a_t, C_{t-}) = C^*(A_t) - C_{t-}$$

where $C^*(A_t) \equiv A_t/a^*$ is defined by

$$C^*(A) = \arg \max_C \{ \ell(A, C)C + (\mu^k - r)C \}. \quad (52)$$

At a^* , the policies are given by

$$s(a^*) = \mu^k, \quad \delta(a^*) = r - \ell(a^*), \quad \text{and} \quad \varphi^* = 1.$$

The value of equity is given by

$$E(A) = \frac{\ell(a^*) + \mu^k - r}{r + \frac{\lambda}{\xi+1} - \mu} \frac{A}{a^*}.$$

The key insight is that in the presence of arbitrageurs, equity holders are able to target an optimal ratio a^* with the stability fee and the interest payment policies without incentives to deviate. Any deviation from the equilibrium policies $s(a^*)$ and $\delta(a^*)$ triggers an immediate adjustment of the supply of stablecoins to a suboptimal level. Thus, a decentralized platform does not require commitment to any of its policies to enforce a stable equilibrium.

6 Conclusion

In this paper, we propose a general model of stablecoins and examine the merits and vulnerabilities of various stabilization mechanisms. Our analysis highlights that, although local equilibria are feasible, platforms that are only partially collateralized are always vulnerable to large negative demand shocks. This result holds even in a full commitment case. Platforms that do not have full commitment over their issuance-redemption rule may still also feature a locally stable equilibrium given a convex coupon issuance policy. The decentralization of a platform acts as a substitute for a commitment technology. Overall, our work has practical implications for the design and regulation of stablecoins.

References

- Admati, Anat R, DeMarzo, Peter M, Hellwig, Martin F, and Pfleiderer, Paul. The leverage ratchet effect. *The Journal of Finance*, 73(1):145–198, 2018.
- Aghion, Philippe and Bolton, Patrick. An incomplete contracts approach to financial contracting. *The Review of Economic Studies*, 59(3):473–494, 1992.
- Arner, Douglas W, Auer, Raphael, and Frost, Jon. Stablecoins: risks, potential and regulation. *Financial Stability Review. N° 39 (Autumm 2020)*, p. 95-123, 2020.
- Berentsen, Aleksander and Schär, Fabian. Stablecoins: The quest for a low-volatility cryptocurrency. *Fatas A., Economics of Fintech and Digital Currencies*, pages 65–71, 2019.
- Black, Fischer and Scholes, Myron. The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3):637–654, 1973. ISSN 00223808, 1537534X.
- Bolton, Patrick and Scharfstein, David S. A theory of predation based on agency problems in financial contracting. *The American Economic Review*, 80(1):93–106, 1990.
- Bullmann, Dirk, Klemm, Jonas, and Pinna, Andrea. In search for stability in crypto-assets: are stablecoins the solution? ECB Occasional Paper 230, European Central Bank, August 2019.
- Cong, Lin William, Li, Ye, and Wang, Neng. Token-based platform finance. Working Paper 27810, National Bureau of Economic Research, September 2020a.
- Cong, Lin William, Li, Ye, and Wang, Neng. Tokenomics: Dynamic Adoption and Valuation. *The Review of Financial Studies*, 08 2020b. ISSN 0893-9454. doi: 10.1093/rfs/hhaa089.
- Del Negro, Marco and Sims, Christopher A. When does a central banks balance sheet require fiscal support? *Journal of Monetary Economics*, 73:1–19, 2015. ISSN 0304-3932. Carnegie-Rochester-NYU Conference Series on Public Policy “Monetary Policy: An Unprecedented Predicament” held at the Tepper School of Business, Carnegie Mellon University, November 14-15, 2014.

DeMarzo, Peter M and He, Zhiguo. Leverage dynamics without commitment. *The Journal of Finance*, 76(3):1195–1250, 2021.

Donaldson, Jason, Piacentino, Giorgia, and Gromb, Denis. Conflicting priorities: A theory of covenants and collateral. *Available at SSRN*, 2020.

ECB. Stablecoins – no coins, but are they stable? IN FOCUS 3, European Central Bank, November 2019.

Eichengreen, Barry. From commodity to fiat and now to crypto: What does history tell us? Working Paper 25426, National Bureau of Economic Research, January 2019.

G30. Digital currencies and stablecoins. Risks, opportunities, and challenges ahead. Technical report, Group of Thirty, July 2020.

Griffin, John M. and Shams, Amin. Is bitcoin really untethered? *The Journal of Finance*, 75(4):1913–1964, 2020.

Gudgeon, L., Perez, D., Harz, D., Livshits, B., and Gervais, A. The decentralized financial crisis. In *2020 Crypto Valley Conference on Blockchain Technology (CVCBT)*, pages 1–15, 2020. doi: 10.1109/CVCBT50464.2020.00005.

Klages-Mundt, Arian and Minca, Andreea. (In) stability for the blockchain: Deleveraging spirals and stablecoin attacks. *arXiv preprint arXiv:1906.02152*, 2019.

Klages-Mundt, Arian and Minca, Andreea. While stability lasts: A stochastic model of stablecoins. *arXiv preprint arXiv:2004.01304*, 2020.

Li, Ye and Mayer, Simon. Money creation in decentralized finance: A dynamic model of stablecoin and crypto shadow banking. 2022.

Malenko, Andrey and Tsoy, Anton. Optimal time-consistent debt policies. *Available at SSRN 3588163*, 2020.

Marcet, Albert and Marimon, Ramon. Recursive contracts. *Econometrica*, 87(5):1589–1631, 2019.

Myers, Stewart C. Determinants of corporate borrowing. *Journal of Financial Economics*, 5(2):147 – 175, 1977. ISSN 0304-405X.

Reis, Ricardo. Comment on: “when does a central bank’s balance sheet require fiscal support?” by marco del negro and christopher a. sims. *Journal of Monetary Economics*, 73:20–25, 2015. ISSN 0304-3932. Carnegie-Rochester-NYU Conference Series on Public Policy “Monetary Policy: An Unprecedented Predicament” held at the Tepper School of Business, Carnegie Mellon University, November 14-15, 2014.

Smith, Clifford W. and Warner, Jerold B. On financial contracting: An analysis of bond covenants. *Journal of Financial Economics*, 7(2):117–161, 1979.

Appendices

A Stablecoin Designs in a Crypto Crisis

This appendix provides a short introduction to the variety of stablecoin pegging mechanisms in practice, emphasizing their performance during the crypto crunch of May 2022. We review two custodial (USDCoin and Tether), a purely algorithmic (Terra), an over-collateralized (DAI), and an under-collateralized (FRAX) stablecoin platforms. At the beginning of May, these five stablecoins accounted for more than 80% of the total stablecoin market.

USD Coin

USD Coin (USDC) is a custodial (fully collateralized) stablecoin managed by the Center consortium on behalf of the peer-to-peer payment technology Circle headquartered in Boston, MA. USDC is effectively acting as a narrow bank by backing its stablecoins exclusively with cash (bank deposits or equivalents) and short-term Treasury securities and providing full redemption. During the May 2022 crypto crash, USDC fared particularly well, as can be seen in Figure 8: It maintained its peg and the quantity of USDC outstanding increased during that time period. Given its conservative reserves management strategy, USDC presumably benefited from a “flight to safety” as investors were fleeing from fast depreciating crypto-currencies and other stablecoins.

Tether

Tether (USDT) is another custodial stablecoin that is native of the Ethereum ledger and issued by the Tether Limited company domiciled in Hong Kong under the umbrella of Tether Holdings Limited in the British Virgin Islands. Although Tether claims to be “fully backed by US dollar reserves”, its definition of reserves appear to be less restrictive than the one applied by USDC and also include privately-issued commercial papers, corporate bonds but also volatile crypto-currencies.²⁴ [Griffin and Shams \(2020\)](#) reports suspicious transaction

²⁴Since 2021 and a \$41 million fine to the Commodity Futures Trading Commission for misleading claims that it was fully backed by the US dollar, Tether Holdings Limited regularly reports a reserves audit from Cayman-based auditing companies

patterns on the blockchain suggesting that the platform has been using unbacked Tether creation to purchase large quantities of Bitcoin to support its price.

Figure 8 displays the time-series price of Tether and quantities outstanding. One can observe a sharp reduction in supply around the crypto crash of May 2022, along with a temporary de-pegging. Tether nonetheless re-anchored within a couple of days and has so far proven to be able to absorb the \$5 bn. of redemption it has faced.

Terra

Terra (UST) is a prime example of a fully algorithmic (uncollateralized) stablecoin. As described in the main text, algorithmic stablecoins such as Terra are uncollateralized and rely exclusively on quantity adjustments through smart contracts specifying rules for stablecoin issuances and buybacks. In the case of Terra, these are ruled through an external module allowing any investor to exchange one unit of stablecoin (Terra) for one dollar worth of governance token (Luna) and conversely. Between its introduction in early 2020 and the crypto crash of May 2022, Terra was among the fastest-growing stablecoin platforms. By May 2022, the quantity of stablecoin Terra outstanding was close to \$20 bn. while the governance token Luna had a peak market capitalization of \$40 Bn.

As can be seen in figure 9, the platform completely collapsed between May 7 and May 12, 2022. In the right panel of figure 9, we see how the platform attempted but failed at defending the peg. On May 12, the platform burnt around 8 bn. of Terra partly through the issuance of additional Luna at an exponential pace. As can be seen in the left panel, this massive issuance of Luna led to the complete free-fall in its price to zero. Simultaneously, the Terra Foundation liquidated around \$3 bn. of Bitcoin it had held in reserves. Given the size of the shock, these adjustments were not sufficient to re-anchor the peg, and the value of Terra eventually also fell very close to zero.

DAI

Dai (DAI) is a fully-decentralized over-collateralized stablecoin platform. By its decentralized nature, Dai is slightly more complex than other stablecoins and requires a longer description. With Dai, every user is able to deposit some Ethereum-based crypto-asset as collateral in a smart contract called a Collateralized Debt Position (CDP). The user can then issue and sell stablecoin tokens Dai against this collateral up to a certain over-

collateralization threshold while effectively retaining an equity tranche in the CDP. In doing so, CDP users acquire a leveraged position in the collateral asset. Initially, it was only possible to use Ethereum as a collateral asset, but the platform migrated to a multiple collateral system at the end of 2019. Since then, the custodial stablecoin USD Coin (see above) has been extensively used as collateral for Dai. To close the CDP and retrieve the locked collateral, its owner has to repurchase and burn all previously issued Dai from the secondary market.

The platform also issues its own governance token called Maker (MKR). Holding Maker allows its user to vote on key policies of the platform and effectively entails a right to future seigniorage revenues. The platform is able to generate revenues for Maker holders by collecting “stability” fees on CDP owners. These fees accrue to a “buffer” fund up to a certain limit and are then distributed to Maker holders as dividends.

The pegging mechanism in Dai is tied to its over-collateralization. When the collateral in a CDP falls below the required threshold, the position is automatically liquidated, and collateral assets are sold in an auction to burn corresponding Dai. When auction proceeds are insufficient to repurchase all Dai issued by the CDP, Makers are automatically issued to cover the shortfall. As shown in Figure 9, one can see that this mechanism was at play during the May 2022 market crash. The platform then liquidated for \$3 bn. worth of collateral in CDPs in order to burn more than \$2 bn. worth of Dai. This process was nonetheless done in an orderly fashion, and the parity was maintained throughout. As can be seen from the rightmost panel, no additional Maker was required to be issued.

FRAX

Frax (FRX) is an undercollateralized platform that can be thought of as a hybrid between Terra and Dai. As with Terra, users can exchange the stablecoin Frax for the platform’s governance token Frax Shares (FRS) and the converse. Because the platform is partly collateralized, the swap module requires users to bring both FRS and collateral in a given proportion. For instance, if the collateralization ratio is 90% and Frax is trading for more than one USD, users can exchange 90 USD Coins and \$10 worth of FRS in exchange for 100 Frax and sell them for a profit. The collateralization ratio in Frax is automatically reduced in expansion and increased in contraction so that with a large surge in issuance, Frax would converge to a fully algorithmic platform like Terra.

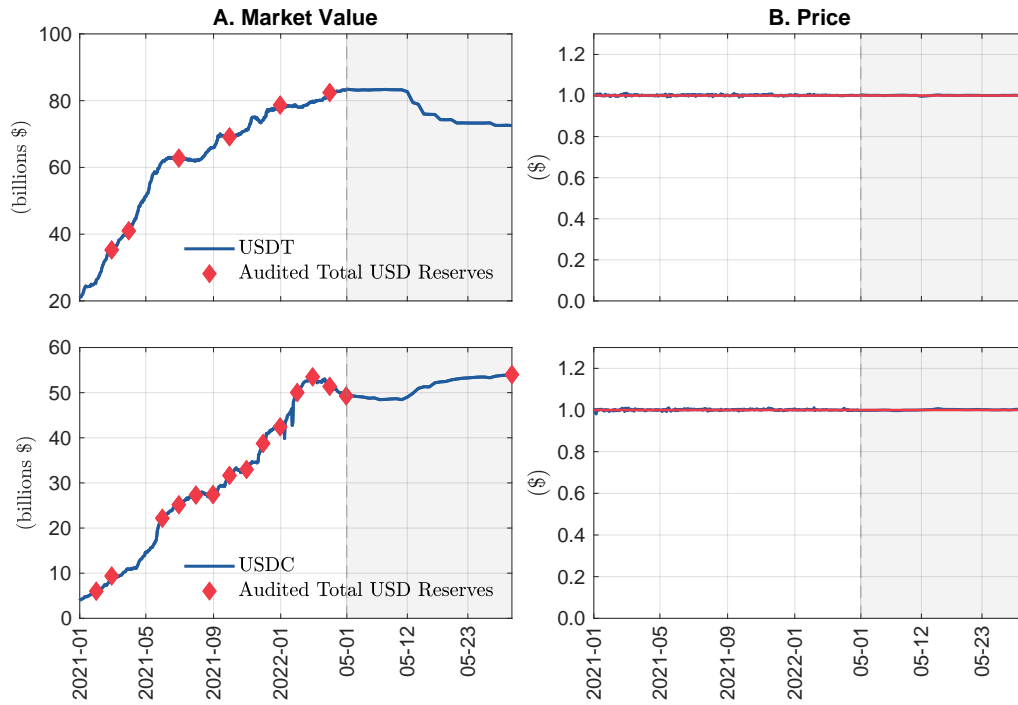


Figure 8: Custodial Stablecoins Time Series. This figure illustrates the daily time series of market capitalization and price for Tether (USDT, first row) and USD Coin (USDC, second row). The first portion of each graph spans the period from January 2021 until April 30 2022, while the gray shaded area zooms on May 2022. The pink diamond markers in Panels A illustrate the total USD value of reserves backing the stablecoin, as certified through external audits made available on the platforms' respective web pages. Data sources: Market capitalization and prices are all retrieved through the CoinGecko API.

In early May 2022, Frax had a collateralization rate of 86.75%. As can be seen in Figure 9, the platform managed to burn around a \$1 bn. without breaking its peg.

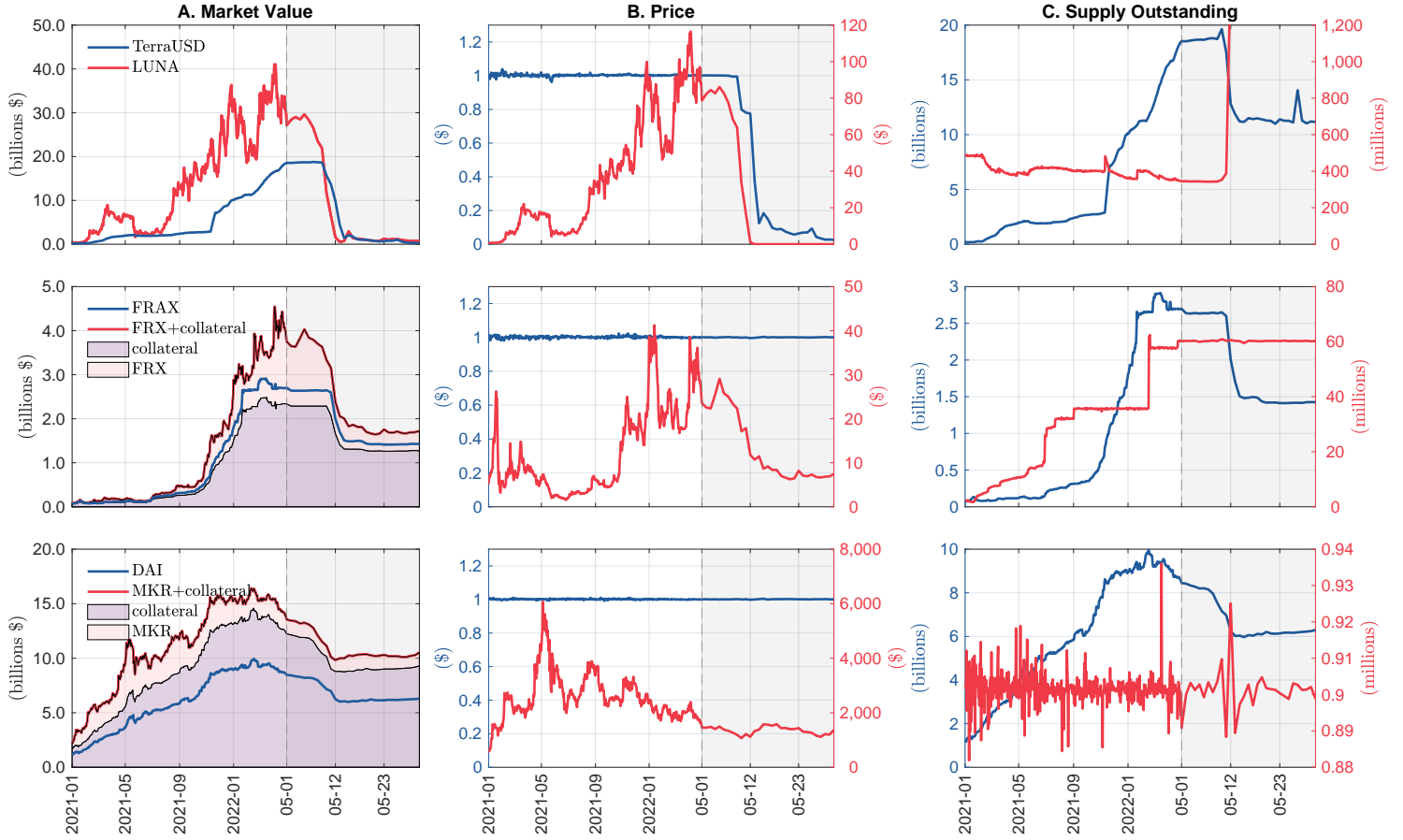


Figure 9: Algorithmic Stablecoins Time Series This figure illustrates the daily time series of market capitalization, price and circulating supply, as denoted in each Column title, for three algorithmic stablecoins. The first portion of each graph spans the period from January 2021 until April 30 2022, while the gray shaded area zooms on May 2022. Each row plots the dynamics for a given stablecoin, as labelled in the first Column; the blue solid line refers the stablecoin asset, while the pink solid line (or light pink shaded area in Panels A) refer to the corresponding governance token. Data sources: Market capitalization, prices and supply outstanding are all retrieved through the CoinGecko API. The total USD value of FRAX collateral, illustrated in light shaded violet in the second row, was manually collected from <https://app.frax.finance>. The amount of DAI collateral was obtained by aggregating across all collateral assets, using the time series debt data made available on Dune Analytics by @adcv via <https://dune.com/queries/865375>; we apply an adjustment factor to account for underestimating measurement error and impute the historical USD value of DAI collateral, illustrated in light violet in the last row, by rescaling the series by the ratio of Total DAI Locked (\$) from <https://daistats.com#/overview> to the aggregated collateral series, both observed as of July 2022, 11, assuming a constant scaling factor.

B Proofs

B.1 Proof of Proposition 2

Substituting for $d\mathcal{G}_t = dC_t - \delta_t C_t dt$, the objective function can be written as

$$E_0 = \max_{\varphi, \{\delta_t, d\mathcal{G}_t\}_{t \geq 0}} \mathbb{E}_0 \left[\int_0^\infty e^{-rt} \left(p_t dC_t - \delta_t p_t C_t dt + \mu^k \varphi C_t dt - \varphi dC_t \right) \right]. \quad (\text{B.53})$$

Integrating the terms in dC_t by parts, we obtain

$$E_0 = \max_{\varphi, \{\delta_t, d\mathcal{G}_t\}_{t \geq 0}} \mathbb{E}_0 \left[\left[(p_t - \varphi) C_t e^{-rt} \right]_0^\infty - \int_0^\infty e^{-rt} C_t \left(dp_t - r(p_t - \varphi) dt + \delta_t p_t dt - \mu^k \varphi \right) \right] \quad (\text{B.54})$$

$$= \max_{\varphi, \{\delta_t, d\mathcal{G}_t\}_{t \geq 0}} \mathbb{E}_0 \left[\int_0^\infty e^{-rt} \left(\ell(A_t, C_t) \mathbb{1}_{\{p_t=1\}} + (\mu^k - r) \varphi \right) C_t dt \right]. \quad (\text{B.55})$$

To obtain the second line, we guess and verify that $\lim_{t \rightarrow \infty} \mathbb{E}_0[(p_t - \varphi) C_t e^{-rt}] = 0$. We use the pricing equation (6) to substitute for $dp_t - (r - \delta)p_t dt$ within the expectation.

Equation (B.55) shows that setting $\varphi = 0$ is optimal. Second δ_t is only determined to the extent that it maintains the price peg and we can rewrite equation (B.55) as

$$E_0 = \max_{\varphi, \{\delta_t, d\mathcal{G}_t\}_{t \geq 0}} \mathbb{E}_0 \left[\int_0^\infty e^{-rt} \left(\ell(A_t, C_t) + (\mu^k - r) \varphi \right) C_t dt \right]. \quad (\text{B.56})$$

Assuming such coupon policy can be chosen, the platform's problem is static and the optimal issuance rule is such that C_t maximizes $\ell(A_t, C_t)C_t$. By Property (iii) in Assumption 1, this maximizer exists, is unique, and is given by (9). The fact that $C_{ul}^*(A) = A/a_{ul}^*$ is linear in A follows from Assumption 1. Finally, our conjecture $\lim_{t \rightarrow \infty} \mathbb{E}_0[(p_t - \varphi) C_t e^{-rt}] = 0$ and the fact that the objective function is bounded follows from the fact that A_t grows at a rate inferior to r . Finally, the coupon policy must be such that $p_t = 1$ for all t , which holds with $\delta(a_{ul}^*) = r - \ell(a_{ul}^*)$.

To conclude, the optimal issuance-repurchase policy $\{d\mathcal{G}_t\}_{t \geq 0}$ features a jump from 0 to $C_{ul}^*(A_0)$ at date 0 and is such that $d\mathcal{G}_t + \delta_t C_t dt = dA_t$ for $t > 0$.

B.2 Proof of Lemma 1

We guess and verify throughout that $p(a) = 1$ if and only if $a \in [\bar{a}, a^*]$ and $p(a) < 1$ otherwise. This implies liquidity benefits are enjoyed by stablecoin users only when $a \in [0, \bar{a}]$. We anticipate the result in Lemma 3 and conjecture an equilibrium with no platform default. We thus set $\underline{a} = 0$ and later prove in Lemma 3 that this feature is optimal. We proceed in three steps, we first show that $e(a) = 0$ is optimal for all $a \leq \bar{a}$ (Step 1). We then derive the optimal issuance policy in the smooth region (Step 2). Finally, we derive the HJB equation for the price in that region (Step 3).

Step 1. Total Platform Value

Consider first the net platform value F . Suppose $a = A/C > \bar{a}$. In this case F only depends on A , not on the outstanding stock of stablecoins C and we denote $\bar{F}(A)$ to avoid confusion. Let τ_S denote the first (stochastic) time when a shock $S \leq \underline{a}/a^*$ hits. We have

$$\begin{aligned} \bar{F}(A_0) = E_{\tau_S} \left[\int_0^{\tau_S} e^{-rt} \left(\ell(A_t, C^*(A_t)) C^*(A_t) + \varphi(\mu^k - r) C^*(A_t) \right) dt \right. \\ \left. + e^{-r\tau_S} \mathbb{E} \left[F(SA_{\tau_S}, C^*(A_{\tau_S})) \mid Sa^* \leq \bar{a} \right] \right]. \end{aligned} \quad (\text{B.57})$$

Given values for (a^*, \bar{a}) , maximizing value $\bar{F}(A_0)$ consists in maximizing the second term of the equation above. We thus explicit the dynamic equation for $F(A, C)$ in the region where $a = A/C \in [0, \bar{a}]$. For a given $a \in [0, \bar{a}]$, denote $\tau(a)$ the first stochastic time when $a_t = \bar{a}$. We have

$$F(A_0, C_0) = \mathbb{E}_{\tau(a_0)} \left[\int_0^{\tau(a_0)} e^{-rt} (\mu^k - r) \varphi C_t dt + e^{-r\tau(a_0)} \bar{F}(A_{\tau(a_0)}) \right], \quad (\text{B.58})$$

$$\text{subject to } (1), \quad dC_t = (\delta_t C_t + G_t) dt. \quad (\text{B.59})$$

The dividend flow for the total platform is negative in the region $[0, \bar{a}]$. Hence, maximizing $F(A, C)$ in region $[0, \bar{a}]$ and thus $\bar{F}(A)$ amounts to minimizing the expected time $\tau(a)$ from any given point a . Given the policies in $[0, \bar{a}]$ in (13), we have

$$\mathbb{E} \left[\frac{da_t}{a_t} \right] = \left(\mu - \frac{\lambda}{\xi + 1} \right) dt - (\delta_t + G_t/C_t) dt. \quad (\text{B.60})$$

Hence the platform should seek to minimize δ_t and G_t subject to the constraint that equity

$E(A, C)$ remains positive for $A/C \in [0, \bar{a}]$. We determine below lower bounds on δ_t and G_t compatible with this constraint.

Step 2. HJB for Equity Value

In the next step, we derive the recursive equation for the equity value in order to pin down the minimum value of $G(A, C)$ such that limited liability holds in region $[0, \bar{a}]$. In doing so, we guess and verify that it holds for $[\bar{a}, \infty)$. Adapting Equation (7), we have

$$\begin{aligned} E(A, C) = & (p(A, C) - \varphi)G(A, C)dt \\ & + (1 - rdt)(1 - \lambda dt)\mathbb{E}[E(A + dA, C + dC) + \mu^k K dt + \varphi G(A, C)dt - \varphi dC] \\ & + (1 - rdt)\lambda dt \mathbb{E}[E(SA, C)], \end{aligned} \quad (\text{B.61})$$

where the terms within the first expectation operator correspond to the difference between the passive and active increases in collateral value ($\mu^k K dt + \varphi G(A, C)dt$) and the change in collateral value required to back the issuance of stablecoins (φdC). Using Ito's Lemma for the term $E(A + dA, C + dC)$ above and keeping only terms of order dt , we obtain the following HJB:

$$\begin{aligned} (r + \lambda)E(A, C) = & (p(A, C) - \varphi)G(A, C) + \mu A E_A(A, C) + \frac{\sigma^2}{2} E_{AA}(A, C) \\ & + (\delta(A, C)C + G(A, C))E_C(A, C) + (\mu^k - \delta(A, C))\varphi C + \lambda \mathbb{E}[E(SA, C)]. \end{aligned} \quad (\text{B.62})$$

We rewrite the equation above as a functional equation for $e(a) = E(A, C)/C$. With $E_A(A, C) = e'(a)$, $E_{AA}(A, C) = e''(a)$, $E_C(A, C) = e(a) - ae'(a)$, and $g(a) \equiv G(A, C)/C$, we get

$$\begin{aligned} (r + \lambda)e(a) = & (p(a) - \varphi)g(a) + \mu a e'(a) + \frac{\sigma^2}{2} e''(a) + (\delta(a) + g(a))(e(a) - ae'(a)) \\ & + (\mu^k - \delta(a))\varphi + \lambda \mathbb{E}[e(Sa)]. \end{aligned} \quad (\text{B.63})$$

It follows from the equation above that the minimum value of $g(a)$ such that $e(a) \geq 0$ for all $a \in [0, \bar{a}]$ is given by

$$g(a) = -\frac{\mu^k - \delta(a)}{p(a) - \varphi}\varphi.$$

Given policy $g(a)$ above and $e(a) = e'(a) = 0$, the impact of $\delta(a)$ is offset in the HJB and

we can set $\delta(a)$ to its minimum at 0 for $a \leq \bar{a}$. This concludes the proof.

Step 3. HJB equation for stablecoin price

Next, we characterize the price dynamics in region $[0, \bar{a}]$. The price equation can be written as

$$p(A, C) = (1 - rdt)(1 - \lambda dt)\mathbb{E}[p(A + dA, C + dC)] + (1 - rdt)\lambda dt\mathbb{E}[p(SA, C)]. \quad (\text{B.64})$$

When $a \in [0, \bar{a}]$, stablecoin owners enjoy no cash flow because the platform optimally sets $\delta(a) = 0$ and liquidity benefits are equal to 0 because the price is not pegged to 1 since $p(a) < 1$. Using $dC = g(a)Cdt$, the first term on the right-hand side can be expanded using Ito's Lemma:

$$\begin{aligned} \mathbb{E}[p(A + dA, C + dC)] &= p(A, C) + p_A(A, C)\mu A dt + \frac{\sigma^2}{2}A^2 p_{AA}(A, C)dt + p_C(A, C)g(a)Cdt \\ &= p(a) + (\mu - g(a))ap'(a)dt + \frac{\sigma^2}{2}a^2 p''(a)dt. \end{aligned} \quad (\text{B.65})$$

To obtain the second line, we used the homogeneity of degree 0 of the price function, that is, $p(A/C) \equiv p(A, C)$ to replace $p_A(A, C) = p'(a)/C$, $p_{AA}(A, C) = p''(a)/C^2$ and $p_C(A, C) = -p'(a)A/C^2$. Plugging (B.65) into (B.64) and keeping only terms of order dt we obtain

$$0 = -(r + \lambda)p(a) + (\mu - g)ap'(a) + \frac{\sigma^2}{2}a^2 p''(a) + \lambda\mathbb{E}[p(Sa)] \quad (\text{B.66})$$

which is equivalent to equation (16).

Finally, the boundary condition $p(\bar{a}) = 1$ obtains by construction in our conjectured equilibrium with $p(a) = 1$ for $a \geq \bar{a}$. The boundary condition $p(\bar{a}) = \varphi$ obtains because the value of a stablecoin in default is equal to the collateral backing it, that is, φ . Note also that issuance policy (15) implies this is a reflecting boundary, which implies $p(a) > \varphi$ for all $a > \underline{a}$. This concludes the proof.

B.3 Proof of Lemma 2

The fact that equity value is equal to 0 in region $[\underline{a}, \bar{a}]$ is shown in Lemma 1. Because the collateralization ratio satisfies $\varphi \leq 1$, the platform's value is also equal to 0 in default

region $[0, \underline{a}]$.

Consider now interval $[\bar{a}, \infty)$. As argued in the main text, by definition of a policy in (13), equation (20) must hold. We can rewrite this relationship as follows

$$Ce(a) = C^*(A)e(a^*) + (p(a^*) - \varphi)(C^*(A) - C). \quad (\text{B.67})$$

Dividing both terms by C and using $C^*(A) = A/a^*$ by definition of a^* , we obtain equation (18).

We are thus left to derive the HJB for the equity value at demand ratio $e(a^*)$. The recursive equation is the following.

$$\begin{aligned} E(a^*C_-, C_-) = & (1 - rdt)(1 - \lambda dt) \mathbb{E} \left[E(a^*C_- + dA, C_- + dC) + \mu^k K dt - \varphi dC \middle| dN_t = 0 \right] \\ & + (1 - rdt)\lambda dt \mathbb{E} [E(Sa^*C_-, C_-) | dN_t = 1], \end{aligned} \quad (\text{B.68})$$

where the term in the first line corresponds to the case in which the adjustment in demand A_t is smooth ($dN_t = 0$) while the second term corresponds to the case in which demand is hit by a Poisson shock ($dN_t = 1$). The term $\mu^k K dt - \varphi dC$ corresponds to the change in collateral value.

We develop the first term corresponding to Brownian shocks. In region $[\bar{a}, \infty)$, we have

$$E(A, C) = C^*(A)e(a^*) + (p(a^*) - \varphi)(C^*(A) - C) = [e(a^*) + p - \varphi] \frac{A}{a^*} - (p - \varphi)C. \quad (\text{B.69})$$

Hence, given that $dC = \delta(a^*)Cdt$, we obtain the following relationship by Ito's Lemma:

$$\begin{aligned} \mathbb{E} [E(a^*C_- + dA, C_- + dC) | dN_t = 0] = & E(a^*C_-, C_-) + \mu [e(a^*) + p - \varphi] C^*(A) dt \\ & - (p - \varphi) \delta(a^*) C^*(A) dt. \end{aligned} \quad (\text{B.70})$$

Keeping only terms of order at least dt and dividing by $C^*(A)$, we obtain

$$e(a^*) = e(a^*) + \left(-(r + \lambda)e(a^*) + \mu [e(a^*) + p(a^*) - \varphi] - p(a^*)\delta(a^*) + \mu^k \varphi + \lambda \mathbb{E}[e(Sa^*)] \right) dt$$

which simplifies to equation (18). For future reference, we further solve for $e(a^*)$ by com-

putting the term $\mathbb{E}[e(Sa^*)]$. Using (17), we get

$$\mathbb{E}[e(Sa^*)] = \int_0^{\ln(a^*/\bar{a})} e(e^{-s}a^*)\xi e^{-\xi s} ds \quad (\text{B.71})$$

$$= \int_0^{\ln(a^*/\bar{a})} \left[(e(a^*) + p(a^*) - \varphi)e^{-s} - p(a^*) + \varphi \right] \xi e^{-\xi s} ds \quad (\text{B.72})$$

$$= \frac{\xi}{\xi + 1} \left(1 - \left(\frac{a^*}{\bar{a}} \right)^{-(\xi+1)} \right) (e(a^*) + p(a^*) - \varphi) - \left(1 - \left(\frac{a^*}{\bar{a}} \right)^{-\xi} \right) (p(a^*) - \varphi). \quad (\text{B.73})$$

Plugging this equation in (18), we get

$$\begin{aligned} & \left(r + \frac{\lambda}{\xi + 1} - \mu + \frac{\lambda\xi}{\xi + 1} \left(\frac{a^*}{\bar{a}} \right)^{-(\xi+1)} \right) e(a^*) \\ &= \left(\mu^k - \mu + \frac{\lambda}{\xi + 1} - \lambda \left(\frac{a^*}{\bar{a}} \right)^{-\xi} + \frac{\lambda\xi}{\xi + 1} \left(\frac{a^*}{\bar{a}} \right)^{-(\xi+1)} \right) \varphi \\ &+ \left(\mu - \delta(a^*) - \frac{\lambda}{\xi + 1} - \frac{\lambda\xi}{\xi + 1} \left(\frac{a^*}{\bar{a}} \right)^{-(\xi+1)} + \lambda \left(\frac{a^*}{\bar{a}} \right)^{-\xi} \right) p(a^*). \end{aligned} \quad (\text{B.74})$$

Using $p(a^*) = 1$, after some manipulations, we can rewrite the objective function as follows

$$\frac{e(a^*) + 1 - \varphi}{a^*} = \frac{(\mu^k - r)\varphi + r - \delta(a^*) + \lambda(1 - \varphi) \left(\frac{a^*}{\bar{a}} \right)^{-\xi}}{r - \mu + \frac{\lambda}{\xi+1} + \frac{\lambda\xi}{\xi+1} \left(\frac{a^*}{\bar{a}} \right)^{-(\xi+1)}}. \quad (\text{B.75})$$

Finally, we derive the value of the interest paid by the platform at the target ration a^* in order to maintain the peg. To do so, we derive the dynamic equation for the price at the target. We have

$$\begin{aligned} p(A, C^*(A)) &= (\delta(a^*) + \ell(A, C^*(A)))p(A, C^*(A))dt \\ &+ (1 - rdt)(1 - \lambda dt)\mathbb{E}[p(A + dA, C^*(A) + dC)] + (1 - rdt)\lambda dt\mathbb{E}[p(SA, C)]. \end{aligned} \quad (\text{B.76})$$

Note that from any point in target region $[\bar{a}, a^*]$, the platform jumps discretely to a^* . Hence, the equilibrium price is $p(a) = p(a^*)$ when $a \in [\bar{a}, a^*]$. We can thus substitute

$\mathbb{E}[p(A + dA, C^*(A) + dC)]$ with $p(a^*)$. Keeping only terms of order dt , we obtain

$$(r + \lambda)p(a^*) = (\delta(a^*) + \ell(a^*))p(a^*) + \lambda\mathbb{E}[p(Sa^*)].$$

Setting $p(a^*) = 1$ and solving for $\delta(a^*)$ we get (19).

B.4 Proof of Lemma 3

Equation (B.75) shows that for given $\{\bar{a}, a^*\}$, the objective function depends on \bar{a} only via the term $-\delta(a^*)$ which is itself increasing with $\mathbb{E}[p(Sa^*)]$. Next, $-\delta(a^*)$ also enters positively the limited liability constraint $e(a^*) \geq 0$ which implies that increasing $\mathbb{E}[p(Sa^*)]$ also allows to relax the constraint. Overall, the platform should set the default threshold \underline{a} to maximize $\mathbb{E}[p(Sa^*)]$. Below, we show that $\underline{a} = 0$ is the optimum.

Suppose the platform does not default, that is, $\underline{a} = 0$. Because the price $p(a)$ must be increasing for $a \in [0, \bar{a}]$ and $p(0) = \varphi$, it follows that $p(a) \geq \varphi$ for all $a \in [0, \bar{a}]$. Defaulting at some threshold $\hat{a} > 0$ implies the price would satisfy $p(a) = \varphi$ for all $a \in [0, \hat{a}]$, which is weakly less than the price when $\underline{a} = 0$. Hence, default cannot increase the price on the interval $[0, \bar{a}]$ and thus cannot increase $\mathbb{E}[p(Sa^*)]$. Indeed, the price of a stablecoin in $[a, \bar{a}]$ is given by $p_t = \mathbb{E}_t [e^{-r(\tau-t)}(\mathbb{1}\{a_\tau \geq \bar{a}\} + \varphi\mathbb{1}\{a_\tau \leq a\})]$ where $\tau \equiv \inf\{s \geq t; a_s \leq \underline{a} \cup a_s \geq \bar{a}\}$. This expectation is strictly decreasing in the default threshold \underline{a} . This proves that setting default threshold $\underline{a} = 0$ is optimal.

B.5 Proof of Proposition 3

Step 1. Our conjecture for the pricing function is

$$p(a) = \begin{cases} \sum_{k=1}^3 b_k a^{-\gamma_k} & \text{if } 0 \leq a < \bar{a}, \\ 1 & \text{if } a \geq \bar{a}. \end{cases} \quad (\text{B.77})$$

The issuance policy in region $[0, \bar{a}]$ is given by (15), that is, $g = 0$ when $\varphi = 0$. We first derive conditions on $\{\gamma_k\}_{k=1..3}$ such that HJB equation (16) is satisfied by our guess. We

have

$$p'(a) = - \sum_{k=1}^3 b_k \gamma_k a^{-(\gamma_k+1)}, \quad (\text{B.78})$$

$$p''(a) = \sum_{k=1}^3 b_k \gamma_k (\gamma_k + 1) a^{-(\gamma_k+2)}, \quad (\text{B.79})$$

$$\mathbb{E}[p(Sa)] = \int_0^\infty p(e^{-s}a) \xi e^{-\xi s} ds = \int_0^\infty \sum_{k=1}^3 b_k e^{s\gamma_k} a^{-\gamma_k} \xi e^{-\xi s} ds = \sum_{k=1}^3 \frac{b_k \xi a^{-\gamma_k}}{\xi - \gamma_k}. \quad (\text{B.80})$$

Replacing into (16) and equalizing terms proportional to $a^{-\gamma_k}$, we obtain that for each $k \in \{1, 2, 3\}$, γ_k must be a root of characteristic equation (21). The roots of this third-order polynomial are

$$\gamma_k = -\frac{1}{2t_1} \left(t_2 + \zeta^\nu R + \frac{\Delta_0}{\zeta^\nu R} \right) \quad (\text{B.81})$$

where

$$\begin{aligned} \Delta_0 &= t_2^2 - 3t_1 t_3, \quad \Delta_1 = 2t_2^3 - 9t_1 t_2 t_3 + 27t_1^2 t_4, \\ R &= \sqrt[3]{\frac{\Delta_1 + \sqrt{\Delta_1^2 - 4\Delta_0^3}}{2}}, \quad \zeta = \frac{-1 + \sqrt{-3}}{2}, \quad \nu = \{0, 1, 2\}, \\ t_1 &= -\frac{\sigma^2}{2}, \quad t_2 = \mu + \frac{\sigma^2}{2}(\xi - 1), \quad t_3 = -\mu\xi + \frac{\sigma^2}{2}\xi + r + \lambda, \quad t_4 = -r\xi. \end{aligned}$$

According to Descartes' rule of sign, this polynomial has 2 positive roots and 1 negative root. Because the price must be bounded below by 0, the coefficients b_k corresponding to positive roots must be 0. We now call γ the negative root of this polynomial. We can show numerically that for $\{r, \sigma, \lambda, \xi, \mu\} \in (0, 1) \times \mathbb{R}_+ \times (0, 1) \times \mathbb{R}_{>0} \times [-\infty, r + \lambda/(\xi + 1)]$, $\gamma < -1$.

The price function is thus given by $p(a) = ba^{-\gamma}$ for $a \in [0, \bar{a}]$. To determine b , we use the continuity of p at \bar{a} . Setting $p(\bar{a}) = 1$ yields $b = \bar{a}^\gamma$.

Step 2. We now show that the maximization problem of the platform at date 0 is given by (22). Rewriting equation (B.74), we obtain

$$e(a^*) + p(a^*) = \frac{r - \delta(a^*) + \lambda \left(\frac{a^*}{\bar{a}}\right)^{-\xi}}{r + \frac{\lambda}{\xi+1} - \mu + \frac{\lambda\xi}{\xi+1} \left(\frac{a^*}{\bar{a}}\right)^{-(\xi+1)}} p(a^*). \quad (\text{B.82})$$

We are left to substitute for $\delta(a^*)$ thanks to equation (19). We have

$$\begin{aligned}
\delta(a^*) &= r - \ell(a^*) + \lambda(1 - \mathbb{E}[p(a^*S)]), \\
&= r - \ell(a^*) + \lambda - \lambda \left[\int_0^{\ln(a^*/\bar{a})} \xi e^{-\xi s} ds + \int_{\ln(a^*/\bar{a})}^{\infty} \left(\frac{a^*}{\bar{a}}\right)^{-\gamma} e^{s\gamma} \xi e^{-\xi s} ds \right], \\
&= r - \ell(a^*) + \lambda - \lambda \left[1 - \left(\frac{a^*}{\bar{a}}\right)^{-\xi} \right] - \lambda \frac{\xi}{\xi - \gamma} \left(\frac{a^*}{\bar{a}}\right)^{-\xi}. \tag{B.83}
\end{aligned}$$

Substituting for $\delta(a^*)$ into (B.82) and setting $p(a^*) = 1$, we obtain

$$e(a^*) + p(a^*) = \frac{\ell(a^*) + \frac{\lambda\xi}{\xi-\gamma} \left(\frac{a^*}{\bar{a}}\right)^{-\xi}}{r + \frac{\lambda}{\xi+1} - \mu + \frac{\lambda\xi}{\xi+1} \left(\frac{a^*}{\bar{a}}\right)^{-(\xi+1)}}.$$

Simple computations show this equation is equivalent to (22) if (23) holds, which we show below.

We are left to derive liability constraint (23). From Lemma 1, we have $e(a) = 0$ for all $a \in [0, \bar{a}]$ and from Lemma 2, $e(a)$ strictly increases with a for $a \in [\bar{a}, \infty)$. Hence, limited liability holds for all a if $e(\bar{a}) = 0$. Using equation (17) with $\varphi = 0$ and $p(a^*) = 1$, this condition writes

$$[e(a^*) + p(a^*)] \frac{\bar{a}}{a^*} - 1 = 0, \tag{B.84}$$

which is equivalent to (23). This concludes the proof.

B.6 Proof of Proposition 4

We first show that $\bar{a} = 0$ when $\varphi = 1$. This result follows from equation (17) in Lemma 2. Setting $\varphi = 1$ and $p(a^*) = 1$, it is clear that $e(\bar{a}) \geq 0$ for all $a \geq \bar{a}$ if $e(a^*) \geq 0$. This latter condition is verified later in the existence result of Proposition 5.

For the second part of the proof, we rewrite equation (B.74) with $\bar{a} = 0$ to obtain

$$e(a^*) = e(a^*) + p(a^*) - 1 = \frac{\mu^k - \delta(a^*)}{r - \mu + \frac{\lambda}{\xi+1}}. \tag{B.85}$$

Substituting for $\delta(a^*)$ thanks to equation (B.83), which becomes $\delta(a^*) = r - \ell(a^*)$ in this case, we obtain equation (24). This concludes the proof.

B.7 Proof of Proposition 5

Consider first the case $\varphi = 0$. Proposition ?? shows that an equilibrium with positive stablecoin value exists if there exist (\bar{a}, a^*) with $\bar{a} \leq a^*$ such that condition (23) holds. Using equation (??) to substitute for $e(a^*) + p(a^*)$, this condition holds if there exists a^* and $x \in [0, 1]$ such that

$$\ell(a^*)x - c - bx^{\xi+1} \geq 0, \quad \text{with} \quad u \equiv r + \frac{\lambda}{\xi+1} - \mu, \quad v(\gamma) \equiv \frac{\lambda\xi}{\xi+1} - \frac{\lambda\xi}{\xi-\gamma}. \quad (\text{B.86})$$

To derive implications from this condition define $H : x \mapsto \frac{x}{u+v(\gamma)x^{\xi+1}}$ and let x_{max} be the argument of the global maximum of H on $[0, 1]$. We have

$$H'(x) \propto u - v(\gamma)\xi x^{\xi+1},$$

which is strictly decreasing with x because $v(\gamma) > 0$ since $\gamma < -1$. Two cases are then possible. Either $H'(1) = u - \xi v(\gamma) \geq 0$ and $x_{max} = 1$ or $H'(1) < 0$ and $x_{max} = \left(\frac{u}{v(\gamma)\xi}\right)^{\frac{1}{\xi+1}}$ so that overall $x_{max} = \min\left\{1, \frac{u}{v(\gamma)\xi}\right\}^{\frac{1}{\xi+1}}$ and, for a given a^* , a necessary condition for the desired equilibrium to exist is

$$\ell(a^*) \geq \frac{u + v(\gamma)x_{max}^{\xi+1}}{x_{max}} = \frac{u + v(\gamma) \min\left\{1, \frac{u}{v(\gamma)\xi}\right\}}{\min\left\{1, \frac{u}{v(\gamma)\xi}\right\}^{\frac{1}{\xi+1}}}. \quad (\text{B.87})$$

A necessary condition for (B.87) to hold is $\ell(a^*) \geq u$ as stated in Proposition 5.

Consider now case $\varphi = 1$. According to Proposition 4, a solution exists if there exists a^* such that $\ell(a^*) + \mu^k - r \geq 0$. Given that $\max_a \ell(a) \geq \ell(a^*)$ by definition, this condition can hold only if (25) holds. This concludes the proof.

B.8 Proof of Proposition 6

We first state a series of Lemma and prove them at the end of this section.

Lemma 4. *The equity value $e(a)$ is weakly convex, continuously differentiable, and stablecoin price function $p(a)$ is continuous and increasing.*

Lemma 5. *If the equity value $e(a)$ is linear over some interval $[a_L, a_U]$, the equilibrium*

issuance policy features a target demand ratio $a^{jump} \in [a_L, a_U]$ such that the issuance policy for any $a \in [a_L, a_U]$ is to jump at a^{jump} .

Lemma 6. *If $e(a)$ is strictly convex over some interval $[a_L, a_U]$, the equilibrium debt policy is smooth in that region. Furthermore, there is no MPE with positive stablecoin price if the equilibrium issuance policy is smooth everywhere.*

Proposition 6 is then a corollary of the next result.

Lemma 7. *If the coupon policy is optimally chosen at date 0, there exists (\bar{a}, a^*) such that the equilibrium issuance policy equilibrium issuance policy is smooth over $[0, \bar{a}]$ and features a jump at a^* when $a \in [\bar{a}, \infty)$.*

We now provide a proof for these lemma.

Proof of Lemma 4. These properties follow from Lemma A.1 in DeMarzo and He (2021). \square

Proof of Lemma 5. We first show that if the equity value $e(a)$ is linearly increasing in a over some segment $[a_L, a_U]$ (with strictly positive slope), the equilibrium issuance policy cannot be smooth over this interval. We then show that for any such interval $[a_L, a_U]$, there is a single jump point.

The proof is by contradiction. Suppose $d\mathcal{G}_t = G(a)dt$ over $[a_L, a_U]$ with $g(a) \equiv G(a)/C$ the stablecoin issuance rate per unit of stablecoins. With a smooth debt policy, use equation (B.62) to rewrite the HJB equation governing stablecoin issuance as follows

$$(r + \lambda)e(a) = \max_{g(a)} \left\{ g(a)(p(a) - \varphi) + \mu a e'(a) + (\mu^k - \delta(a))\varphi + (g(a) + \delta(a))(e(a) - e'(a)a) + \frac{\sigma^2}{2} a^2 e''(a) + \lambda \mathbb{E}[e(Sa)] \right\}. \quad (\text{B.88})$$

A smooth debt policy is optimal if the first-order condition with respect to g is satisfied, that is, if

$$p(a) - \varphi = e'(a)a - e(a). \quad (\text{B.89})$$

The assumption that $e(a)$ is linear in a further implies that $p'(a) = e''(a)a = 0$ and we

denote $p(a) = p$ in what follows. Hence, equation (B.88) simplifies to

$$(r + \lambda)e(a) = \mu^k \varphi - \delta(a)p + \mu a e'(a) + \lambda \mathbb{E}[e(Sa)]. \quad (\text{B.90})$$

We now establish a contradiction between equation (B.89) and (B.90) when $e(a)$ is linear. Taking the first-order-derivative with respect to a of the terms in (B.90), we obtain

$$(r + \lambda)e'(a) = -\delta'(a)p + \mu e'(a) + \lambda \mathbb{E}[e'(Sa)S]. \quad (\text{B.91})$$

The HJB equation for the stablecoin price is given by

$$(r + \lambda)p(a) = \ell(a)p(a) + \delta(a)p(a) - (g(a) + \delta(a))ap'(a) + \mu ap'(a) + \frac{\sigma^2}{2}a^2 p''(a) + \lambda \mathbb{E}[p(Sa)]. \quad (\text{B.92})$$

which, for a constant $p(a) = p$, simplifies to

$$(r + \lambda)p = \ell(a)p + \delta(a)p + \lambda \mathbb{E}[p(Sa)]. \quad (\text{B.93})$$

Combining equations (B.90), (B.91) and (B.93), we obtain

$$\begin{aligned} 0 &= (r + \lambda)(p(a) - \varphi + e(a) - e'(a)a) = \ell(a)p + \delta(a)p + \lambda \mathbb{E}[p(Sa)] - (r + \lambda)\varphi + \mu^k \varphi - \delta(a)p + \mu a e'(a) \\ &\quad + \lambda \mathbb{E}[e(Sa)] + \delta'(a)ap - \mu a e'(a) - \lambda \mathbb{E}[e'(Sa)Sa], \\ &= (\mu^k - r)\varphi + \ell(a)p + \delta'(a)ap(a) \\ &\quad + \lambda \mathbb{E}[p(Sa) - \varphi + e(Sa) - e'(Sa)Sa], \\ &= (\mu^k - r)\varphi + \ell(a)p + \delta'(a)ap(a). \end{aligned} \quad (\text{B.94})$$

The last equality follows from equation (B.89). We proved this relationship for segments where the equilibrium issuance policy is smooth. For segments over which the issuance policy features jumps, equation (??) shows that for any a, a' in this segment, we have

$$e(a') = [e(a) + p - \varphi] \frac{a'}{a} - (p - \varphi). \quad (\text{B.95})$$

Taking the first-order derivative with respect to a' and then setting $a' = a$ we obtain equation (B.89).

We now establish a contradiction. Suppose first $a_L = 0$. Thus $p = \varphi$ and $\mathbb{E}[p'(Sa)S] = 0$ for $a \in [a_L, a_U]$. Then, if $p \neq 1$ and thus $\ell(a) = 0$, it is immediate that equations (B.94) and (B.93) are inconsistent. If instead $p = 1$, these two equations imply that

$$(\mu^k - r)\varphi + \ell(a) - \ell'(a)a = 0$$

but the functional form of $\ell(a)$ cannot be pinned down by these equilibrium conditions because it is a primitive of the problem.

Suppose now $a_L > 0$. Suppose first $p(a) = p \neq 1$ in which case $\ell(a) = 0$ by definition. Equations (B.94) and (B.93) then imply that

$$\delta'(a) = \frac{r - \mu^k}{ap}\varphi = -\lambda\mathbb{E}[p'(Sa)S]. \quad (\text{B.96})$$

This equation cannot hold because $r > \mu^k$ while $p' \geq 0$ by Lemma 4. Finally, suppose $p(a) = 1$. Equations (B.93) and (B.94) imply together that

$$\frac{(\mu^k - r)\varphi + \ell(a) - \ell'(a)a}{a} = \lambda\mathbb{E}[p'(Sa)S]. \quad (\text{B.97})$$

We have

$$\mathbb{E}[p'(Sa)S] = \int_0^\infty p'(e^{-s}a)\xi e^{-s(\xi+1)}ds \quad (\text{B.98})$$

$$= \int_{\ln(a/a_L)}^\infty p'(e^{-s}a)\xi e^{-s(\xi+1)}ds = \kappa a^{-(\xi+1)} \quad (\text{B.99})$$

where $\kappa \equiv a_L^{\xi+1} \int_0^\infty p'(e^{-s}a_L)\xi e^{-s(\xi+1)}ds$ is a positive constant. To obtain the second line, we used the fact that p is constant over $[a_L, a_U]$. Thus, we must have

$$\ell(a) = \ell'(a)a - (\mu^k - r)\varphi a + \lambda\kappa a^{-\xi} \quad (\text{B.100})$$

for $a \in [a_L, a_U]$. A general solution to this equation is of the form

$$\ell(a) = \alpha a + \beta + f a^{-\xi-1}$$

with $f \geq 0$. Hence, assuming the issuance policy is smooth pins down a function form for $\ell(a)$. This leads to a contradiction because $\ell(a)$ is an exogenous function in this problem.

We now show there can only be one jump point $a_{jump} \in [a_L, a_U]$ if $e(a)$ is linear over $[a_L, a_U]$. Suppose there are two such jump points (the argument generalizes for more jump points) labeled a_{jump}^1 and a_{jump}^2 . Then, the single-peak property in Assumption 1 ensures there must be one jump point, say, a_{jump}^1 for which liquidity benefits $\ell(a)/a * A$ are larger than at a_{jump}^2 . Hence, to maximize its date-0 value, the platform would strictly prefer jumping to a_{jump}^1 from any point in $[a_L, a_U]$ rather than to a_{jump}^2 .

We are left to show that jumping to a_{jump}^1 instead of a_{jump}^2 is compatible with the equilibrium issuance policy. By Lemma 6 and 5, the issuance policy features jumps on $[a_L, a_U]$ only if equity value is linear and price is constant. Hence, from any state a with jump point a_{jump}^2 , we have

$$e(a) = [e(a_{jump}^2) + p(a_{jump}^2) - \varphi] \frac{a}{a_{jump}^2} = [e(a_{jump}^1) + p(a_{jump}^1) - \varphi] \frac{a}{a_{jump}^1}.$$

Hence, jumping to a_{jump}^1 is also an optimal equilibrium issuance policy. This equality simply reflects the fact that the platform is indifferent ex-post between all points in $[a_L, a_U]$. At date-0, however, the platform would choose jump point a_{jump}^1 as the sole jump point. \square

Proof of Lemma 6. We first show that if the equity value is strictly convex in C over some interval, the issuance policy is smooth in this region. Given any debt level \hat{C} , equity holders have the option to adjust the stock of stablecoins to C by issuing $C - \hat{C}$ at the price of $p(A, C)$. Therefore, by optimality of the debt issuance policy, the equity value at \hat{C} must satisfy

$$E(A, \hat{C}) \geq E(A, C) + p(A, C)(C - \hat{C}). \quad (\text{B.101})$$

To show that discrete repurchases are suboptimal, we prove that inequality (B.101) is strict if the equity value is strictly convex with respect to its second argument. Suppose to the contrary there exists $C' \neq C$ such that $E(A, C') = E(A, C) + p(A, C)(C - C')$. By strict convexity of E , we get that for all $x \in]0, 1[$

$$E(A, xC + (1 - x)C') < xE(A, C) + (1 - x)E(A, C') = E(A, C) + (1 - x)p(A, C)(C - C'). \quad (\text{B.102})$$

Using then condition (B.101) for $\hat{C} = xC + (1 - x)C'$, we obtain

$$E(A, xC + (1 - x)C') \geq E(A, C) + (1 - x)p(A, C)(C - C'), \quad (\text{B.103})$$

which is a contradiction with (B.102). Thus, it must be that

$$E(A, C') > E(A, C) + p(A, C)(C - C'). \quad (\text{B.104})$$

Hence, any discrete issuance with $|C - C'| > 0$ would be suboptimal for shareholders, that is, the debt policy must be smooth if E is strictly convex in C .

Second, we show that there cannot be an equilibrium with positive platform value and a smooth debt policy for all a . For the equilibrium issuance policy to be smooth it must be that equation (B.89) holds. The platform starts at date 0 if liquidity benefits can be captured in equilibrium. Two cases are possible given that p is weakly increasing with a . First, there exists an interval $[a_L, a_U]$ over which the price is constant with $p(a) = 1$. Equation (B.89) then implies that e is linear. We can then use Lemma 5 to show that the equilibrium debt policy features jump, a contradiction. The second case is that of a single point \hat{a} for which $p(\hat{a}) = 1$ and such that the platform spends strictly positive time at \hat{a} . Such a feature requires the platform to perform a control at \hat{a} . The same arguments used in DeMarzo and He (2021), however, show that such a policy cannot be part of an equilibrium in a region where the equity value is strictly convex.

□

Proof of Lemma 7. From Lemma 4, we know that since the equity value $e(a)$ is weakly convex, there must be a strictly ordered sequence $\{a^{(n)}\}_{n \geq 0}$ such that $a_0 = \underline{a}$ is the default threshold and $\lim_{n \rightarrow \infty} a^{(n)} = \infty$ such that on each segment $[a^{(n)}, a^{(n+1)}]$, e is either strictly convex or linear, with different convexity on two consecutive segments.

Our second step is to show that there is at least 1 segment with $e(a)$ strictly convex (possibly empty), and one segment with $e(a)$ linear. We first establish that the equity value cannot be linear on segment $[a^{(0)}, a^{(1)}]$ unless $a^{(0)} = 0$ and $\varphi = 1$. Suppose first $a^{(0)} > 0$ so that the platform may default in equilibrium. If $e(a)$ is linear over $[a^{(0)}, a^{(1)}]$, there is a kink in the equity value at $a^{(0)}$ such that $\lim_{a \downarrow a^{(0)}} e'(a) \neq 0$, which is incompatible with an optimal default decision and the corresponding smooth pasting condition. Suppose now $a^{(0)} = 0$ so that the platform never defaults in equilibrium. If $e(a)$ is linear on $[0, a^{(1)}]$,

there must be $a^{jump} \in [0, a^{(1)}]$ such that the issuance policy is to jump at a^{jump} from any point in $[0, a^{(1)}]$ by Lemma 5. This implies that for any $a \in [0, a^{(1)}]$

$$e(a) = [e(a^{jump}) + p(a^{jump}) - \varphi] \frac{a}{a^{jump}} - (p(a^{jump}) - \varphi),$$

with $p(a^{jump})$ constant over $[0, a^{(1)}]$ and $p(a^{jump}) > \varphi$ unless $\varphi = 1$. Hence, when $a \rightarrow 0$ limited liability is violated except in the case $\varphi = 1$. This proves the equity value is strictly convex over $[0, a^{(1)}]$ unless $\varphi = 1$ and $\underline{a} = 0$. In that case, the equilibrium equity value may be linear for all a .

Second, Lemma 6 implies there must exist a segment over which $e(a)$ is linear. The last step of the proof is to show there exists \bar{a} such that the equity value is strictly convex over $[\underline{a}, \bar{a}]$ and linear over $[\bar{a}, \infty)$. The characterization of the equilibrium issuance policy as a targeted Markov policy then follows from Lemma 4, 5, and 6. Let $\delta(a)$ be an interest policy that induces a non-zero MPE with issuance policy $d\mathcal{G}$ such that there exists a segment $[a^{(2)}, a^{(3)}]$ over which e is strictly convex. Call it the original (coupon) policy for short. We want to show that there exists an alternative coupon policy $\hat{\delta}(a)$ that induces a Markov equilibrium with issuance policy $d\hat{\mathcal{G}}$ such that $e(a)$ has the desired properties and the date-0 platform value is strictly higher.

We first construct an alternative policy and its induced equilibrium. Let a^* be the target value in the first linear region $[a^{(1)}, a^{(2)}]$ for equity in the equilibrium induced by the original policy. Construct the alternative policy and the induced equilibrium as follows. Set $\hat{\delta}(a) = \delta(a)$ for all a and $d\hat{\mathcal{G}}(a, C) = d\mathcal{G}(a, C)$ for $a \leq a^*$ and $d\hat{\mathcal{G}}(a, C) = A/a^* - C$ for $a \geq a^*$. Next, set the same default policy $\hat{\underline{a}} = \underline{a}$. Finally, conjecture that in the equilibrium induced by the alternative policy, the equity value $\hat{e}(a)$ is linear and the price $\hat{p}(a)$ is constant for all $a \in [a^{(1)}, \infty)$.

Next, we argue that the issuance policy $d\hat{\mathcal{G}}(a, C)$ and the default policy $\hat{\underline{a}}$ are equilibrium policies induced by the alternative coupon policy $\hat{\delta}(a)$. The subspace $[0, a^*]$ is absorbing for the equilibrium induced by the original policy because there are only downward jumps to A and the platform jumps to a^* from any $a \in [a^{(1)}, a^{(2)}]$. Hence, the fact that $d\mathcal{G}(a, C)$ for $a \in [0, a^{(2)}]$ is an equilibrium issuance policy induced by the original coupon policy implies that $d\hat{\mathcal{G}}(a, C)$ for $a \in [0, a^{(2)}]$ is an equilibrium issuance policy induced by the alternative coupon policy. The same argument applies to the default threshold $\hat{\underline{a}} = \underline{a}$. This argument also implies that $\hat{e}(a) = e(a)$ and $\hat{p}(a) = p(a)$ for all $a \in [0, a^*]$. We are thus left to show

that $d\hat{\mathcal{G}}(a, C)$ is an equilibrium issuance policy on the rest of the state space, $a \in [a^{(2)}, \infty)$. This result follows from the observation that $\hat{e}(a)$ is linear over $a \in [a^{(1)}, \infty)$ and $\hat{p}(a)$ is constant. This implies that jumping to any point in $a \in [a^{(1)}, \infty)$ including a^* can be part of an equilibrium issuance policy as we showed above.

Third, we show that $p(a) = 1$ for $a \in [a^{(1)}, a^{(2)}]$ in the equilibrium induced by the original policy, and thus $\hat{p}(a) = 1$ for all $a \in [a^{(1)}, \infty)$. Equity value is linear over $[a^{(1)}, a^{(2)}]$ and the equilibrium issuance policy is to jump at $a^* \in [a^{(1)}, a^{(2)}]$ when $a \in [a^{(1)}, a^{(2)}]$. Hence, the price $p(a) = p$ must be constant over $[a^{(1)}, a^{(2)}]$. Since $[0, a^*]$ is an absorbing subspace for the equilibrium induced by the original policy, it must be that $p = 1$. If not, investors never enjoy any liquidity benefit for $a \in [0, a^*]$ and thus $p(a) = e(a) = 0$ for all $a \in [0, a^*]$, which is a contradiction. To see this, suppose first $p < 1$. By monotonicity of p , we have $p(a) < 1$ for all $a \in [0, a^{(2)}]$ which implies investors never enjoy the liquidity benefit. Conversely, if $p > 1$ over $[a^{(1)}, a^{(2)}]$, we have $p(a) = 1$ for a unique $a \in [0, a^{(1)})$ because $p(a)$ is strictly increasing over $[0, a^{(1)})$ since $e(a)$ is strictly convex (see the proof of Lemma 6). With a smooth equilibrium issuance policy on $[0, a^{(1)}]$ this state is not visited with positive probability and thus investors enjoy liquidity benefit with zero probability, which again leads to a contradiction. Hence $p(a) = 1$ for $a \in [a^{(1)}, a^{(2)}]$. This implies $\hat{p}(a) = 1$ for all $a \in [a^{(1)}, \infty)$ in the equilibrium induced by the alternative policy.

Finally, we can show that the platform value at date 0 is higher under the alternative policy than under the original policy. The platform's value at date 0 is given by equation (10), which we rewrite here for convenience.

$$E_0 = \mathbb{E} \left[\int_0^\tau e^{-rt} \ell(A_t, C_t) C_t \mathbf{1}_{p(A_t, C_t)=1} + (\mu^k - r) \varphi C dt \middle| A_0, C_0 = 0 \right].$$

In any equilibrium, liquidity benefits are only enjoyed when $a \in [a^{(1)}, a^{(2)}]$ because $p(a) = 1$ for $a \in [a^{(1)}, a^{(2)}]$. Under the alternative policy $a^* \in [a^{(1)}, a^{(2)}]$ is reached immediately at date 0 by design because the equilibrium issuance policy is to jump to a^* when no stablecoins are outstanding ($a = \infty$). In the equilibrium induced by the original policy though, the optimal choice at date 0 is some $a^{**} > a^{(2)}$ by design of the original policy. Denote τ_f the first (stochastic) time the platform enters the region $[a^{(1)}, a^{(2)}]$ under the

original policy. We have

$$E_0 = \mathbb{E}[E^{-r\tau_f}] \hat{E}_0 + \mathbb{E} \left[\int_0^\infty e^{-rt} (\mu^k - r) \varphi C dt \right] < E_0$$

because no liquidity benefit is enjoyed before the platform reaches $[a^{(1)}, a^{(2)}]$. The inequality follows from the fact that $\mathbb{E}[\tau_f] > 0$ by design of the original policy and $\mu^k < r$.

We showed that the original policy is strictly dominated. Hence, in an equilibrium induced by an optimal coupon policy, the issuance policy must belong to the class of targeted Markov policies. \square

This concludes the proof of Proposition [Proposition 6](#).

B.9 Proof of Proposition [7](#)

Point 1. We derive the equilibrium stablecoin issuance rate in the smooth region $[\underline{a}, \bar{a}]$. Our analysis in the proof of Proposition [6](#) shows that a smooth debt issuance policy is optimal if and only if equation [\(B.89\)](#) holds. We will solve for the equilibrium value of g thanks to this equation. Taking the first-order derivative of e in equation [\(B.88\)](#) at $g = 0$, we obtain

$$(r + \lambda)e'(a) = \mu(e'(a) + ae''(a)) - \delta'(a)p(a) - p'(a)\delta(a) + \frac{\sigma^2}{2}a(2e''(a) + ae'''(a)) + \lambda\mathbb{E}[Se'(Sa)] \quad (\text{B.105})$$

The HJB for the stablecoin price is given by equation [\(B.92\)](#) with $\ell(a) = 0$ because the price p is strictly below one by construction in the smooth region. We can then use [\(B.89\)](#)

to obtain a condition on g . We have

$$0 = (r + \lambda)(p(a) - \varphi + e(a) - e'(a)a) \quad (\text{B.106})$$

$$= \delta(a)p(a) - (g(a) + \delta(a))p'(a)a + \mu ap'(a) + \frac{\sigma^2}{2}a^2p''(a) + \lambda\mathbb{E}[p(Sa)] - (r + \lambda)\varphi \quad (\text{B.107})$$

$$+ \mu^k\varphi - \delta(a)p(a) + \mu ae'(a) + \frac{\sigma^2}{2}a^2e''(a) + \lambda\mathbb{E}[e(Sa)] \quad (\text{B.108})$$

$$+ \delta(a)p'(a)a + \delta'(a)p(a)a - \mu a^2e''(a) - \mu ae'(a) - \frac{\sigma^2}{2}a^3e'''(a) - \sigma^2a^2e''(a) - \lambda\mathbb{E}[e'(Sa)Sa] \quad (\text{B.109})$$

$$= (\mu^k - r)\varphi - g(a)ap'(a) + \delta'(a)ap(a) + \mu a \underbrace{(p'(a) - ae''(a))}_{=0} + \frac{\sigma^2}{2}a^2 \underbrace{(p''(a) + e''(a) - ae'''(a))}_{=0} \quad (\text{B.110})$$

$$+ \lambda\mathbb{E}[\underbrace{p(Sa) - \varphi + e(Sa) - e'(Sa)Sa}_{=0}] \quad (\text{B.111})$$

$$= (\mu^k - r)\varphi - g(a)ap'(a) + \delta'(a)ap(a). \quad (\text{B.112})$$

To obtain the last equation, we used (B.89) to set the last term to 0. Differentiating equation (B.89) further shows

$$p'(a) = e''(a)a, \quad (\text{B.113})$$

$$p''(a) = e'''(a)a + e''(a), \quad (\text{B.114})$$

which allow us to set other terms to 0. This proves our claim.

Point 2 Next, we derive smooth-pasting condition at \underline{a} and \bar{a} . Consider first default threshold \underline{a} if $\underline{a} > 0$. Equation (7) shows that equity value in default is equal to 0 because the collateralization rate φ is lower than 1 by assumption. Hence, the default threshold is chosen optimally if and only if condition (31) holds. Consider now the lower bound of the target region \bar{a} . For $a \geq \bar{a}$, equity value is given by (B.120). Hence, we obtain for all $a = a/C \geq \bar{a}$

$$e'(a) = E_A(A, C) = \frac{e(a^*) + 1 - \varphi}{a^*} \quad (\text{B.115})$$

Hence, continuity of the derivative of e with respect to a at \bar{a} implies condition (30).

Point 3 Finally, we derive conditions to rule out ex-post deviations from the conjectured equilibrium policy in the target region $[\bar{a}, \infty)$. The conjectured issuance policy features a jump to a^* from any point in the target region. To derive conditions for this policy to be ex-post optimal, we consider “one-step” deviations whereby the platform deviates and then follows the equilibrium policy from the value of the demand ratio following the deviation.

We first show that we only need to consider smooth deviations. Proposition 6 shows that in the target region, the equilibrium equity value must be given by

$$E(A, C) = E(A, C^*(A)) + (p(a^*) - \varphi)(C^*(A) - C) \quad (\text{B.116})$$

with $p(A, C) = p(a^*)$ for any $a = A/C \geq \bar{a}$. The value when jumping to a ratio $\hat{a} \in [\bar{a}, \infty)$ is thus

$$\hat{E}(A, C) = E(A, \hat{C}(A)) + (p(a^*) - \varphi)(\hat{C}(A) - C) = E(A, C^*(A)) + (p(a^*) - \varphi)(C^*(A) - C), \quad (\text{B.117})$$

with $\hat{C}(A) \equiv A/\hat{a}$. Hence, from $a \in [\bar{a}, \infty)$, jumping to \hat{a} gives the platform the same utility as the equilibrium policy. The platform cannot gain from jumping to a different point of the target region because it then jumps instantaneously to target demand ratio a^* . Next, a jump to some ratio $\hat{a} \leq \bar{a}$ can be ruled out because the equity value is strictly convex in C for $a \in [\underline{a}, \bar{a}]$. The value from such a jump is indeed

$$\hat{E}(A, C) = E(A, \hat{C}(A)) + (p(\hat{a}) - \varphi)(\hat{C}(A) - C) < E(A, C) \quad (\text{B.118})$$

We are thus left to derive conditions such that for any $a \in [\bar{a}, \infty)$, deviating with a smooth issuance policy is suboptimal under condition (32). Given that the return to issuance is zero by construction, it is enough to check that equity owners prefer the equilibrium policy over inaction during time interval dt . For state (A, C) with $A/C \geq \bar{a}$, the equilibrium value of equity is given by (B.116). If instead equity owners stay inactive during time interval dt before reverting to the equilibrium policy, they enjoy

$$\begin{aligned} \hat{E}(A, C) = & \mu^k \varphi C dt - \varphi \delta(a) C dt + (1 - r dt)(1 - \lambda dt) \mathbb{E}[E(A + dA, C + \delta(a) C dt)] \\ & + (1 - r dt) \lambda dt \mathbb{E}[E(SA, C)] \end{aligned} \quad (\text{B.119})$$

When $a \in [\underline{a}, \infty)$, rewriting (B.116) the equilibrium equity value is given by

$$E(A, C) = \frac{A}{a^*} e(a^*) + (p(a^*) - \varphi)(C^*(A) - C) = \frac{e(a^*) + p(a^*) - \varphi}{a^*} A - (p(a^*) - \varphi)C \quad (\text{B.120})$$

Hence, we get

$$\mathbb{E}[E(A + dA, C + \delta(a)C dt)] = E(A, C) + \mu \frac{e(a^*) + p(a^*) - \varphi}{a^*} A dt - (p(a^*) - \varphi)\delta(a)C dt \quad (\text{B.121})$$

Plugging (B.121) into (B.119) and keeping only terms of order at least dt , we obtain

$$\begin{aligned} \hat{E}(A, C) = E(A, C) &- (r + \lambda)E(A, C)dt + \mu[e(a^*) + p(a^*) - \varphi]C^*(A)dt \\ &- p(a^*)\delta(a)C dt + \mu^k \varphi C dt + \lambda \mathbb{E}[E(SA, C)]dt \end{aligned} \quad (\text{B.122})$$

Equity owners do not deviate if and only if $\hat{E}(A, C) < E(A, C)$, that is, if

$$(r + \lambda)E(A, C) \geq \mu[e(a^*) + p(a^*) - \varphi]C^*(A) - p(a^*)\delta(a)C + \mu^k \varphi C + \lambda \mathbb{E}[E(SA, C)], \quad (\text{B.123})$$

which is equivalent to

$$(r + \lambda)[e(a^*)C^*(A) + (p(a^*) - \varphi)(C^*(A) - C)] \geq \mu[e(a^*) + p(a^*) - \varphi]C^*(A) - p(a^*)\delta(a)C + \mu^k \varphi C + \lambda \mathbb{E}[E(SA, C)]. \quad (\text{B.124})$$

where we used equation (B.120) to substitute for $E(A, C)$. Rearranging terms, (B.124) can be written

$$\begin{aligned} (r + \lambda - \mu)e(a^*)C^*(A) &\geq -(r + \lambda)(p(a^*) - \varphi)(C^*(A) - C) + \mu(p(a^*) - \varphi)C^*(A) \\ &- p(a^*)\delta(a)C + \mu^k \varphi C + \lambda \mathbb{E}[E(SA, C)]. \end{aligned} \quad (\text{B.125})$$

Using now equation (18) to substitute for $e(a^*)$, we get

$$\begin{aligned} \mu^k \varphi C^*(A) - p(a^*)\delta(a^*)C^*(A) + \mu(p(a^*) - \varphi)C^*(A) + \lambda \mathbb{E}[E(SA, C^*(A))] &- \lambda \mathbb{E}[E(SA, C)] \geq \\ - (r + \lambda)(p(a^*) - \varphi)(C^*(A) - C) + \mu(p(a^*) - \varphi)C^*(A) &- p(a^*)\delta(a)C + \mu^k \varphi C, \end{aligned} \quad (\text{B.126})$$

which we can finally rewrite as

$$\begin{aligned} \left[(r + \lambda)(p(a^*) - \varphi) + \mu^k \varphi \right] (C - C^*(A)) &\leq \lambda \mathbb{E}[E(SA, C^*(A))] - \lambda \mathbb{E}[E(SA, C)] \\ &\quad + p(a^*)(\delta(a)C - \delta(a^*)C^*(A)). \end{aligned} \quad (\text{B.127})$$

Setting $p(a^*) = 1$ and dividing all terms by C , (B.127) is equivalent to (32). This concludes the proof.

B.10 Equity and Price Characterization with Limited Commitment

We first verify our guess for the equity value and the price function. Using HJB equation (B.88) together with condition (33) and Assumption 4, we obtain the following HJB equation for the equity value

$$(r + \lambda)e(a) = (\mu^k - \underline{\delta})\varphi + \underline{\delta}(e(a) - ae'(a)) + \mu ae'(a) + \frac{\sigma^2}{2}a^2e''(a) + \lambda \mathbb{E}[e(Sa)]. \quad (\text{B.128})$$

Then, we compute the term $\mathbb{E}[e(Sa)]$ using the conjectured $e(a)$. We have

$$\begin{aligned} \mathbb{E}[e(Sa)] &= \int_0^\infty \left\{ e(e^{-s}a) \xi e^{-\xi s} \right\} ds = \int_0^{\ln(a/\underline{a})} \left[\underline{e} + \left\{ \sum_{k=1}^3 c_k e^{s\gamma_k} a^{-\gamma_k} \right\} \right] \xi e^{-\xi s} ds \\ &= \underline{e} \left(1 - \left(\frac{a}{\underline{a}} \right)^{-\xi} \right) + \sum_{k=1}^3 \frac{c_k \xi}{\xi - \gamma_k} a^{-\gamma_k} \left(1 - \left(\frac{a}{\underline{a}} \right)^{-(\xi - \gamma_k)} \right). \end{aligned} \quad (\text{B.129})$$

We then plug in guess (??) into the HJB to obtain

$$\begin{aligned} (r + \lambda - \underline{\delta}) \left[\underline{e} + \sum_{k=1}^3 c_k a^{-\gamma_k} \right] &= (\mu^k - \underline{\delta})\varphi - (\mu - \underline{\delta}) \sum_{k=1}^3 \gamma_k c_k a^{-\gamma_k} \\ &\quad + \frac{\sigma^2}{2} \sum_{k=1}^3 (1 + \gamma_k) \gamma_k c_k a^{-\gamma_k} + \lambda \mathbb{E}[e(Sa)]. \end{aligned} \quad (\text{B.130})$$

Several conditions are necessary for this equation to hold. Equating first constant terms on each side of (B.130), we can solve for \underline{e} :

$$\underline{e} = \frac{\mu^k - \underline{\delta}}{r - \underline{\delta}} \varphi. \quad (\text{B.131})$$

Next, as the terms in $a^{-\gamma_k}$ must be equal on each side of (B.130), γ_k must solve equation (41) for $k \in \{1, 2, 3\}$. The roots of that polynomial are given by

$$\gamma_k = -\frac{1}{2t_1} \left(t_2 + \zeta^k R + \frac{\Delta_0}{\zeta^k R} \right) \quad (\text{B.132})$$

where

$$\begin{aligned} \Delta_0 &= t_2^2 - 3t_1 t_3, \quad \Delta_1 = 2t_2^3 - 9t_1 t_2 t_3 + 27t_1^2 t_4, \\ R &= \sqrt[3]{\frac{\Delta_1 + \sqrt{\Delta_1^2 - 4\Delta_0^3}}{2}}, \quad \zeta = \frac{-1 + \sqrt{-3}}{2}, \quad k = \{0, 1, 2\}, \\ t_1 &= -\frac{\sigma^2}{2}, \quad t_2 = \mu - \underline{\delta} + \frac{\sigma^2}{2}(\xi - 1), \quad t_3 = -(\mu - \underline{\delta})\xi + \frac{\sigma^2}{2}\xi + r - \underline{\delta} + \lambda, \quad t_4 = -(r - \underline{\delta})\xi. \end{aligned}$$

where γ_k is decreasing with k . According to Descartes' rule of sign, this polynomial has 2 positive roots and 1 negative root if $\underline{\delta} < r$, 1 positive root and 1 negative root if $\underline{\delta} = r$, and 1 positive root and 2 negative roots if $r < \underline{\delta}$. We can show numerically that for $\{r, \sigma, \lambda, \xi, \mu, \underline{\delta}\} \in (0, 1) \times \mathbb{R}_+ \times (0, 1) \times \mathbb{R}_{>0} \times [-\infty, r + \lambda/(\xi + 1)] \times \mathbb{R}_+$, it always holds that $\gamma_2 > -1$, $\gamma_3 < -1$, and $\partial\gamma_3/\partial\underline{\delta} < 0$. (If $\lambda = 0$, it can be shown algebraically that only 1 root is strictly lower than 1 and the derivative of that root with respect to $\underline{\delta}$ is negative.)

To solve for the parameters c_k 's, we use the matching conditions imposed by continuity of $e(\cdot)$ at \underline{a} and \bar{a} and the memoryless property of the exponential distribution of downward jumps. Continuity at \underline{a} and \bar{a} imply that

$$\underline{e} + \sum_{k=1}^3 c_k \underline{a}^{-\gamma_k} = 0, \quad (\text{B.133})$$

$$\underline{e} + \sum_{k=1}^3 c_k \bar{a}^{-\gamma_k} = (e(a^*) + p(a^*) - \varphi) \frac{\bar{a}}{a^*} - (p(a^*) - \varphi). \quad (\text{B.134})$$

Next, the terms in $a^{-\xi}$ on each side of (B.128) must cancel out:

$$\underline{e} + \sum_{k=1}^3 \frac{c_k \xi}{\xi - \gamma_k} \underline{a}^{-\gamma_k} = 0, \quad (\text{B.135})$$

which is equivalent to

$$\mathbb{E}[e(S\underline{a})] = 0. \quad (\text{B.136})$$

That is, thanks to the memoryless property of the exponential distribution, we only need condition (B.135) to solve for the expected value of a downward jump below \underline{a} .

Finally, for \underline{a} and \bar{a} to be optimal, smooth pasting conditions (30) and (31) from Proposition 7 must be satisfied:

$$-\sum_{k=1}^3 c_k \gamma_k \bar{a}^{-(\gamma_k+1)} = \frac{e^* + 1 - \varphi}{a^*}, \quad (\text{B.137})$$

$$-\sum_{k=1}^3 c_k \gamma_k \underline{a}^{-(\gamma_k+1)} = 0. \quad (\text{B.138})$$

We proceed similarly for the price function. Using $\delta(a) = \underline{\delta}$ by Assumption 4 and plugging the equilibrium issuance equation (29) into the HJB for the price, equation (B.92), we obtain

$$(r + \lambda - \underline{\delta})p(a) = (r - \mu^k)\varphi + (\mu - \underline{\delta})ap'(a) + \frac{\sigma^2}{2}a^2p''(a) + \lambda\mathbb{E}[p(Sa)]. \quad (\text{B.139})$$

Then, we compute the term $\mathbb{E}[p(Sa)]$ using the conjectured $p(a)$. We have

$$\begin{aligned} \mathbb{E}[p(Sa)] &= \int_0^\infty \left\{ p(e^{-s}a) \xi e^{-\xi s} \right\} ds \\ &= \int_0^{\ln(a/\underline{a})} \left[\underline{p} + \left\{ \sum_{k=1}^3 b_k e^{s\gamma_k} a^{-\gamma_k} \right\} \right] \xi e^{-\xi s} ds + \int_{\ln(a/\underline{a})}^\infty \varphi \xi e^{-\xi s} ds \\ &= \underline{p} \left(1 - \left(\frac{a}{\underline{a}} \right)^{-\xi} \right) + \sum_{k=1}^3 \frac{b_k \xi}{\xi - \gamma_k} a^{-\gamma_k} \left(1 - \left(\frac{a}{\underline{a}} \right)^{-(\xi - \gamma_k)} \right) + \varphi \left(\frac{a}{\underline{a}} \right)^{-\xi}. \end{aligned} \quad (\text{B.140})$$

Equation (B.139) holds if the constant term \underline{p} in (40) solves

$$\underline{p} = \frac{r - \mu^k}{r - \underline{\delta}} \varphi, \quad (\text{B.141})$$

and if γ_k is a solution to equation (41) for $k \in \{1, 2, 3\}$. Next, the matching conditions

$p(\underline{a}) = \varphi$ and $p(\bar{a}) = 1$ imply respectively

$$\underline{p} + \sum_{k=1}^3 b_k \underline{a}^{-\gamma_k} = \varphi \quad (\text{B.142})$$

$$\underline{p} + \sum_{k=1}^3 b_k \bar{a}^{-\gamma_k} = 1 \quad (\text{B.143})$$

Finally, from the memoryless property of the exponential distribution, we get

$$\underline{p} + \sum_{k=1}^3 \frac{b_k}{\xi - \gamma_k} \underline{a}^{-\gamma_k} = \varphi. \quad (\text{B.144})$$

Next, we derive the platform's objective function as a function of parameters. From equation (18) and (19), we obtain

$$e(a^*) + 1 - \varphi = \frac{\ell(a^*) + (\mu^k - r)\varphi + \lambda \mathbb{E}[e(Sa^*) + p(Sa^*) - \varphi]}{r + \lambda - \mu}. \quad (\text{B.145})$$

Now, we solve for $\mathbb{E}[e(Sa^*)]$ and $\mathbb{E}[p(Sa^*)]$ using the functional forms (39) and (40). We have

$$\begin{aligned} \mathbb{E}[e(Sa^*)] &= \int_0^{\ln(a^*/\bar{a})} \left[(e(a^*) + 1 - \varphi)e^{-s} - (1 - \varphi) \right] \xi e^{-\xi s} ds \\ &\quad + \int_{\ln(a^*/\bar{a})}^{\ln(a^*/\underline{a})} \left[\underline{e} + \sum_{k=1}^3 c_k (a^*)^{-\gamma_k} e^{s\gamma_k} \right] \xi e^{-\xi s} ds \\ &= \frac{\xi}{\xi + 1} \left(1 - \left(\frac{a^*}{\bar{a}} \right)^{-(\xi+1)} \right) (e(a^*) + 1 - \varphi) - \left(1 - \left(\frac{a^*}{\bar{a}} \right)^{-\xi} \right) (1 - \varphi) + \left[\left(\frac{a^*}{\bar{a}} \right)^{-\xi} - \left(\frac{a^*}{\underline{a}} \right)^{-\xi} \right] \underline{e} \\ &\quad + \sum_{k=1}^3 \frac{\xi c_k}{\xi - \gamma_k} (a^*)^{-\gamma_k} \left[\left(\frac{a^*}{\bar{a}} \right)^{-(\xi-\gamma_k)} - \left(\frac{a^*}{\underline{a}} \right)^{-(\xi-\gamma_k)} \right]. \end{aligned} \quad (\text{B.146})$$

Turning now to the price term, we have

$$\begin{aligned}
\mathbb{E}[p(Sa^*)] &= \int_0^{\ln(a^*/\bar{a})} \xi e^{-\xi s} ds + \int_{\ln(a^*/\bar{a})}^{\ln(a^*/\underline{a})} \left[\underline{p} + \sum_{k=1}^3 b_k (a^*)^{-\gamma_k} e^{s\gamma_k} \right] \xi e^{-\xi s} ds + \int_{\ln(a^*/\underline{a})}^{\infty} \varphi \xi e^{-\xi s} ds \\
&= \left(1 - \left(\frac{a^*}{\bar{a}} \right)^{-\xi} \right) + \left[\left(\frac{a^*}{\bar{a}} \right)^{-\xi} - \left(\frac{a^*}{\underline{a}} \right)^{-\xi} \right] \underline{p} \\
&\quad + \sum_{k=1}^3 \frac{\xi b_k}{\xi - \gamma_k} (a^*)^{-\gamma_k} \left[\left(\frac{a^*}{\bar{a}} \right)^{-(\xi-\gamma_k)} - \left(\frac{a^*}{\underline{a}} \right)^{-(\xi-\gamma_k)} \right] + \varphi \left(\frac{a^*}{\underline{a}} \right)^{-\xi}. \quad (\text{B.147})
\end{aligned}$$

Using equations (B.146) and (B.147), we can thus express the platform's objective in (B.145) as a function of the parameters $\{\underline{a}, \bar{a}, a^*, \underline{\delta}\}$ of the TMP and the parameters of the functional forms for $e(a)$ and $p(a)$ in (39) and (40) which themselves depend on the TMP's parameters via equations (B.131), (B.133), (B.134), (B.135), (B.137), (B.138), (B.141), (B.142), (B.143), and (B.144).

B.11 Proof of Proposition 8

We first prove that the platform does not default, that is, $\underline{a} = 0$. From (B.131) the constant term \underline{e} in the equity value function (39) is equal to 0 and thus (weakly) positive for any value of $\underline{\delta}$. This implies that the option value to default has no value so the default threshold is $\underline{a} = 0$.

Second, the fact that $\underline{a} = 0$ implies that the only relevant root of characteristic equation (41) is the root strictly below -1 . Consider indeed the conditions on the equity value function. Equation (B.134) which imposes continuity at 0 implies that γ_k must be negative. Smooth-pasting condition (B.138) further implies that $\gamma_k < -1$. The same conclusion applies to the price function because (33) holds in the smooth region. For ease of notation, we now call γ this root and b and c the corresponding coefficients for the price function and the equity value function respectively.

We now restate continuity conditions (B.134) and (B.143) as well as smooth-pasting

conditions (B.137) at \bar{a} using the simplest function form we obtained above. We have

$$c\bar{a}^{-\gamma} = (e(a^*) + 1)\frac{\bar{a}}{a^*} - 1, \quad (\text{B.148})$$

$$-c\gamma\bar{a}^{-(\gamma-1)} = \frac{e(a^*) + 1}{a^*}, \quad (\text{B.149})$$

$$b\bar{a}^{-\gamma} = 1. \quad (\text{B.150})$$

Other conditions at \underline{a} for the equity value and the price are satisfied by construction as well as the memoryless property condition. Note also that because $e(a) = ca^{-\gamma}$ for $a \in [0, \bar{a}]$ and $e(a)$ increases with a for $a \in [\bar{a}, \infty)$, limited liability holds for all a if $e(\bar{a}) \geq 0$, which is implied by optimization constraint (43) in the statement of the proposition.

To obtain objective function (42) from (36), we derive $\mathbb{E}[e(Sa^*) + p(Sa^*)]$ using (B.146) and (B.147). Setting $\varphi = 0$ and using $\underline{a} = 0$ and $\underline{e} = 0$, we obtain

$$\begin{aligned} \mathbb{E}[e(Sa^*) + p(Sa^*)] &= \frac{\xi}{\xi+1} \left(1 - \left(\frac{a^*}{\bar{a}} \right)^{-(\xi+1)} \right) (e(a^*) + 1) - \left(1 - \left(\frac{a^*}{\bar{a}} \right)^{-\xi} \right) + \frac{\xi c \bar{a}^{-\gamma}}{\xi - \gamma} \left(\frac{a^*}{\bar{a}} \right)^{-\xi} \\ &\quad + \left(1 - \left(\frac{a^*}{\bar{a}} \right)^{-\xi} \right) + \frac{\xi b \bar{a}^{-\gamma}}{\xi - \gamma} \left(\frac{a^*}{\bar{a}} \right)^{-\xi} \\ &= \left[\frac{\xi}{\xi+1} \left(1 - \left(\frac{a^*}{\bar{a}} \right)^{-(\xi+1)} \right) + \frac{\xi}{\xi - \gamma} \left(\frac{a^*}{\bar{a}} \right)^{-(\xi+1)} \right] (e(a^*) + 1) \end{aligned} \quad (\text{B.151})$$

where to obtain the second line, we used conditions (B.148) and (B.150). Substituting (B.151) into (36), we obtain expression (42) for the platform's objective function. This expression shows that $(e(a^*) + 1)/(a^*)$ is a function only of variables $\{\bar{a}, a^*, \gamma\}$. Hence, equations (B.148) to (B.150) define b and c as a function of these parameters and leave one constraint for the optimization problem. Combining (B.148) and (B.149), we obtain (43). This concludes the proof.

B.12 Proof of Proposition 9

First, we prove the result about $\underline{\delta}$. We can write the Lagrangian of (42) as

$$\mathcal{L}(\underline{\delta}, \bar{a}, a^*, \theta) = \frac{e(a^*) + 1}{a^*} + \theta \left([e(a^*) + 1] \frac{\bar{a}}{a^*} - 1 + \frac{1}{1 + \gamma} \right). \quad (\text{B.152})$$

The partial derivatives are given by

$$\mathcal{L}_{\underline{\delta}} = (1 + \theta \bar{a}) \frac{\partial}{\partial \underline{\delta}} \left(\frac{e(a^*) + 1}{a^*} \right) - \frac{\theta}{(1 + \gamma)^2} \frac{\partial \gamma}{\partial \underline{\delta}}, \quad (\text{B.153})$$

$$\mathcal{L}_{\bar{a}} = \frac{\partial}{\partial \bar{a}} \left(\frac{e(a^*) + 1}{a^*} \right) + \theta \left(\frac{e(a^*) + 1}{a^*} \right), \quad (\text{B.154})$$

$$\mathcal{L}_{a^*} = (1 + \theta \bar{a}) \frac{\partial}{\partial a^*} \left(\frac{e(a^*) + 1}{a^*} \right), \quad (\text{B.155})$$

$$\mathcal{L}_{\theta} = [e(a^*) + 1] \frac{\bar{a}}{a^*} - 1 + \frac{1}{1 + \gamma}. \quad (\text{B.156})$$

Since

$$\frac{\partial}{\partial \bar{a}} \left(\frac{e(a^*) + 1}{a^*} \right) < 0, \quad (\text{B.157})$$

given $0 < \bar{a} < \infty$, $\mathcal{L}_{\bar{a}} = 0$ yields $\theta > 0$. We can show numerically that for $\{r, \sigma, \lambda, \xi, \mu, \underline{\delta}, \bar{a}, a^*\} \in (0, 1) \times \mathbb{R}_+ \times (0, 1) \times \mathbb{R}_{>0} \times [-\infty, r + \lambda/(\xi + 1)] \times \mathbb{R}_+ \times (0, a^*] \times \mathbb{R}_{>0}$, if $(e(a^*) + 1)/a^* > 0$, it always holds that $\mathcal{L}_{\underline{\delta}} > 0$.

Second, we prove the result about a^* . Given policy choice $\underline{\delta} = \infty$ such that $\gamma = -\infty$, the maximization problem of the platform given by (42) becomes

$$\max_{\bar{a}, a^*} \frac{\ell(a^*)/a^*}{r + \frac{\lambda}{\xi + 1} - \mu + \frac{\lambda \xi}{\xi + 1} \left(\frac{a^*}{\bar{a}} \right)^{-(\xi + 1)}} \quad (\text{B.158})$$

$$\text{subject to} \quad e(\bar{a}) \equiv [e(a^*) + 1] \frac{\bar{a}}{a^*} - 1 = 0. \quad (\text{B.159})$$

This problem is similar to that under full commitment, given by (22) except that the term $\lambda \xi / (\xi + 1) - \lambda \xi / (\xi - \gamma)$ at the denominator of (22) is replaced by $\lambda \xi / (\xi + 1)$ at the denominator of (42). This term loads negatively on a^* and it is larger in the limited commitment case, which implies a^* should be higher under limited commitment than under full commitment.

We now prove that the necessary condition for a platform to exist is tighter under limited commitment. The optimization problems under full commitment, given by (22) and that under no commitment, given by (B.158) can be nested under the following specification

$$\max_{\bar{a}, a^*} \frac{\ell(a^*)/a^*}{u + v(\gamma) \left(\frac{a^*}{\bar{a}}\right)^{-(\xi+1)}} \quad (\text{B.160})$$

$$\text{subject to} \quad e(\bar{a}) \equiv [e(a^*) + 1] \frac{\bar{a}}{a^*} - 1 = 0. \quad (\text{B.161})$$

with

$$u = r + \frac{\lambda}{\xi + 1} - \mu, \quad v(\gamma) = \frac{\lambda\xi}{\xi + 1} - \frac{\lambda\xi}{\xi - \gamma}$$

We have $\gamma = -\infty$ in the limited commitment case, as shown above, while $\gamma > -\infty$ is the solution to (21) in the full commitment case. We can thus use the analysis in the proof of Proposition 5 which shows that a necessary condition for existence of an uncollateralized platform is (B.87). Observe that $v(\gamma)$ is increasing with $\gamma < 0$. Hence, to show that the existence condition is tightest under limited commitment, we are left to show that the right-hand side of (B.87) increases with $v(\gamma)$. We have

$$\frac{u + v(\gamma) \min \left\{ 1, \frac{u}{v(\gamma)\xi} \right\}}{\min \left\{ 1, \frac{u}{v(\gamma)\xi} \right\}^{\frac{1}{\xi+1}}} = \begin{cases} u + v(\gamma) & \text{if } v(\gamma) \leq \frac{u}{\xi} \\ \frac{u(\xi+1)}{\xi} \left(\frac{v(\gamma)\xi}{u} \right)^{\frac{1}{\xi+1}} & \text{if } v(\gamma) \geq \frac{u}{\xi} \end{cases}. \quad (\text{B.162})$$

The desired result follows immediately from inspection of (B.162).

B.13 Proof of Proposition 11

First, note that in equilibrium, we need $p(a) \leq 1$. If that is not the case, then there is an arbitrage opportunity as the liquidation of a vault consider the value of debt at par value. Indeed, one could buy a vault priced at $v(a) = \varphi - p(a)$ and liquidate the vault to get $\varphi - 1$ units of collateral. Thus, in equilibrium it cannot be profitable for vault owners to default as $v(a) \geq \varphi - 1$. If $p_t < 1$, then default would incur a loss for vault owners and $\tau^i > t$. If $p_t = 1$, then $\varphi \in [0, 1]$ must be equal to 1 otherwise $v_t = \varphi - p_t$ is negative. Thus, $K_t^i - C_t^i = 0$. Therefore, $\int_i \min\{K_s^i - C_s^i, 0\} \mathbb{1}\{\tau^i = s\} di = 0$.

The objective function can then be written as

$$E_t = \max_{\tau, \delta, s, C} \mathbb{E}_t \left[\int_t^\tau e^{-r(s-t)} (s_s - \delta_s) p_s C_s ds \right] \quad (\text{B.163})$$

subject to the no-arbitrage condition (51) and the pricing function of a stablecoin (49).

Given that $\varphi \leq 1$, at default $V_t^i = 0$ and we can rewrite the value of a vault as

$$V_t^i = \max_{d\mathcal{G}^i} \mathbb{E}_t \left[\int_t^\tau e^{-r(s-t)} (p_s d\mathcal{G}_s^i - d\mathcal{M}_s^i) \right]. \quad (\text{B.164})$$

Without loss of generality, we solve for the equilibrium where vault owners never default as we showed that default can never be beneficial. For collateral to be such that $K^i = \varphi C^i$, we need $d\mathcal{M}^i = \varphi dC^i - \mu^k \varphi C^i dt$. Thus, the HJB for the value of a vault at a can be written as

$$v(a)C_-^i = \max_{d\mathcal{G}^i} \left\{ p(a)\mathbb{E}[d\mathcal{G}^i] - \mathbb{E}[d\mathcal{M}^i] + (1 - rdt)\mathbb{E}[v(a + da)(C_-^i + dC^i)] \right\}. \quad (\text{B.165})$$

Substituting for dC^i , we get

$$\begin{aligned} v(a)C_-^i = \max_{d\mathcal{G}^i} \left\{ p(a)\mathbb{E}[d\mathcal{G}^i] - \varphi \mathbb{E} \left[s(a)C_-^i dt + d\mathcal{G}^i - \mu^k C_-^i dt \right] \right. \\ \left. + (1 - rdt)\mathbb{E} \left[v(a + da)(C_-^i + s(a^*)C^i dt + d\mathcal{G}^i) \right] \right\}. \end{aligned} \quad (\text{B.166})$$

The first-order condition for $d\mathcal{G}^i$ is given by

$$p(a) - \varphi + v(a) = 0, \quad (\text{B.167})$$

which is always satisfied in equilibrium. Thus,

$$v(a)C_-^i = \varphi(\mu^k - s(a))C_-^i dt + (1 - rdt)\mathbb{E} \left[v(a + da)(C_-^i + s(a)C^i dt) \right]. \quad (\text{B.168})$$

Substituting for $v(a)$ using the first-order condition (B.167), we get

$$(\varphi - p(a))C_-^i = \varphi(\mu^k - s(a))C_-^i dt + (1 - rdt)\mathbb{E} \left[(\varphi - p(a + da))(C_-^i + s(a)C^i dt) \right]. \quad (\text{B.169})$$

Thus, we can solve for $s(a)$ that satisfies the no-arbitrage condition:

$$s(a) = r + \varphi \frac{\mu^k - r}{p(a)} - \mu^p(a) \quad (\text{B.170})$$

where $\mu^p(a) \equiv \mathbb{E}[dp(a)/(p(a)dt)]$. Similarly, we can write the HJB of the price as

$$p(a) = \ell(a)p(a)dt + \delta(a)p(a)dt + (1 - rdt)\mathbb{E}[p(a + da)]. \quad (\text{B.171})$$

Further algebra yields

$$rp(a) = \ell(a)p(a) + \delta(a)p(a) + \mu^p(a)p(a). \quad (\text{B.172})$$

Using equation (B.170) and (B.172), we get

$$(s(a) - \delta(a))p(a) = \ell(a)p(a) + \varphi(\mu^k - r). \quad (\text{B.173})$$

The maximization problem is then given by

$$E_t = \max_{\tau, s, \delta} \mathbb{E}_t \left[\int_0^\tau e^{-r(s-t)} \left(\ell_s p_s C_s + (\mu^k - r) \varphi C_s \right) ds \right] \quad (\text{B.174})$$

subject to $\varphi - p_t \geq 0$.

Because arbitrageurs are always indifferent in their issuance policy, equityholders can target a desired level of stablecoins $C^* = A/a^*$ by submitting a policy schedule:

$$s(a) = \begin{cases} s(a^*) + \varepsilon & \text{if } a < a^*, \\ s(a^*) & \text{if } a = a^*, \\ s(a^*) - \varepsilon & \text{if } a > a^*, \end{cases} \quad (\text{B.175})$$

where $\varepsilon > 0$, $s(a^*) = r + \varphi \frac{(\mu^k - r)}{p(a^*)} - \mu^p(a^*)$. The only equilibrium supply of stablecoin is then C^* . Maximizing $\ell_t p_t C_t + (\mu^k - r) \varphi_t$ results in setting δ_t such that $p_t = 1$. Thus, $\varphi^* = 1$ and C^* is given by

$$C^*(A) = \arg \max_C \{ \ell(A, C)C + (\mu^k - r)C \}. \quad (\text{B.176})$$

Because the problem is the same at every time t , all policies are constant over time.

The HJB for the value of equity at a^* is given by

$$E(a^*C_-, C_-) = (s(a^*) - \delta(a^*))p(a^*)C_-dt \quad (\text{B.177})$$

$$+ (1 - rdt)(1 - \lambda dt)\mathbb{E}[E(a^*C_- + dA, C_- + dC)|dN_t = 0] \quad (\text{B.178})$$

$$+ (1 - rdt)\lambda dt\mathbb{E}[E(Sa^*C_-, C_-)|dN_t = 1]. \quad (\text{B.179})$$

Given the policy (B.176), equityholders earn a constant spread per unit of stablecoin. Thus $E(A, C) = e(a^*)A/a^*$ and

$$e(a^*)C_- = (s(a^*) - \delta(a^*))p(a^*)C_-dt \quad (\text{B.180})$$

$$+ (1 - rdt)(1 - \lambda dt)e(a^*)C_-(1 + \mu dt) \quad (\text{B.181})$$

$$+ (1 - rdt)\lambda dt \left(e(a^*)C_- \frac{\xi}{\xi + 1} \right). \quad (\text{B.182})$$

Removing terms in dt and scaling by C_-dt , we have

$$\left(r + \frac{\lambda}{\xi + 1} - \mu \right) e(a^*) = (s(a^*) - \delta(a^*))p(a^*). \quad (\text{B.183})$$

Substituting for $s(a^*) - \delta(a^*)$ and $p(a^*)$, we get

$$e(a^*) = \frac{\ell(a^*) + \mu^k - r}{r + \frac{\lambda}{\xi + 1} - \mu}. \quad (\text{B.184})$$

C No Loss of Generality for Policies without Brownian Component

In this section, we show that considering a policy function $d\mathcal{G}_t = g_t C_t dt$ instead of a more general functional form $d\mathcal{G}_t = g_t C_t dt + \kappa_t C_t dZ_t$ is without loss of generality. We proof the case for the centralized uncollateralized protocol in the smooth region but the proof can be adapted to any case. The intuition of the results is straightforward: If fighting brownian shocks with κ_t has any expected impact on the value of equity, it will also be taken into account in the smooth issuance decision g_t and cancel out. With a stochastic term in $d\mathcal{G}_t$

we can write the value of equity in the smooth region as

$$E(A_t, C_t) = \mathbb{E}[p(A_t + dA_t, C_t + d\mathcal{G}_t)d\mathcal{G}_t] \quad (\text{C.185})$$

$$+ (1 - rdt - \lambda dt)\mathbb{E}[E(A_t + dA_t, C_t + d\mathcal{G}_t)] + (1 - rdt)\lambda dt\mathbb{E}[E(SA_t, C_t)]. \quad (\text{C.186})$$

Using Ito's lemma and the fact that terms in $dt dt$ converge to 0 faster than terms in dt , we can get

$$\mathbb{E}[p(A_t + dA_t, C_t + d\mathcal{G}_t)d\mathcal{G}_t] = \mathbb{E}\left[p(A_t, C_t)g_t C_t dt + \sigma A p_A(A_t, C_t)\kappa_t C_t dt + \kappa_t^2 C_t^2 p_C(A_t, C_t)dt\right] \quad (\text{C.187})$$

and

$$\mathbb{E}[E(A_t + dA_t, C_t + d\mathcal{G}_t)] = \mathbb{E}[E(A_t, C_t) + \mu A E_A(A_t, C_t)dt + g_t C_t E_C(A_t, C_t)dt] \quad (\text{C.188})$$

$$+ \frac{\sigma^2}{2} A_t^2 E_{AA}(A_t, C_t)dt + \frac{\kappa_t^2}{2} C_t^2 E_{CC}(A_t, C_t)dt + \sigma A_t \kappa_t C_t E_{AC}(A_t, C_t)dt] \quad (\text{C.189})$$

The first order condition for g_t is still given by

$$p(A, C) + E_C(A, C) = 0 \quad (\text{C.190})$$

while the first order condition for κ_t is given by

$$\sigma A p_A(A, C) + \kappa C p_C(A, C) + \kappa C E_{CC}(A, C) + \sigma A E_{AC}(A, C) = 0. \quad (\text{C.191})$$

As

$$p_A(A, C) + E_{AC}(A, C) = 0 \quad (\text{C.192})$$

and

$$p_C(A, C) + E_{CC}(A, C) = 0 \quad (\text{C.193})$$

the first order condition for κ_t is satisfied if and only if the first order condition for g_t is satisfied. The HJB for $p(A, C)$ becomes

$$(r + \lambda - \delta(A, C))p(A, C) = \mu A p_A(A, C) + (g(A, C) + \delta(A, C))C p_C(A, C) \quad (\text{C.194})$$

$$+ \frac{\sigma^2}{2} A^2 p_{AA}(A, C) + \frac{\kappa^2}{2} C^2 p_{CC}(A, C) + \sigma A \kappa C p_{AC}(A, C) + \lambda \mathbb{E}[p(SA, C)]. \quad (\text{C.195})$$

Given that $p(A/C) = p(A, C)$, we get

$$(r + \lambda - \delta(a))p(a) = \ell(a) + \mu a p'(a) - (g(a) + \delta(a))a p'(a) \quad (\text{C.196})$$

$$+ \frac{\sigma^2}{2} a^2 p''(a) + \frac{\kappa(a)^2}{2} (p''(a)a^2 + 2p'(a)a) - \sigma \kappa(a)(p'(a)a^2 + p'(a)a) + \lambda \mathbb{E}[p(Sa)]. \quad (\text{C.197})$$

Similarly,

$$e(a) = -\delta(a)p(a) + \mu a e'(a) + \frac{\sigma^2}{2} a^2 e''(a) + \lambda \mathbb{E}[e(Sa)] \quad (\text{C.198})$$

and

$$e'(a) = -\delta'(a)p(a) - \delta(a)p'(a) + \mu a e''(a) + \mu e'(a) + \frac{\sigma^2}{2} a^2 e'''(a) + \sigma^2 a e''(a) + \lambda \mathbb{E}[e'(Sa)]. \quad (\text{C.199})$$

Using the first order condition for $g(a)$ and its derivatives:

$$p(a) = -e(a) + e'(a)a, \quad (\text{C.200})$$

$$p'(a) = e''(a)a, \quad (\text{C.201})$$

$$p''(a) = e'''(a)a + e''(a), \quad (\text{C.202})$$

we get

$$0 = (r + \lambda)(p(a) + e(a) - e'(a)a), \quad (\text{C.203})$$

$$= \ell(a) + \delta(a)p(a) - (g(a) + \delta(a))ap'(a) + \mu ap'(a) + \frac{\sigma^2}{2}a^2p''(a) \quad (\text{C.204})$$

$$+ \frac{\kappa(a)^2}{2}(p''(a)a^2 + 2p'(a)a) - \sigma\kappa(p'(a)a^2 + p'(a)a) + \lambda\mathbb{E}[p(Sa)] \quad (\text{C.205})$$

$$- \delta(a)p(a) + \mu ae'(a) + \frac{\sigma^2}{2}a^2e''(a) + \lambda\mathbb{E}[e(Sa)] \quad (\text{C.206})$$

$$+ \delta'(a)ap(a) + \delta(a)p'(a)a - \mu a^2e''(a) - \mu ae'(a) - \frac{\sigma^2}{2}a^3e'''(a) - \sigma^2a^2e''(a) - \lambda\mathbb{E}[e'(Sa)a] \quad (\text{C.207})$$

$$= \ell(a) + \delta'(a)ap(a) - g(a)ap'(a) + \kappa(a)^2/2(p''(a)a^2 + 2p'(a)a) - \sigma\kappa(a)(p'(a)a^2 + p'(a)a). \quad (\text{C.208})$$

Thus, in the smooth part of the equilibrium, it must be that

$$g(a) = \frac{\ell(a) + \delta'(a)ap(a) + \kappa(a)^2/2(p''(a)a^2 + 2p'(a)a) - \sigma\kappa(a)(p'(a)a^2 + p'(a)a)}{ap'(a)}. \quad (\text{C.209})$$

Therefore, the HJB for $p(a)$ is given by

$$(r + \lambda)p(a) = \delta(a)p(a) - \delta'(a)ap(a) + \mu ap'(a) + \frac{\sigma^2}{2}a^2p''(a) + \lambda\mathbb{E}[p(Sa)] \quad (\text{C.210})$$

and none of the equilibrium price functions are affected by $\kappa(a)$.

D No Commitment

In the main text, we assume that a centralized platform has some commitment power with respect to the coupon policy and the collateralization rule. As claimed in Section ??, we show that the platform has no value if it cannot commit at all.

Lemma 8. *Without commitment, there is no MPE with strictly positive equity value $E(A, C, K) > 0$ and stablecoin price $p(A, C, K) > 0$.*

The problem of a platform without any commitment to policies is similar to that of

a firm that can choose whether or not to make coupon payments on perpetuity debt without defaulting. Once stablecoins/debt are issued, the firm strictly prefers not to make coupon payments because it already captured any benefits from issuance. As a result, the platform would always set the coupon payment to 0 ex-post, which means that stablecoin have no value ex-ante because the peg is not guaranteed. Lemma 8 thus shows that some commitment to a coupon policy is necessary; otherwise the platform and the stablecoin it issues have no value.

Proof of Lemma 8. Note that we have

$$dC_t = \delta_t C_t dt + G_t dt + (\mathcal{G}_t - \mathcal{G}_{t^-}) \quad (\text{D.211})$$

and

$$dK_t = \mu K_t dt + \sigma K_t dZ_t + M_t dt + K_{t^-} (S_t - 1) dN_t + (\mathcal{M}_t - \mathcal{M}_{t^-}). \quad (\text{D.212})$$

If $\mathcal{G}_t = \mathcal{G}_{t^-}$ and $\mathcal{M}_t = \mathcal{M}_{t^-}$ ²⁵, using Ito's lemma we get

$$(r + \lambda)E(A_t, C_{t^-}, K_{t^-}) = p(A_t, C_t, K_t)G_t - M_t + \mu A_t E_A(A_t, C_t, K_t) \quad (\text{D.214})$$

$$+ (G_t + \delta_t C_t)E_C(A_t, C_t, K_t) + (M_t + \mu K_t)E_K(A_t, C_t, K_t) \quad (\text{D.215})$$

$$+ \frac{\sigma^2}{2} A_t^2 E_{AA}(A_t, C_t, K_t) + \frac{\sigma^2}{2} K_t^2 E_{KK}(A_t, C_t, K_t) + \sigma^2 A_t K_t E_{AK}(A_t, C_t, K_t) \quad (\text{D.216})$$

$$+ \lambda \mathbb{E}[E(SA_t, C_t, SK_t)]. \quad (\text{D.217})$$

Therefore, if $E_C(A, C, K)$ is strictly negative, given a strategy $\delta(A, C)$, there is always an optimal deviation to a lower interest payment $\delta(A, C) - \Delta$ where $\Delta > 0$ until $\delta(A, C) = 0$. By Proposition I of [DeMarzo and He \(2021\)](#), $E(A, C, K)$ is strictly decreasing in C when $p(A, C, K) > 0$.

Similarly, without commitment to $K(A, C) = \varphi C$, it is always optimal to put no collateral in the platform as $\mu^k < \mu$ and $K(A, C) = 0$. \square

²⁵Otherwise, we get

$$E(A_t, C_{t^-}, K_{t^-}) = E(A_t, C_t + \mathcal{G}_t - \mathcal{G}_{t^-}, K_{t^-} + \mathcal{M}_t - \mathcal{M}_{t^-}) + p(A_t, C_t + \mathcal{G}_t - \mathcal{G}_{t^-}, K_{t^-} + \mathcal{M}_t - \mathcal{M}_{t^-})(\mathcal{G}_t - \mathcal{G}_{t^-}) - (\mathcal{M}_t - \mathcal{M}_{t^-}), \quad (\text{D.213})$$

which is not impacted by δ_t .