Firm Characteristics and Stock Price Levels: A Long-Term Discount Rate Perspective

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We study how firm characteristics are correlated with stock price *levels* by measuring the long-term discount rates (defined as the internal rate of return) of anomaly portfolios over a long horizon. Utilizing a simple novel non-parametric estimation methodology, which proxies ex-ante equity payout expectations with ex-post realizations, we reveal that the patterns of long-term discount rates are out-of-line with the average short-term holding period returns for multiple prominent anomalies. The set of stylized facts uncovered correspondingly shed new light on the mechanisms underlying various asset-pricing anomalies. Moreover, they indicate that long-term discount rates better characterize firms' equity financing cost than short-term expected returns; with a representative example, we demonstrate how structural models that posit a tight connection between the two could imply counterfactual patterns in price levels.

JEL Classification: G12

Key Words: Stock Price Level, Discount Rate, Asset Pricing Anomaly

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1 Introduction

Since the seminal work of Fama and French (1992, 1993), an extensive literature has been established to study asset pricing factors/anomalies[1], focusing on the relation between firm characteristics and expected short-term stock returns. The expected short-term return of a stock captures how the price tends to move shortly after portfolio formation, but fairly little is understood regarding what determines the *level* of stock prices. We aim to study this subject in our paper.

The standard protocol to investigate the relation between firm characteristics and equity financing cost is to sort stocks into portfolios along a certain dimension, rebalance them frequently, and then compare the average holding period return from one dynamically-rebalanced decile to another. However, despite its wide adoption, this procedure has significant limitations, because it only measures the short-term changes in stock prices one period following portfolio formation. Yet, the notion of equity financing cost should inherently concern the long run, as it is supposed to reflect the level of stock prices relative to the amount of all the cash flows that firms are expected to pay shareholders over a long horizon stretching into the distant future. Motivated by this observation, we develop a novel non-parametric methodology to estimate the long-term cash flow discount rates (defined below) of equity claims, and show that over-interpreting the performances of the dynamically-rebalanced portfolios can lead to misperceptions of firms' equity financing costs due to a disconnect between the short run and the long run – namely, there can be substantial differences between the cross-sectional patterns of the average short-term returns and the long-term cash flow discount rates, so that the former only represent the tendency of the immediate changes in stock prices, while the latter properly measure stock price levels and firms' overall equity financing costs.

To fix ideas, consider the well-known idiosyncratic volatility anomaly documented by Ang et al. (2006), whereby stocks with high idiosyncratic volatility tend to generate low returns shortly after portfolio formation. But we reveal that those stocks also face higher long-term discount rates, so that, contrary to conventional wisdom, even though they deliver lower returns in the short run, they are also cheaper in price levels. We will further show below that the idiosyncratic volatility anomaly is not the only example; in fact, such a disconnect between the short run and the long run is not uncommon among prominent anomalies.

Our objective is to compare equity financing costs across different firms, and thus the valuation levels of their equity claims. However, prices of these claims are not directly comparable, because their cash flows might have different patterns. To resolve this issue, we draw an analogy with

[1]In this paper, we use the term "anomaly" or "factor" to simply refer to the spread in the expected stock return along a certain firm characteristic. We do not take a stand on whether such a spread is caused by rational or behavioral forces. We thus use these two terms interchangeably.

the fixed-income literature: similar to quoting bond prices with the yield–to-maturity metric, we normalize stock prices by quoting them with the internal rate of return (IRR) over the long run:

$$P_0 = \sum_{t=1}^{\infty} \frac{\mathbb{E}_0\left(D_t\right)}{\left(1+y\right)^t} \approx \sum_{t=1}^{T} \frac{\mathbb{E}_0\left(D_t\right)}{\left(1+y\right)^t} + \frac{\mathbb{E}_0\left(P_T\right)}{\left(1+y\right)^T},$$

where P_0 (P_T) is the stock price observed at time 0 (T); $\mathbb{E}_0\left(D_t\right)$ is the expected total net payout of the equity claim occurring in time t; T represents a long horizon; and y is the internal rate of return.

Analogous to the yield-to-maturity measure of a bond claim, the long-horizon IRR is not a return that can be earned by investors, but a metric that quotes the price level of an equity claim. In other words, the long-horizon IRR does not translate into the actual performance that an investor can achieve, but it simply normalizes stock price by all expected future payouts, so that it can be directly compared across different equity claims.[2]

While equity discount rates can be different for different installments within the cash flow stream, the IRR is a non-linear weighted average across all payouts over different horizons and captures the overall equity financing cost of the firm.[3] To reflect the fact that the long-horizon IRR metric summarizes firm's cost of equity capital and to contrast it with the short-term holding period returns generated by dynamic trading strategies, we refer to this metric as the "long-term discount rate". Moreover, it is also worth noting that the long-term discount rate offers a model-free characterization of the equity financing costs for firms in the cross section. We focus on this metric as we intentionally aim to document the cross-sectional patterns firms' cost of equity without taking a stand on which specific asset pricing model to adopt.

Acknowledging that the long-term discount rate can be time-varying, in this paper we choose to focus on its time average; in parallel to the anomaly literature, we study how firm characteristics are correlated with the long-term discount rate in the cross section of stocks. In other words, we don't require the long-term discount rate to be constant in a dynamic setting. y is the *unconditional* average of discount rate in a richer setting where the discount rate is time-varying. And our paper is the long-term counterpart to the strand of the anomaly literature where the goal is to understand whether stocks with certain characteristics feature higher or lower *unconditional* premium in short-

[2]Another analogous metric is the implied volatility of an option contract. The implied volatility is not identical to the actual volatility of stock returns, but it is a metric that financial economists use to quote option prices so that options with different features can be compared.

[3]The long-term discount rate metric captures the average cost of capital across all projects and potential future investment opportunities of the existing firm. It does not necessarily equal to the cost of capital of a particular marginal investment opportunity. That said, the two would coincide if the marginal project has the same risk profile as the existing firm.

term returns than other stocks.

The challenge of estimating the long-term discount rate from the data is that cash flow expectations are not directly observable. The existing literature attempts to circumvent this issue by using either analysts' subjective forecasts as proxies for market expectations or by building structural models to predict cash flows. However, both approaches have limitations, as analysts' forecasts are known to be inaccurate and have limited coverage and models can be mis-specified.[4] To address these issues, we propose a simple non-parametric methodology that approximates cash flow expectations and estimates the long-term discount rate with realized corporate payouts and stock prices.

Our estimation starts by replacing the ex-ante expectations of the cash flows with their ex-post realizations and inferring the ex-post discount rate $\tilde{y}$:

$$P_0 = \sum_{t=1}^{T} \frac{D_t}{(1+\tilde{y})^t} + \frac{P_T}{(1+\tilde{y})^T}.$$

We then utilize the time series and measure $\tilde{y}$ repeatedly with portfolios formed at different points in time.[5] Naturally, the sample mean of these ex-post rates serve as an estimate of the true ex-ante rate y.

Alas, such an estimate is biased. For a given price, the true discount rate is a non-linear function of the cash flow expectations (i.e. $y = f\left(\{\mathbb{E}_0(D_t)\}_{t=1}^{T}, \mathbb{E}_0(P_T), P_0\right)$, where $f(\cdot)$ is the non-linear IRR function that uncovers y from current price and future cash flow expectations); and to precisely recover the discount rate, one needs to take the expectations of the cash flows inside the non-linear IRR function. However, by taking the average after inferring the rates from the realized cash flow paths, we essentially are taking the expectation outside of the IRR function (i.e. $\mathbb{E}_0(\tilde{y}) = \mathbb{E}_0\left[f\left(\{D_t\}_{t=1}^{T}, P_T, P_0\right)\right]$). The order of the expectation is changed, and as a result, a Jensen's term arises and biases the estimated rate. To address this issue, as the final step of the estimation procedure, we non-parametrically estimate the Jensen's term and correct for such a bias with Taylor approximations. Using simulations, we verify that our methodology indeed produces accurate estimates for our purposes.

We focus on a number of prominent anomalies; utilizing our methodology, we show how the

[4] It is well-documented that analysts' forecasts are overly optimistic and exhibit heterogenous biases for different stocks in the cross section. See, for example, Das, Levine, and Sivaramakrishnan (1998), Clement (1999), Lim (2001), Hong, Kubik, and Solomon (2000), Bradshaw, Richardson, and Sloan (2001), Hong and Kubik (2003), etc. Moreover, long-term growth forecasts, which are important for recovering the long-term discount rates, are only available for a small set of large firms.

[5] We apply our methodology to portfolios instead of individual stocks because we intend to study the discount rate over the long run and individual stocks might only exist for a short period in the sample.

pattern of the association between firm characteristics and the long-term discount rates can be substantially different from that of the average short-term holding period returns. For example, among the Fama-French five factors, instead of displaying a monotonic pattern across characteristics deciles, the long-term discount rates of the portfolios sorted along gross profitability and investment show a strong U-shape. In another interesting case, as briefly mentioned earlier, while Ang et al. (2006) document that high idiosyncratic volatility stocks tend to generate lower returns than low idiosyncratic volatility stocks in the short term; once we consider the long term, the pattern is inverted with the high idiosyncratic volatility stocks featuring significantly higher discount rates than the low idiosyncratic volatility stocks. Moreover, the estimation of the long-term discount rate also uncovers firm characteristics that are important for equity pricing but were previously overlooked from studies centered around short-term stock returns. For example, while average short-term holding period returns are almost flat across portfolios sorted by credit rating, as soon as we turn to the long term, we reveal that high rating firms face much lower long-term discount rates than low rating firms. Therefore, the equity market is much more integrated with the credit market than one previously would conclude by only focusing on the short-term holding period returns.

Our finding of the disconnect between the short term and the long term has important implications for a large class of structural asset pricing models that aim to reconcile anomalies with rational forces. Using Kogan and Papanikolaou (2013) as an example, we illustrate how this class of models interprets the spreads in the short-term expected returns as manifestations of the differences in firms' overall long-term discount rates or stock price levels, and why this can be problematic. By replicating their simulations, we first verify that the mechanism of their paper indeed generates realistic short-term premia for multiple anomalies. However, once we apply our methodology to the simulated environment, we reveal that the long-term discount rates implied by their model always take the same shape as the short-term expected returns, which is apparently counterfactual for several anomalies such as idiosyncratic volatility, gross profitability and investment. Therefore, this exercise highlights how drawing inferences about firms' equity financing costs from average short-term stock returns can be misleading when the patterns of the two diverge, with these models being hardwired to produce counterfactual patterns of the long-term discount rates.

Overall, from analyzing a host of anomalies, our empirical findings illustrate two important insights. First, the patterns of the long-term discount rates in the cross section can be very different from that of the average short-term holding period returns. It is not always the case that the spread in the long-term discount rates is even in the same direction as the short-term expected returns. Often times, the spread is inverted, or the shape in the long-term discount rates is non-monotonic. In these cases, simply assuming the short-term expected returns follow mean-reversion processes, as in Van Binsbergen and Opp (2019) for example, cannot reconcile the difference between the patterns in the long-term discount rates and the short-term expected returns. Second, even though the average short-term holding period return is an important metric that informs the profitability of a dynamically-rebalanced trading strategy, it can be misleading in representing the long-term equity financing cost of a firm. We argue that, compared with the short-term expected returns, the long-term discount rate or discount factor serve as more appropriate measures of firm's equity cost of capital.

Related Literature

Our paper belongs to a growing literature that studies stock price levels. Notable examples in the literature include Cohen, Polk, and Vuolteenaho (2009), Van Binsbergen and Opp (2019), Cho and Polk (2019) and van Binsbergen et al. (2021). Cohen, Polk, and Vuolteenaho (2009) and Cho and Polk (2019) propose testing asset pricing models with price levels instead of short-term returns, and argue that the CAPM cannot be rejected in price level tests. Van Binsbergen and Opp (2019) build a structural model to estimate the cost to the real economy due to asset price distortions with counterfactual analysis. By examining the price wedges of anomaly portfolios relative to rational benchmarks, van Binsbergen et al. (2021) classify asset pricing anomalies into the ones that resolve mispricing versus the ones that exacerbate mispricing. Compared to these earlier work, we present the novel finding that there can be substantial differences between the patterns of the long-term discount rates and that of the expected short-term returns, which cannot be generated by simply assuming mean reversion in factor premia as in Van Binsbergen and Opp (2019). Moreover, our paper adopts a novel model-free approach. Different from the aforementioned papers, we intentionally avoid building our empirical analysis based on a specific asset pricing model. And we focus on estimating the long IRR because it is a discount rate metric that intuitively summarizes firms' overall equity financing costs, as opposed to a test statistic targeting a specific model. In a way, our methodology is the cross-sectional counterpart of Shiller (1981). Shiller (1981) studies the time variations of stock prices by comparing the price levels of the stock market with the ex-post realized dividends. Our focus is on the cross section; we estimate the long-term discount rates of anomaly portfolios with realized cash flows, and document how stock price levels in the cross section are correlated with various firm characteristics. As a result, we produce multiple new stylized facts that could advance our understanding about the determinants of stock price levels.

Our paper is also related to a large literature in finance and accounting that aims to estimate cost of equity capital, including Botosan (1997), Fama and French (1997), Gebhardt, Lee, and Swaminathan (2001), Easton (2004), Easton and Monahan (2005), Ohlson and Juettner-Nauroth

(2005), Pástor, Sinha, and Swaminathan (2008), Richardson, Tuna, and Wysocki (2010), Hou, Van Dijk, and Zhang (2012), Lu (2016), Levi and Welch (2017) etc. The long-term discount rate studied in this paper can be regarded as the average cost of capital of a firm's equity claim. However, our paper differs from existing literature in a number of ways. First and foremost, we stress the distinction between the long-term cost of equity capital and the average short-term holding period return immediately after portfolio formation. In other words, we emphasize that the factor premia of dynamically-rebalanced trading strategies do not necessarily reflect the long-term equity financing costs of firms. Also, the existing implied cost of capital literature uses analyst subjective forecasts or model predictions as proxies for market expectations of future cash flows. Such approaches suffer from issues of cross-sectionally heterogenous analyst biases or model mis-specifications. We circumvent these issues by working with ex-post cash flow realizations and leveraging repeated observations to make inference about the properties of ex-ante cost of capital. Therefore, the wedge, that we identify, between the patterns of the long-term discount rates and the short-term expected returns is not attributable to imperfect specifications of cash flow expectations. In addition, the existing literature often computes cost of capital with cash flow projections over a short horizon of three to five years. Our methodology, on the other hand, takes advantage of the benefit of hindsight and utilizes long realized payout streams that stretch out by as far as 15 years.

In addition, our paper belongs to the cross-sectional asset pricing literature. We contribute to this literature by offering a new perspective. Structural models in this literature often regard the spread in the short-term expected returns of firms as a manifestation of the differences in the their equity cost of capital. Notable examples include Berk, Green, and Naik (1999), Gomes, Kogan, and Zhang (2003), Zhang (2005), etc. However, this assumed nexus between short-term expected returns and long-term discount rates has not been thoroughly studied and validated. In other words, the models make statements on long-term discount rates, yet researchers evaluate these models by conducting tests centered around short-term expected returns. We show empirically that this common approach is not always reliable. The cross-sectional patterns found in one do not always conform to the other. Moreover, our paper is closely related to a recent growing strand in this literature that studies the evolution of factor premia over a long horizon. For example, Liu, Moskowitz, and Stambaugh (2021) show that the CAPM is preferred to multi-factor models in explaining the returns of anomaly portfolios formed a decade ago. In a contemporaneous paper, Baba-Yara, Boons, and Tamoni (2020) show that the evolutions of factor prema are not tied to the dynamics of firm characteristics. We differ from these papers by revealing that the long-term discount rates in the cross section can take substantially different shapes as the average short-term stock returns, so that simply appealing to mean reversion in factor premia is insufficient to reconcile the wedge between the two.

Lastly, our paper is closely related to Keloharju, Linnainmaa, and Nyberg (2021), who study the relation between firm characteristics and the one-period expected stock return measured at a time far away from portfolio formation. Instead of focusing on the one-period effect years *after* portfolio formation, we complement their work by studying the cumulative effect in equity discount rates *during* the period from portfolio formation to the distant future. While Keloharju, Linnainmaa, and Nyberg (2021) find that the one-period expected stock return long after portfolio formation does not vary much across anomaly portfolios, we find significant heterogeneity in firms' discount rates *over* a long horizon, which have distinct patterns compared to the one-period expected return shortly after portfolio formation.

The remaining of the paper is organized as the following. Section 2 explains our empirical methodology in detail and presents the relevant derivations. Section 3 reports the empirical findings. Section 4 conducts a case study with the Kogan and Papanikolaou (2013) model to illustrate the implications of our findings for asset pricing theories. Section 5 concludes.

2 Empirical Method

2.1 Data

We obtain stock returns from CRSP, and our sample includes firms listed on NYSE, AMEX and NASDAQ with a share code of 10 or 11. We follow Boudoukh et al. (2007) to calculate the total net payout of common stock for each firm every year using Compustat data. Total net payout adds up dividend payments and share repurchases, and subtracts stock issuance.[6] Our sample is from 1970 to 2018. We start the sample from 1970 because this is when share repurchase and issuance data first became available in Compustat. The term structure of zero-coupon risk-free rates are constructed from the data downloaded from the federal reserve website[7]. All rates and cash flows are in nominal terms.

When a firm exits the market, we treat its delisting market cap as its last payout. Following Shumway (1997) and Shumway and Warther (1999), when the delisting return is missing from CRSP, we assign a delisting return of -35%(-55%) for NYSE and AMEX stocks (for NASDAQ stocks) if the delisting code is 500 or between 520 and 584, and zero otherwise.

[6]Following Boudoukh et al. (2007), we adopt Compustat item "DVC" as dividend payment. Share repurchase is measured as purchase of common and preferred stocks ("PRSTKC") plus the change in preferred stock redemption value (the difference between "PSTKRV" of the current fiscal year versus the previous fiscal year). Share issuance is measured as sale of common and preferred stock ("SSTK") minus the change in preferred stock redemption value. The total net payout is dividend plus share repurchase minus share issuance plus the change in preferred stock redemption value. All missing values are treated as zero.

[7]See https://www.federalreserve.gov/data/nominal-yield-curve.htm.

Following Cohen, Polk, and Vuolteenaho (2009) and Cho and Polk (2019), we construct a three-dimensional dataset to keep track of the performances and characteristics of anomaly portfolios. Each observation is identified with a triplet (c, t, i), where c denotes the cohort of the portfolio, i.e. the year of portfolio formation; t is the year of observation; i is the portfolio index. For example, $D^i_{c,t}$ refers to the total net payout in year t by the ith portfolio formed in year c. We assume all payouts occur at fiscal year ends.[8] As in Cohen, Polk, and Vuolteenaho (2009) and Cho and Polk (2019), all portfolios are not rebalanced. We keep track of the portfolios for 15 years, and construct a new cohort of value-weighted portfolios by the end of every year (month) for each annual (monthly) frequency anomaly. By the end of the 15-year period, we liquidate the portfolios and treat the liquidating market values of the portfolios as their last payouts. Notice that under such a construction, the market capitalization of a portfolio upon formation is commensurate to its subsequent net payout, i.e. the market capitalization is the price paid by all investors in the market for the equity claims issued by the constituent firms, whereas the net payout of the portfolio is the total amount of cash flows distributed by these firms to the same investors ex-post. Table 1 reports the list of well-known anomalies that we study in this paper, which includes the Fama-French 5 factors, momentum, idiosyncratic volatility, long-term reversal, etc.

2.2 Estimating the Long-Term Discount Rate

We define the long-term discount rate of an equity claim as the internal rate of return implied by its expected net payouts. An equity claim can, in theory, have an infinite horizon. For our empirical exercises, we follow the literature (Cohen, Polk, and Vuolteenaho (2009), Cho and Polk (2019), etc.) and truncate the horizon at 15 years, and treat the liquidating value of the portfolio by the end of the 15-year period as its last payout.[9] In Appendix C, we also show the key results with 10-year and 5-year horizons . Most of the results are qualitatively similar to the 15-year specification. We next define the long-term discount rate of an equity claim, y, with the following equality:

$$P_0 = \sum_{t=1}^{T} \frac{\mathbb{E}_0(D_t)}{(1+y)^t} + \frac{\mathbb{E}_0(P_T)}{(1+y)^T},$$
$$\Rightarrow y \equiv f\left(\{\mathbb{E}_0(D_t)\}_{t=1}^{T}, \mathbb{E}_0(P_T), P_0\right) \tag{1}$$

[8] Assuming that payouts occur at the middle of the fiscal year does not materially change results.

[9] Since we apply our methodology to anomaly portfolios consisting of many stocks, all our portfolios exist for at least 15 years. When a constituent stock disappears in the sample, we treat its delisting market cap as part of the portfolio payout in that period.

where T is the horizon of the cash flows, which is specified as 15 years; P_T is the liquidating value of the equity claim at T; D_t is the total net payout of the stock, which incorporates dividends, share repurchases and share issuance; $f(\cdot)$ is the internal rate of return function that uncovers y from expected cash flows and prices.

The challenge of estimating the long-term discount rate y from Equation (1) is that the expected payouts and liquidating value of a stock is not directly observable. The existing literature attempts to circumvent this problem by proxying market expectations with either analysts' subjective forecasts (Gebhardt, Lee, and Swaminathan (2001), Easton and Monahan (2005), Easton and Sommers (2007), Guay, Kothari, and Shu (2011)) or predictions from parametric models (Ohlson and Juettner-Nauroth (2005), Hou, Van Dijk, and Zhang (2012), Levi and Welch (2017)). However, both approaches have limitations, because they may suffer from issues of cross-sectionally heterogenous analyst biases or model mis-specifications.

We propose a new methodology to estimate y non-parametrically by replacing the ex-ante expectations with the ex-post realizations. Define the counterfactual price $\hat{P}_0$ as the present value of the realized cash flows:

$$\begin{aligned}\hat{P}_0 &\equiv \sum_{t=1}^{T} \frac{D_t}{(1+y)^t} + \frac{P_T}{(1+y)^T} \\ &= \sum_{t=1}^{T} \frac{\mathbb{E}_0(D_t) + \varepsilon_t}{(1+y)^t} + \frac{\mathbb{E}_0(P_T) + e_T}{(1+y)^T} \\ &= P_0 + \sum_{t=1}^{T} \frac{\varepsilon_t}{(1+y)^t} + \frac{e_T}{(1+y)^T} \\ &\equiv P_0 + \xi, \end{aligned} \tag{2}$$

where ε_t and e_T are the innovations in payouts and liquidating value, respectively. $\xi \equiv \sum_{t=1}^{T} \frac{\varepsilon_t}{(1+y)^t} + \frac{e_T}{(1+y)^T}$, is the aggregated innovation term, which has zero expectation: $\mathbb{E}_0(\xi) = 0$.

According to Equations (1) and (2), the discount rate y can be represented as either a non-linear function of the true price or the counterfactual price:

$$\begin{aligned} y &= f\left(\{\mathbb{E}_0(D_t)\}_{t=1}^{T}, \mathbb{E}_0(P_T), P_0\right) \\ &= f\left(\{D_t\}_{t=1}^{T}, P_T, \hat{P}_0\right) = f\left(\{D_t\}_{t=1}^{T}, P_T, P_0 + \xi\right), \end{aligned} \tag{3}$$

where $f(\cdot)$ is the internal rate of return function that uncovers y from cash flows and prices.

Equation (3) shows that y can be uncovered from either the true price and the expected cash flows, or the counterfactual price and the realized cash flows. And in the second representation, only the counterfactual price is not directly observable.

To illustrate the intuition of our methodology, let's take the first-order approximation of Equation (3) around P_0:

$$
\begin{aligned}
& y \approx f\left(\{D_t\}_{t=1}^{T}, P_T, P_0\right) + f_P\left(\{D_t\}_{t=1}^{T}, P_T, P_0\right)\xi \equiv f(\omega) + f_P(\omega)\xi \\
\Rightarrow\ & y \approx \mathbb{E}_0\left(f\left(\{D_t\}_{t=1}^{T}, P_T, P_0\right)\right) = \mathbb{E}_0\left(f(\omega)\right) = \mathbb{E}_0\left(\tilde{y}\right),
\end{aligned}
\tag{4}
$$

where, in order to ease notation, we use $\omega \equiv \left\{\{D_t\}_{t=1}^{T}, P_T, P_0\right\}$ to represent a cash flow trajectory; $\tilde{y} \equiv f\left(\{D_t\}_{t=1}^{T}, P_T, P_0\right) = f(\omega)$ is the ex-post discount rate inferred from the realized cash flows for a given trajectory. The second line above takes the expectation on both sides of the first line and utilizes $\mathbb{E}_0(\xi) = 0$ to eliminate the unobservable ξ.

Therefore, y and ξ can be approximated to the first order by:

$$
\hat{y}^{1st} = \hat{\mathbb{E}}\left(f\left(\left\{\{D_t\}_{t=1}^{T}, P_T, P_0\right\}^{j}\right)\right) = \hat{\mathbb{E}}\left(f\left(\omega^{j}\right)\right) = \frac{1}{J}\sum_{j=1}^{J}\tilde{y}^{j},
$$

$$
\xi^{j} = \frac{\hat{y}^{1st} - f\left(\omega^{j}\right)}{f_P\left(\omega^{j}\right)}. \tag{5}
$$

where $\hat{\mathbb{E}}(\cdot)$ denotes the sample mean; j indexes the trajectory of payouts and prices of a portfolio formed in period j; J is the total number of trajectories.

Intuitively, $\hat{y}^{1st}$ is the sample average of $f\left(\{D_t\}_{t=1}^{T}, P_T, P_0\right)$, which is the internal rate of return of the realized cash flows. Therefore, as a first step, one can approximate y by simply computing the internal rate of return of the realized cash flows for portfolios formed at different points in time and then take an average.

However, the first-order approximation is biased. In order to uncover the true discount rate, one needs to take expectations of the cash flows inside the IRR function ($y \equiv f\left(\{\mathbb{E}_0(D_t)\}_{t=1}^{T}, \mathbb{E}_0(P_T), P_0\right)$), but the first-order approximation essentially takes the expectation outside of the IRR function ($\mathbb{E}_0\left(f\left(\{D_t\}_{t=1}^{T}, P_T, P_0\right)\right)$). The order of expectation is changed, so that the first-order approximation differs from the true discount rate by a Jensen's term due to the nonlinearity of the IRR function. On the other hand, the Jensen's term also corresponds to the high-order terms omitted by the first-order approximation of Equation (4). Therefore, one can better approximate y and try to make up for the Jensen's term by expanding Equation (3) to higher orders:

$$
y = f(\omega) + f_P(\omega)\xi + \frac{1}{2}f_{PP}(\omega)\xi^{2} + \frac{1}{6}f_{PPP}(\omega)\xi^{3} + \cdots, \tag{6}
$$

or

$$
\begin{aligned}
y = & \mathbb{E}_0\left(f\left(\omega\right)\right) + \mathbb{E}_0\left(f_P\left(\omega\right)\xi\right) \\
& + \frac{1}{2}\mathbb{E}_0\left(f_{PP}\left(\omega\right)\xi^2\right) + \frac{1}{6}\mathbb{E}_0\left(f_{PPP}\left(\omega\right)\xi^3\right) + \cdots
\end{aligned} \tag{7}
$$

where $f_P\left(\cdot\right)$, $f_{PP}\left(\cdot\right)$ and $f_{PPP}\left(\cdot\right)$ denote the first, second and third partial derivative of $f\left(\cdot\right)$ with respect to P_0.

Motivated by Equation (7), we adopt the third-order approximation as the estimate of y:[10]

$$
\begin{aligned}
\hat{y} \equiv \hat{y}^{3rd} = & \hat{\mathbb{E}}\left(f\left(\omega^j\right)\right) + \hat{\mathbb{E}}\left(f_P\left(\omega^j\right)\tilde{\xi}^j\right) \\
& + \frac{1}{2}\hat{\mathbb{E}}\left(f_{PP}\left(\omega^j\right)\left(\tilde{\xi}^j\right)^2\right) + \frac{1}{6}\hat{\mathbb{E}}\left(f_{PPP}\left(\omega^j\right)\left(\tilde{\xi}^j\right)^3\right) \\
= & \hat{y}^{1st} + \frac{1}{2}\hat{\mathbb{E}}\left(f_{PP}\left(\omega^j\right)\left(\tilde{\xi}^j\right)^2\right) + \frac{1}{6}\hat{\mathbb{E}}\left(f_{PPP}\left(\omega^j\right)\left(\tilde{\xi}^j\right)^3\right),
\end{aligned}
$$

where $\hat{\mathbb{E}}\left(\cdot\right)$ denotes the sample mean; $\tilde{\xi}^j = \frac{\hat{y}^{1st} - f\left(\omega^j\right)}{f_P\left(\omega^j\right)}$ is from the first-order approximation of Equation (5).[11]

Notice that, even though we use ex-post realized cash flows to estimate the long-term discount rate, our estimation is not subject to look-ahead bias due to the aggregation over repeated observations. It is true that for a specific realization of the ex-post rate $\tilde{y}^j = f\left(\omega^j\right)$, which is the rate inferred from the jth path of the cash flows $\omega^j = \left\{\left\{D_t\right\}_{t=1}^T, P_T, P_0\right\}^j$, a positive (negative) unexpected component of ex-post realized cash flows would generate a downward (upward) bias in the ex-post rate $\tilde{y}^j$ relatively to the ex-ante rate y. However, as we average across all paths in the first-order approximation $\hat{y}^{1st} = \frac{1}{J}\sum_{j=1}^J \tilde{y}^j$, the unexpected component in each $\tilde{y}^j$ is averaged out asymptotically. Also notice that our estimation does not assume that investors have ex-ante knowledge about which firms would survive the 15-year horizon. The investors simply buy and hold a set of stocks with certain characteristics observed at the time of portfolio formation. Once a firm disappears from the portfolio (due to bankruptcy or privatization), they receive its delisting value as part of the payout of the portfolio in that period.

We evaluate the accuracy of our methodology using both bootstrap exercises and simulations

[10] Simulations show that approximating y to the fourth order and beyond does not improve the accuracy of estimation any more.

[11] One could estimate $\left\{\tilde{\xi}^j\right\}$ recursively in each iteration of the Taylor approximation, i.e. re-estimate $\tilde{\xi}^j$ when an estimate of y from a higher order approximation becomes available. But (untabulated) simulations show that the difference is negligible.

with structural models. We discuss the results in Section 2.3.

Once the long-term discount rate, $\hat{y}$, is estimated, we define its difference with the 15-year zero-coupon risk-free rate, r_f^{15}, as the long-term risk premium.

As explained earlier, by drawing an analogy with the fixed-income literature, we define the equity long-term discount rate as the IRR over a long horizon. Conceptually, IRR normalizes the price level of a portfolio by its expected future payouts. One might wonder how we compare the long-term IRR versus the long-term holding period return as a metric to represent firm's equity financing cost. We prefer long-term IRR over long-term holding period return in this context for the following two reasons. First, over a long horizon, the holding period return of a portfolio substantially depends on the reinvestment policy for its intermediate cash flow payouts (Van Binsbergen and Koijen (2010)). By using IRR, we avoid the issue of having to commit to a specific reinvestment strategy. Second, the long-term holding period return essentially weights the single-period returns equally in a path. On the other hand, different cash flows within a stream are presumably associated with different horizon-specific discount rates (Lettau and Wachter (2007), Gormsen and Lazarus (2019)). The long-term IRR is a non-linear weighted average of these horizon-dependent rates, and it assigns higher weights to the rates that are associated with more cash flows. Therefore, the long-term IRR is a more appropriate measure of firm's equity financing cost than the long-term holding period return.

Compared to the existing methods proposed in the implied cost of capital literature where cash flow expectations are proxied with analyst forecasts or model predictions, our new methodology has the advantage of being objective and non-parametric so that it does not suffer from analyst biases or model mis-specifications. Granted that the implied discount rates derived from analyst forecasts or parametric models are ex-ante measures so that they are useful for predicting future returns, the objective of our paper is to investigate the underlying mechanisms of asset pricing anomalies instead of return forecasting. Therefore, our discount rate estimates, albeit being available only ex-post, well serves our purposes. In addition, our exercise focuses on uncovering discount rate under the objective measure of future cash flow expectations. We do not take a stand on whether investors are able to form correct subjective expectations. Any potential biases in investors subjective beliefs about future cash flows could be a force generating the wedge between objective discount rates that we observe in the data versus the rates suggested by the asset pricing models that they employ (Porta et al. (1997)).

It is also worth noting that a stream of cash flows might, in general, feature multiple IRRs. However, the multiplicity of IRR is not of a concern in our study because our cash flows are from portfolio payout. In most cases, the realized payout of an anomaly portfolio is strictly positive, so that there is only a unique IRR. Indeed, out of a total of 54,950 cash flow paths in our sample, only

8 paths feature multiple IRRs because total issuance outweighs total dividends and repurchases so that the net payout becomes negative in a specific period. We drop these 8 paths in our study.

2.3 Accuracy of the Estimation Methodology

We provide two sets of analyses to evaluate the accuracy of our estimation methodology. The first approach measures the magnitude of the biases of our discount rate estimates with canonical structural cash flow processes. The second approach is model-free, whereby we non-parametrically evaluate the accuracy of our estimates with realized cash flows in the data using block bootstrap techniques. In the rest of the subsection, we show that these two approaches arrive at the same conclusion that our estimation methodology is sufficiently accurate to study the cross-sectional patterns of long-term discount rates.

2.3.1 The Structural Approach

We study a number of parametric cash flow processes in this section, and show that the biases of our estimations are sufficiently small across these models with parameters iterated over large grids.

We run simulations with the following 7 cash flow specifications: 1) the original Fama and Babiak (1968) process; 2) the adjusted Fama and Babiak (1968) process; 3) the original Leary and Michaely (2011) process; 4) the adjusted Leary and Michaely (2011) process; 5) the long-run risk process of Bansal and Yaron (2004); 6) the rare disaster process of Barro (2006); 7) the adjusted Berk, Green, and Naik (1999) process.

We study these specifications due to the following considerations. Fama and Babiak (1968) and Leary and Michaely (2011) propose processes that capture the co-integration between the earnings and the dividends of a firm. Their original models do not feature cash flow growth. We thus modify their models to settings with growing cash flows. Bansal and Yaron (2004) propose a specification where cash flows contain a persistent long-run component. Barro (2006) propose a setting where cash flows are subject to rare large drops. Berk, Green, and Naik (1999) specify and calibrate a model where firms are able to realize growth options by making new investments. We further simplify the risk adjustment and pricing mechanism of their model for our purposes. We choose these models because they capture different features of firms' payout processes that have been studied in the literature. The details of the model specifications and parameter choices are presented in Appendix A.

For each model, we iterate parameters across large grids. For each set of parameters, we measure the magnitude of estimation bias by taking the difference between the estimated discount rate and the true discount rate in the simulation. Table 2 reports the statistics of the biases across different models. The table shows that, in the most extreme case, the Barro (2006) process is associated with a bias of 46 bps, and the biases are much smaller than that in most cases. Across all models and all parameter values, the average bias in the simulation is 7 bps, the median is 3bps, and the 95th percentile is 29 bps. These biases are usually much smaller compared to the empirically-estimated spreads of discount rates across anomaly portfolios as will be presented in Section 3. Therefore, our methodology produces accurate long-term discount rate estimates in structural models.

2.3.2 The Model-Free Approach

In addition to the structural approach, we also estimate the biases non-parametrically with a block bootstrap exercise using realized cash flows in the data for each anomaly portfolio.

Figure 1 illustrates the block bootstrap procedure. For a given anomaly decile, we construct a trajectory by forming a portfolio by the end of each year or month, and tracking its cash flows for 15 years since formation. We then divide the trajectories into 5-year blocks, and randomly draw the blocks across trajectories of the same anomaly decile to form new simulated trajectories.

To preserve the evolution of portfolio characteristics, blocks are indexed by levels. Levels 1, 2 and 3 represent the first, second and third 5-year episodes since portfolio formation. Each block records the time series of capital appreciation $\left\{Rx_t \equiv \frac{P_t}{P_{t-1}}\right\}_t$ and net-payout-to-price ratio $\left\{DP_t \equiv \frac{D_t}{P_t}\right\}_t$ of the original portfolio. A simulated trajectory is constructed from one random block from each of the three levels respectively. When blocks are recombined into simulated trajectories, the time series of prices and cash flows are constructed from the capital appreciation and net-payout-to-price series within the blocks.

Once the cash flows of the simulated trajectories are constructed, their initial price P_0 is determined by discounting the average cash flows across the simulated trajectories with the discount rate estimate, $\hat{y}$, from the true trajectories. In other words, the estimated discount rate from the true trajectories acts as the true rate in the simulated environment and prices the simulated trajectories.

The block bootstrap procedure preserves the evolution of portfolio characteristics precisely within each 5-year block. The indexing of the blocks also attempts to capture portfolio evolution across 5-year episodes. Nonetheless, the breaking points at every 5-year-end when one block is stitched to another might introduce an inevitable wedge between the simulated trajectories and the true data generating process. To alleviate such a concern, Appendix B shows that such a wedge is small across the structural cash flow processes that we consider with parameters iterated over large grids.

To estimate the bias in $\hat{y}$, we take the difference between the discount rate estimated from the simulated trajectories and the true rate in the simulation. Table 3 reports the estimated biases for all the anomaly deciles. The table shows that the biases due to higher-order term truncation are in general small. In the worst case scenario, the high-minus-low spread of the cash-flow-duration-sorted portfolios has an estimated bias of -58 bps. The median absolute bias is much smaller and is about 21 bps.

Therefore, the model-free approach and the structural approach reach the same conclusion that the biases of our estimation methodology are sufficiently small in order to detect the patterns of long-term discount rates in the cross section of stocks.

2.4 Estimating the Standard Errors

Following the literature studying stock price levels, we keep track of our portfolios for 15 years, and construct a new cohort of portfolios by the end of every year or month. As a result, our sample features non-trivial overlap, and we have much fewer independent observations of long-term rates than short-term returns. Conceptually, though, the estimation of the long-term discount rates is not necessarily less accurate than the short-term expected returns. On the one hand, since our focus is on the long term, we naturally have fewer independent observations in the data. On the other hand, due to the amplification effect of cash flow duration, the long-term discount rates are also much less volatile than the short-term returns in the time-series.[12] Therefore, compared with the estimation of short-term expected returns, there are fewer observations to use in the estimation, but the long-term discount rates are also much less volatile than the short-term returns, which makes them easier to estimate.

To correctly compute the standard errors of our long-term discount rate and discount factor estimates, we again perform the block bootstrap procedure presented in Section 2.3.2. To estimate the standard error of $\hat{y}$, we construct batches of the simulated trajectories. Each simulated batch contains the same number of trajectories as the true sample, and yields a discount rate estimate $\hat{y}^b$. And the standard error of $\hat{y}$ is estimated as the standard deviation of the discount rate estimates across batches, i.e. $se(\hat{y}) = std\left(\hat{y}^b\right)$.

[12] It is well documented that the long-horizon stock returns, which are closely related to our long-term discount rates, are less volatile than the short-term returns. See, for example, Fama and French (1988), Poterba and Summers (1988), Lo and MacKinlay (1988), Richardson and Stock (1989), Kim, Nelson, and Startz (1991), Richardson (1993), Seigel (2020), etc.

3 Empirical Findings

Table 1 lists the set of prominent asset pricing anomalies that we study in this paper. We broadly classify these anomalies into three groups: 1) anomalies motivated by models of funding cost; 2) anomalies based on historical trading prices and volume; and 3) several additional anomalies. Our classification is not a systematic approach, but simply an arrangement to facilitate the presentation of our empirical findings.[13]

3.1 Anomalies Based on Funding Cost

We first consider a number of anomalies motivated by models of funding cost. They include the five factors summarized by Fama and French (2015), a credit rating factor that sorts stocks by firm's credit rating and the financial distress factor proposed by Campbell, Hilscher, and Szilagyi (2008). Table 4 presents our findings. Following the convention in the literature, we form 10 value-weighted portfolios for each anomaly, and calculate the difference between decile 10 and decile 1, i.e. HML. The indices of the portfolios increase with the sorting characteristic. In order to identify potential U-shape/hump-shape across portfolios, we also construct the extreme-minus-middle (EMM) portfolio, which is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5,6.

3.1.1 The Fama-French Three Factors

Panels (a), (b) and (c) of Table 4 show that the shapes of the long-term discount rates of the Fama-French three factors are consistent with the shapes of the short-term factor premia.

The Capital Asset Pricing Model (CAPM) predicts that the cost of capital of a stock should be linear and increasing with its market beta. Graham and Harvey (2001) also show survey evidence that most CFOs adopt the CAPM for capital budgeting. However, empirical studies generally find that the average holding period returns of stocks in the cross section are too insensitive to their market betas according to the CAPM predictions (Black et al. (1972), Baker, Bradley, and Wurgler (2011), Frazzini and Pedersen (2014)). Panel (a) of Table 4 shows that the point estimates of the long-term discount rates only slightly increase with market beta, and the high-minus-low spread is insignificant. Therefore, similar to the findings with the short-term expected returns, the long-term discount rate estimation does not generate a steep security market line.

[13] We thank Andrew Chen and Tom Zimmermann for making their anomaly replication code publicly available (Chen and Zimmermann (2022)). We borrowed some of their code in our exercise.

Banz (1981) and Bhandari (1988) first show that the market capitalization and the book-to-market ratio of a stock are significantly correlated with its expected return. The size and value factors are then included in the celebrated Fama and French (1992) three-factor model. Moreover, Berk, Green, and Naik (1999) propose a production-based asset pricing model and illustrate how size and book-to-market ratio can be connected to firm's cost of capital. Panels (b) and (c) of Table 4 confirm the large differences in the short-term average returns for stocks with different sizes and book-to-market ratios. The panels also show that there are large spreads in long-term discount rates along these two dimensions, consistent with the spread in short-term average holding period returns. Therefore, our exercise confirms that market capitalization and book-to-market ratio are significantly correlated with firm's equity financing cost.

3.1.2 Gross Profitability and Investment

Novy-Marx (2013) and Titman, Wei, and Xie (2004) show, respectively, that gross profitability and investment rate are correlated with the expected short-term stock returns in the cross section.[14] Fama and French (2015) incorporate these two findings into their five-factor model on the grounds that these two factors can be justified by a discount-cash-flow model and should be correlated with firm's equity financing cost.[15] Panels (d) and (e) show that, indeed, the average short-term holding period returns display a strong gradient along gross profitability and investment rate with the same sign as Novy-Marx (2013) and Titman, Wei, and Xie (2004). However, the estimation of long-term discount rates reveals interesting and surprising findings. The panels show that, despite the large spread in the average short-term holding period returns, these is little or no HML spread in the long-term discount rates across portfolios sorted by these two characteristics. Instead, these two sets of portfolios show strong U-shape in the long-term discount rates. Both the most profitable and least profitable firms tend to face higher equity financing costs than others. And the findings are similar for firms with extreme investment rates. These findings thus caste doubt on the rationale of explaining these two anomalies with a discount-cash-flow model as in Novy-Marx (2013) and Fama and French (2015).

Our exercise produces the novel finding that the long-term discount rates across anomaly portfolios can take an entirely different shape compared to the short-term expected returns. We do not attempt to reconcile the wedge between the shapes of the long-term rates and the short-term average returns in our study. We await future research for a complete reconciliation. That said, the

[14] Also see Ball and Brown (1968), Bernard and Thomas (1990), Fama and French (2006), Cooper, Gulen, and Schill (2008), Polk and Sapienza (2008).

[15] Motivated by the investment-based asset pricing literature, Hou, Xue, and Zhang (2015) also propose a four-factor model that incorporates investment and profitability effects.

wedge between the two can be explained by the following accounting identity:

$$
\begin{aligned}
\sum_{t=1}^{\infty} \frac{\mathbb{E}_0 (D_t)}{(1+y)^t} = P_0 &= \mathbb{E}_0 \left[\frac{D_0 + P_1}{(1+h_1)} \right] \\
&= \sum_{t=1}^{\infty} \mathbb{E}_0 \left[\frac{D_t}{\Pi_{s=1}^{t} (1+h_s)} \right] \\
&= \underbrace{\sum_{t=1}^{\infty} \frac{\mathbb{E}_0 (D_t)}{\Pi_{s=1}^{t} \left(1+\bar{h}_s\right)}}_{\text{Term Structure of Anomaly Premium}} + \underbrace{\sum_{t=1}^{\infty} \left\{ \mathbb{E}_0 \left[\frac{D_t}{\Pi_{s=1}^{t} (1+h_s)} \right] - \frac{\mathbb{E}_0 (D_t)}{\Pi_{s=1}^{t} \left(1+\bar{h}_s\right)} \right\}}_{\text{Co-movement between Payouts and Returns}},
\end{aligned} \tag{8}
$$

where $\bar{h}_s$ is the expected holding period return from period $s-1$ to s.

The last line of the accounting identity above offers a decomposition that speaks to the wedge between the long-term rates and the short-term expected returns. The first term suggests that the long-term discount rate, y, is not only related to the one-period anomaly premium following portfolio formation, $\bar{h}_1$, but it is also connected to the entire term structure of the one-period anomaly premium going forward. The second term is akin a Jensen's term, and it suggests that the long-term rate is also affected by the co-movement between the cash flow process and the return process. Therefore, the accounting identity suggests that the disconnect between the long-term discount rates and the short-term expected returns can be attributed to either term structure effect or an effect induced by the co-movement between the payout process and the return process.

In addition, since the long-term discount rates show a different shape compared to the average short-term holding period returns, our findings cannot be easily reconciled by the mean reversion of factor premia alone as the mechanism assumed in Van Binsbergen and Opp (2019). If the one-period risk premium simply follows a mean-reversion process tied to the characteristics as in Van Binsbergen and Opp (2019), then the accounting identity above suggests that the long-term discount rates in the cross section are likely to take the same shape, although with weakened magnitude, as the short-term expected returns, rather than displaying an entirely different pattern. Moreover, our discoveries strengthen the findings in a contemporaneous paper by Baba-Yara, Boons, and Tamoni (2020), where they show that the most recent observable characteristics of a firm are not sufficient statistics of its expected short-term return, and that there is a disconnect between the evolution of characteristic premia and the mean-reversion of the characteristics themselves. Our findings of the U-shape in the long-term discount rates for gross profitability and investment portfolios further suggest that the characteristic premia might not follow mean-reversion processes at all.

3.1.3 Credit Rating and Risk of Financial Distress

Voluminous previous research shows that a firm's credit rating is strongly correlated with its borrowing cost. High credit rating firms pay lower yields on their corporate bonds than low credit rating firms. However, there is only fairly little evidence regarding the relation between a firm's credit rating and its equity financing cost. As one of the few exceptions, Avramov et al. (2009) argue that, surprisingly, firms with high credit ratings also generate higher average short-term holding period returns. Panel (f) of Table 4 replicates their work by sorting stocks based firms' S&P credit ratings. We confirm that firms with high credit ratings generate slightly higher average holding period returns than the low credit rating firms during the first year since portfolio formation, although the difference is small and insignificant because we have a different and longer sample compared to Avramov et al. (2009). However, the investigation of the long-term discount rates, again, reveals a striking finding that, in contrast to the weak pattern in short-term average stock returns, there is a large spread along firm's credit rating. According to our estimates, the high-credit-rating firms face much lower equity financing costs than the low-credit-rating firms.

This exercise demonstrates how our methodology reveals important patterns in firms' equity financing costs or stock price levels that would have been overlooked with tests centered around the short-term holding period returns of dynamic trading strategies.

Campbell, Hilscher, and Szilagyi (2008) estimate firm-level probability of financial distress with a dynamic logit model and produce the puzzling finding that the average short-term holding period return in cross section decreases with firm's probability of distress. Panel (g) of Table 4 confirms their finding regarding the surprising gradient in the average short-term holding period returns – portfolios containing stocks with higher risks of financial distress tend to generate lower returns in the short term. However, the panel also shows that the long-term discount rates are almost flat across portfolios sorted by fail probability with a modest U-shape. Therefore, the panel shows that the Campbell, Hilscher, and Szilagyi (2008)'s distress risk anomaly is mostly a short-term phenomenon and it does not have surprising implications for firms' long-term equity financing costs.

3.2 Anomalies Based on Historical Trading Prices and Volume

Table 5 presents the findings regarding anomalies based on historical trading prices and volume. The panels in the table reveal an interesting pattern that low-frequency signals are more informative about long-term discount rates than high-frequency signals. The shape of the long-term discount rates is, again, not necessarily consistent with the shape of the average short-term holding period returns.

3.2.1 Short-Term Momentum and Long-Term Reversal

Jegadeesh and Titman (1993) show that stock return momentum is a salient anomaly that features large spread in the holding period returns of the stocks that have recently performed well compared to the stocks that have performed poorly. Panel (a) of Table 5 confirms the large momentum premium in our sample. The momentum anomaly has received significant amount of attention in the literature not only because of its striking magnitude, but also because it is difficult for structural models to generate a sizable momentum effect in firm's equity financing cost. Panel (a) shows that the long-term discount rates across momentum portfolios are almost flat. The shape is slightly non-monotonic with the discount rates of both the winning and losing stocks being higher than the stocks in the middle. The losing stocks also have a slightly higher a long-term discount rate than the winning stocks, by about 78 bps.

Again, our findings cannot be explained by the mean-reversion mechanism in the expected short-term holding period returns, but imply a reversal in momentum premium. Figure 2 plots the term structure of the momentum premium, i.e. the one-year average return difference between high and low momentum portfolios with respected to the years since portfolio formation. Consistent with the empirical findings regarding the reversal of momentum premium such as Lee and Swaminathan (2000) and Jegadeesh and Titman (2001), the figure shows that winning stocks generate significantly higher returns during the first year since portfolio formation than losing stocks, but they also generate significantly lower returns during the second year. Such a finding echoes the accounting identity of Equation (8) in the sense that it illustrates how the term structure in the expected holding period returns can drive a wedge between the shape of the long-term discount rates and the short-term expected returns. The almost flat long-term discount rates across momentum portfolios combined with the reversal in momentum premium is inconsistent with the mechanical channel proposed by Conrad and Kaul (1998) where the momentum effect arises mostly due to the cross-sectional heterogeneity in expected returns. Our findings are consistent with the overreaction channel proposed in behavioral models by Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), Hong and Stein (1999), etc, in that these models suggest that the momentum effect is caused by investors' temporary overreaction so that prices will eventually revert.

De Bondt and Thaler (1985) argue that the prices of stocks with poor performances over the most recent 5 years are too depressed compared to the long-term winning stocks and show that the long-term historical performance of a stock negatively predicts its short-term holding period return going forward. Panel (b) of Table 5 confirms their findings regarding the average short-term returns and also shows the presence of large spread in the long-term discount rates in the same direction as the short-term premium. Thus, the shape of the long-term discount rates is consistent with the behavioral channel proposed by De Bondt and Thaler (1985).

3.2.2 Idiosyncratic Volatility

Ang et al. (2006) present the striking finding that stocks with high idiosyncratic volatility tend to perform poorly compared to stocks with low idiosyncratic volatility. The premium of the idiosyncratic volatility anomaly is large and surprising as the more volatile stocks generate much lower returns the less volatile stocks. Panel (c) of Table 5 replicates their exercise and confirms that, indeed, high idiosyncratic volatility stocks tend to underperform significantly compared low idiosyncratic volatility stocks short after portfolio formation. However, the pattern in the long-term discount rates is opposite to the average short-term holding period returns. Our estimates show that firms with more volatile stocks face about 2% higher equity cash flow discount rates than firms with less volatile stocks.

Our finding of the opposite pattern between the short-term expected returns and the long-term discount rates across idiosyncratic volatility portfolios is consistent with Rachwalski and Wen (2016) where they show that: controlling for idiosyncratic volatility measured from long time ago, stocks with higher recent idiosyncratic volatility underperforms stocks with lower recent idiosyncratic volatility; on the other hand, controlling for recent idiosyncratic volatility, stocks with higher distant idiosyncratic volatility generate higher returns on average. Figure 3 presents an exercise similar to theirs and shows that, indeed, the idiosyncratic volatility premium tends to revert after 10 years since portfolio formation in our sample. Rachwalski and Wen (2016) reconciles the empirical findings by appealing to an under-reaction mechanism. According to their model, idiosyncratic volatility is associated with positive price of risk, so that the increase in idiosyncratic volatility of a stock is accompanied by a rise in risk premium and a drop in price. However, due to under-reaction, the price of such a stock responds to idiosyncratic volatility news with a lag, so that the price keeps falling after idiosyncratic volatility has risen, causing a temporary negative price pressure shortly after portfolio formation. Our finding of the positive association between idiosyncratic volatility and long-term discount rates supports their under-reaction channel. And we further complement their exercise by showing that the long-term positive price of risk associated with idiosyncratic volatility dominates the short-term negative price pressure so that overall idiosyncratic volatility is positively correlated with firms' equity financing costs.

3.2.3 Abnormal Volume

Gervais, Kaniel, and Mingelgrin (2001) find that stocks experiencing unusually high (low) trading volume over a day or a week tend to appreciate (depreciate) over the course of the following month. Panel (d) of Table 5 confirms their finding of the significant high-volume premium. While Gervais, Kaniel, and Mingelgrin (2001) show that a substantive high-volume-premium spread persists for at least 6 months, the panel shows the effect dissipates over longer horizons. The long-term discount rates across abnormal volume portfolios are almost flat; although the 46 bps spread between deciles 10 and 1 is statistically significant, the economic magnitude is small. Consequently, the impact on long-term equity financing costs is modest.

3.3 Additional Anomalies

3.3.1 Mispricing Score

Stambaugh, Yu, and Yuan (2015) propose the practice to aggregate stock characteristics across 11 different anomalies and construct a composite score (MISP) that captures market's mispricing of stocks.[16] They argue that stocks with high MISP scores are over-priced, so that they tend to generate lower returns following portfolio formation. Panel (a) of Table 6 confirms their finding. Indeed, their mispricing score significantly predicts short-term stock returns. The decile with the highest score on average underperforms the lowest decile by 13% per year with a t-statistic of 5. However, the panel also shows that such a pattern is flipped as soon as we turn to the long term. Interestingly, contrary to the findings regarding the short-term returns, the stocks with high mispricing scores face relatively higher long-term discount rates than the stocks with low scores. Therefore, the panel reals that the stocks with high MISP scores are only over-priced in the short term; over the long run, these stocks actually have cheaper price levels relative to their expected payouts.

3.3.2 Liquidity

One important theme in the asset pricing literature is to study the relation between liquidity and the expected return of a stock. For example, Amihud (2002) finds that less liquid stocks are associated with higher returns on average.[17] On the other hand, the investigation of the role of liquidity has been focused on its relation with short-term stock returns, but less is understood regarding its

[16] Specifically, Stambaugh, Yu, and Yuan (2015) calculate the ranking percentiles of the characteristics associated with the anomalies including: financial distress, net stock issues, composite equity issues, total accruals, net operating assets, momentum, gross profit-to-assets, asset growth, return-on-assets (ROA), and investment-to-assets. Then the mispricing score (MISP) is constructed by taking the arithmetic average of the percentile rankings across these 11 anomalies.

[17] Also see Amihud and Mendelson (1986), Chordia, Subrahmanyam, and Anshuman (2001), Acharya and Pedersen (2005), Liu (2006), and others.

influence on valuation levels. Panel (b) of Table 6 shows that both the short-term expected return and the long-term discount rate are positively correlated with Amihud (2002)'s illiquidity measure, and the spread between the most liquid and most illiquid deciles are large for both. The finding is consistent with the intuition that, all else equal, investors would be willing to pay a higher price for a stock that can be more easily traded. Such a liquid stock would then feature a higher price level and deliver lower expected return.

In addition to the level of liquidity, the literature has also studied how a stock's exposure to systematic liquidity risk would affect its expected return. For example, Pástor and Stambaugh (2003) show that stocks with better performances when aggregate liquidity dries up tend to deliver lower returns on average. They explain the finding by suggesting that investors are willing to pay a premium for a stock that hedges the aggregate liquidity risk. Panel (c) of Table 6 replicates their key finding and shows that the liquidity beta proposed in Pástor and Stambaugh (2003) is indeed positively correlated with short-term stock returns.[18] However, the panel also shows that liquidity risk is only associated with short-term returns, but it does not seem to affect price levels and long-term discount rates.

3.3.3 Return Coskewness

Motivated by the weak empirical performance of the CAPM, researchers have expanded the framework of mean-variance analysis and investigated the relation between the expected return of a stock and its higher moments. For instance, Harvey and Siddique (2000) show that the coskewness between a stock and the market is significantly correlated with its average short-term returns.[19] The intuition is that investors have to be compensated for holding a stock whose performance deteriorates during periods of severe market downturns. Numerous subsequent papers provide additional findings that embody similar intuitions, including: Ang and Chen (2002), Dittmar (2002), Ang, Chen, and Xing (2006), Chang, Christoffersen, and Jacobs (2013), Conrad, Dittmar, and Ghysels (2013), Farago and Tédongap (2018), Bollerslev, Li, and Zhao (2020), and others. Therefore, it is well-established that the systematic tail risk of a stock is significantly correlated with its one-period expected return. But how does such coskewness affect the price levels of stocks?

Panel (d) of Table 6 replicates Harvey and Siddique (2000) and extends the result to a longer sample. Consistent with Harvey and Siddique (2000), there is a large spread of 5.18% in the expected returns across stocks sorted by coskewness.[20] However, the pattern in long-term discount

[18] Li et al. (2019) and Pontiff, Singla et al. (2019) also verified the liquidity risk premium documented in Pástor and Stambaugh (2003), although they also raised skepticism regarding the robustness of the paper's specification.

[19] Also see Kraus and Litzenberger (1976) and Friend and Westerfield (1980) for early work in this direction.

[20] Following the convention of Harvey and Siddique (2000), stocks in lower deciles are the ones with more negative coskewness and are associated with higher expected returns.

rates is almost flat across coskewness portfolios. The finding shows that coskewness with the market does not seem to be significantly correlated with the price level of a stock. It is thus important to understand how other measures of higher moments of stock return are related to price levels. We leave this intriguing question to future research.

3.3.4 Cash Flow Duration

By studying the pricing of derivatives contracts, Van Binsbergen, Brandt, and Koijen (2012); Van Binsbergen and Koijen (2017) show that the term structure of equity yields is downward-sloping for the entire market, i.e. dividend strips of S&P 500 with shorter duration are discounted with a higher rate on average compared to long-duration strips. In the meantime, a parallel research agenda aims to apply the same logic to individual stocks in the cross section (Dechow, Sloan, and Soliman (2004), Lettau and Wachter (2007), Lettau and Wachter (2011), Chen and Li (2018), Weber (2018), Gormsen and Lazarus (2019), Gonçalves (2021)). The literature finds that, using various proxies for individual stock cash flow duration, stocks with shorter duration tend to deliver higher returns on average. Such a finding again connects cash flow duration to short-term expected stock returns, but do stocks with shorter duration really have lower price levels as implied by a downward-sloping equity yield curve?

Panel (e) of Table 6 sorts stocks according to the cash flow duration measure proposed in Gonçalves (2021) and shows that, indeed, stocks with shorter duration are associated with both higher expected returns in the short term and higher long-term discount rates.[21] The findings thus complements the existing literature and more directly verifies the link between cash flow duration and price levels.

3.4 Out-of-Sample Performances

We decide to start our main sample from 1970 due to the availability of accounting variables, which are needed to construct most of the anomalies. That said, for a small set of anomalies that are based exclusively on prices or trading volume (momentum, long-term reversal, idiosyncratic volatility, abnormal volume, market beta and size), we are able to extend the sample back to 1926. We use the early data (1926-1970) to perform an out-of-sample test with these anomalies.

Table 7 presents the results, where for convenient comparisons we also reiterate the relevant results from the main sample (1970-2018). For momentum and long-term reversal, average short-term returns and long-term discount rates patterns are similar in both samples. For idiosyn-

[21] Gonçalves (2021) argues that his duration measure constructed with VAR improves upon the measure used in Dechow, Sloan, and Soliman (2004) and Weber (2018).

cratic volatility, the short-term premium disappears in the early sample, yet the long-term discount rates show the same pattern in both samples. For abnormal volume, the short-term premium is stronger in the early sample, but the spread in the long-term discount rates becomes weaker and insignificant. For market beta, the spread in the average short-term returns is insignificant for both samples; the long-term discount rates spread becomes significant in the early sample. Lastly, the size anomaly has a similar short-term premium for both samples, but the spread in the long-term discount rates is insignificant in the early sample. For all of these anomalies, the point estimates of the spread in the long-term discount rates always have the same sign in both samples. Overall, these findings suggest that the long-term discount rates of this set of anomalies generally display similar patterns in the early and main samples.

3.5 Validation with the Relative Discount Factor

In order to validate our estimates of the long-term discount rate as well as the cross-sectional patterns of firms' equity financing costs that we document, we propose and estimate the "relative discount factor" (or *RDF*) defined as the following:

$$RDF \equiv \frac{P_0}{\sum_{t=1}^{T} \frac{\mathbb{E}_0(D_t)}{\left(1+r_{f,0,t}\right)^t} + \frac{\mathbb{E}_0(P_T)}{\left(1+r_{f,0,T}\right)^T}},$$

where $r_{f,0,t}$ is the t-year zero-coupon risk-free rate observed at time 0 from the term structure of interest rates.

The relative discount factor is a ratio between two prices: the actual price of the stock and its counterfactual price if cash flows were priced with the risk-free rates. Therefore, the relative discount factor measures the difference between the equity financing cost of a firm and the borrowing cost of the US government when issuing a default-free security with the same expected cash flows. As a complement to the long-term discount rate, it is an alternative measure for summarizing firms' equity financing costs.

Since the relative discount factor is based on cash flow expectations, its empirical counterpart can be easily estimated with sample average:

$$\frac{1}{RDF} = \mathbb{E}_0 \left(\frac{\sum_{t=1}^{T} \frac{D_t}{\left(1+r_{f,0,t}\right)^t} + \frac{P_T}{\left(1+r_{f,0,T}\right)^T}}{P_0} \right)$$

$$\widehat{\left(\frac{1}{RDF}\right)} = \hat{\mathbb{E}} \left(\frac{\sum_{t=1}^{T} \frac{D_t}{\left(1+r_{f,0,t}\right)^t} + \frac{P_T}{\left(1+r_{f,0,T}\right)^T}}{P_0} \right)$$

$$\widehat{RDF} = 1/\widehat{\left(\frac{1}{RDF}\right)}.$$

When sample size increases, the sample mean $\hat{\mathbb{E}}(\cdot)$ converges to the expectation $\mathbb{E}_0(\cdot)$, so that the estimate of the relative discount factor is consistent. The standard error of the relative discount factor is estimated with a block bootstrap procedure similar to the long-term discount rate, as described in Section 2.4.

Appendix D compares our estimates of the long-term discount rate versus the relative discount factor across various anomaly portfolios. The tables show that the cross-sectional patterns in the long-term discount rate is almost always consistent with the relative discount factor in the sense that a higher discount rate is paired with a lower discount factor. Therefore, the estimation of the relative discount factor further strengthens the robustness of our estimates of the long-term discount rate.

4 Implications for Asset Pricing Theories, A Case Study with Kogan and Papanikolaou (2013)

To illustrate the implications of our empirical exercise for asset pricing theories, we conduct a case study with Kogan and Papanikolaou (2013), which is representative of a large class of structural models, by evaluating their production-based asset pricing model with our methodology.

Kogan and Papanikolaou (2013) propose a relatively simple channel that is able to generate *quantitatively* realistic short-term expected returns for multiple anomalies, including: idiosyncratic volatility, investment rate, gross profitability, etc.[22] The mechanism of their model can be summarized as the following. According to their theory, the value of a firm has two components – the

[22] Kogan and Papanikolaou (2013) is also able to match patterns along Tobin's Q, earnings-to-price ratio and market beta. We only focus on these three anomalies because we identified inconsistent patterns between short-term expected returns and the long-term discount rates in the data.

value of asset in place (VAP_f) and the present value of growth opportunities ($PVGO_f$):

$$V_f = VAP_f + PVGO_f.$$

The value of asset in place is the present value of the cash flows generated by all existing projects that have already been installed in the firm; and the present value of growth opportunities reflects the total NPV of the investment opportunities of the firm, which is the present value of all cash flows coming from potential future projects that have not yet been adopted by the firm.

Kogan and Papanikolaou (2013) argue that with the existence of investment-specific-technology shocks, the cash flows generated by these two components might have different risk profiles and would be discounted with different rates. Since the firm is a portfolio of asset in place and growth opportunities, the overall discount rate of the firm is thus determined by the ratio between $PVGO_f$ and VAP_f. They further show that, under certain assumptions, the ratio between $PVGO_f$ and VAP_f is monotonically related to various observable characteristics.

Therefore, in their model, firm characteristics reflect the ratio between $PVGO_f$ and VAP_f, which, in turn, determines the overall cash flow discount rate of the firm. The heterogeneity in firms' the overall discount rates in the cross section then generates the spreads in the short-term expected returns along various characteristics.[23]

Kogan and Papanikolaou (2013) show that a calibrated model is able to quantitatively match numerous anomalies regarding short-term expected returns. However, the mechanism of their model relies crucially on the nexus between long-term cash flow discount rates and short-term expected returns. In the model, firms have different short-term expected returns because they have different long-term discount rates. In other words, Kogan and Papanikolaou (2013) interpret the spreads in the short-term expected returns as a manifestation of the differences in the long-term discount rates, and then propose a mechanism to generate heterogeneous long-term discount rates in order to match the patterns in short-term expected returns. Therefore, an important implication of their model is that long-term discount rates must have the same pattern as short-term expected returns in the cross section.

Table 8 replicates Kogan and Papanikolaou (2013)'s simulation results and shows that their model is indeed able to closely match the spreads in the short-term expected returns along portfolios sorted by idiosyncratic volatility, investment rate and gross profitability. However, the table also shows that the long-term discount rates in the model always take the same shape as the short-term expected returns, which is evidently counterfactual. In the data, the shape of long-term discount rates is inverted for idiosyncratic volatility and exhibits strong U-shape for investment

[23]We refer the reader to their paper for the details of model specification and calibration.

and gross profitability.

Kogan and Papanikolaou (2013) is not just a single instance, but rather, it represents the view regarding asset pricing anomalies of a large class of structural models including: Berk, Green, and Naik (1999), Gomes, Kogan, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), Zhang (2005), Li, Livdan, and Zhang (2009), Liu, Whited, and Zhang (2009), Li and Zhang (2010), Papanikolaou (2011), Belo, Lin, and Bazdresch (2014), Kogan and Papanikolaou (2014), Belo et al. (2017), Barrot, Loualiche, and Sauvagnat (2019), Belo, Lin, and Yang (2019), Gofman, Segal, and Wu (2020), Dou, Ji, and Wu (2021), etc. This class of models interprets the observed spreads in the average returns of dynamically-rebalanced trading strategies as manifestations of the differences in firms' equity financing costs and valuation levels. However, such a connection between short-term expected returns and long-term discount rates, which serves as the crucial premise for all of these models, has not yet been carefully examined. Our simple non-parametric methodology fills this void. And our findings regarding the disconnect between the short term and the long term for a number of well-known anomalies present a challenge for this line of thinking.

5 Conclusion

In this paper, we propose a simple novel non-parametric methodology to estimate firms' long-term discount rates/discount factors with ex-post realized cash flows. Compared to the existing approach of proxying cash flow expectations with analyst forecast or model predictions, our methodology has the advantage of being objective and model-free. Therefore, our methodology produces "clean" results that are not contaminated by analyst subjective biases or model mis-specifications.

By applying our methodology to the cross section of stocks, we discover that the patterns in long-term discount rates are substantially different from short-term expected returns for multiple well-known anomalies. The empirical findings of our paper provide a number of key implications for the cross-sectional asset pricing literature. First, our findings shed new light on the interpretation of several famous puzzles such as the idiosyncratic volatility anomaly by Ang et al. (2006) or the distress risk anomaly by Campbell, Hilscher, and Szilagyi (2008). For example, while high idiosyncratic volatility stocks do tend to generate much lower returns than low volatility stocks shortly after portfolio formation, we show that the long-term discount rates of the high volatility stocks are actually significantly higher than the low volatility stocks. Therefore, the idiosyncratic volatility anomaly is not a statement about firms' equity financing costs, but rather a short-term effect in the stock returns. Granted that we do not fully resolve the puzzle by explaining the cause of the pattern in short-term average returns, the new refined interpretation of this puzzle makes it arguably less puzzling. Second, our methodology enables us to identify new characteristics that are important for firms' equity financing costs but were previously overlooked by studies centered around short-term returns. For example, while an annually rebalanced trading strategy on firm's credit rating does not generate significant trading profits, we show that firms with low credit ratings do indeed face much higher long-term discount rates than high-credit-rating firms. Last but not least, our findings provide a new insight for a large class of structural models that aims to explain anomalies with rational forces. These models interpret the spreads in short-term average returns as manifestations of the differences in firms' equity financing costs, and propose various mechanisms relying on the nexus between short-term expected returns and long-term discount rates. However, we find that this assumed link between the short term and the long term is not always tenable, as we identify a group of prominent anomalies whose long-term discount rates exhibit substantially different patterns compared to the short-term expected returns. Therefore, reconciling the short term and the long term for these inconsistent anomalies would be a new challenge for this class of models.

Overall, our paper aims to stress that average returns generated by dynamically-rebalanced trading strategies do not necessarily reflect the equity financing costs faced by firms in the cross section. The short term and the long term are both important, but in different ways. Short-term expected returns inform the profitability of dynamic trading strategies, whereas long-term discount rates reflect firms' equity financing costs and valuation levels. And contrary to common beliefs, the patterns in the short term do not always coincide with the long term. Moreover, there need not be a unified framework that explains the disconnect between the short term and the long term for all anomalies. Indeed, multiple factors can be at play, such as investors' under-reaction or over-reaction to information, various frictions in the market that temporarily disrupt the risk-return relation, or convoluted interactions between multiple firm characteristics that follow different dynamics. We leave providing a full-fledged explanation for the wedge between the short and the long term to future research.

As a methodological contribution, our simple non-parametric estimation can be regarded as the long-term counterpart of the Fama-French approach for investigating long-term discount rates in the cross section of stocks. Just as the Fama-French exercise reveals the cross-sectional heterogeneity in short-term expected stock returns, our new methodology enables us to uncover a host of new stylized facts regarding firms' equity financing costs. It is also worth noting that we intentionally propose a model-free approach to avoid taking a stand on which specific asset pricing model to adopt. We simply document new empirical facts, and our findings could encourage future research to identify the closest model that explains long-term asset pricing anomalies as well as to deepen our understanding about the determinants of stock price levels.

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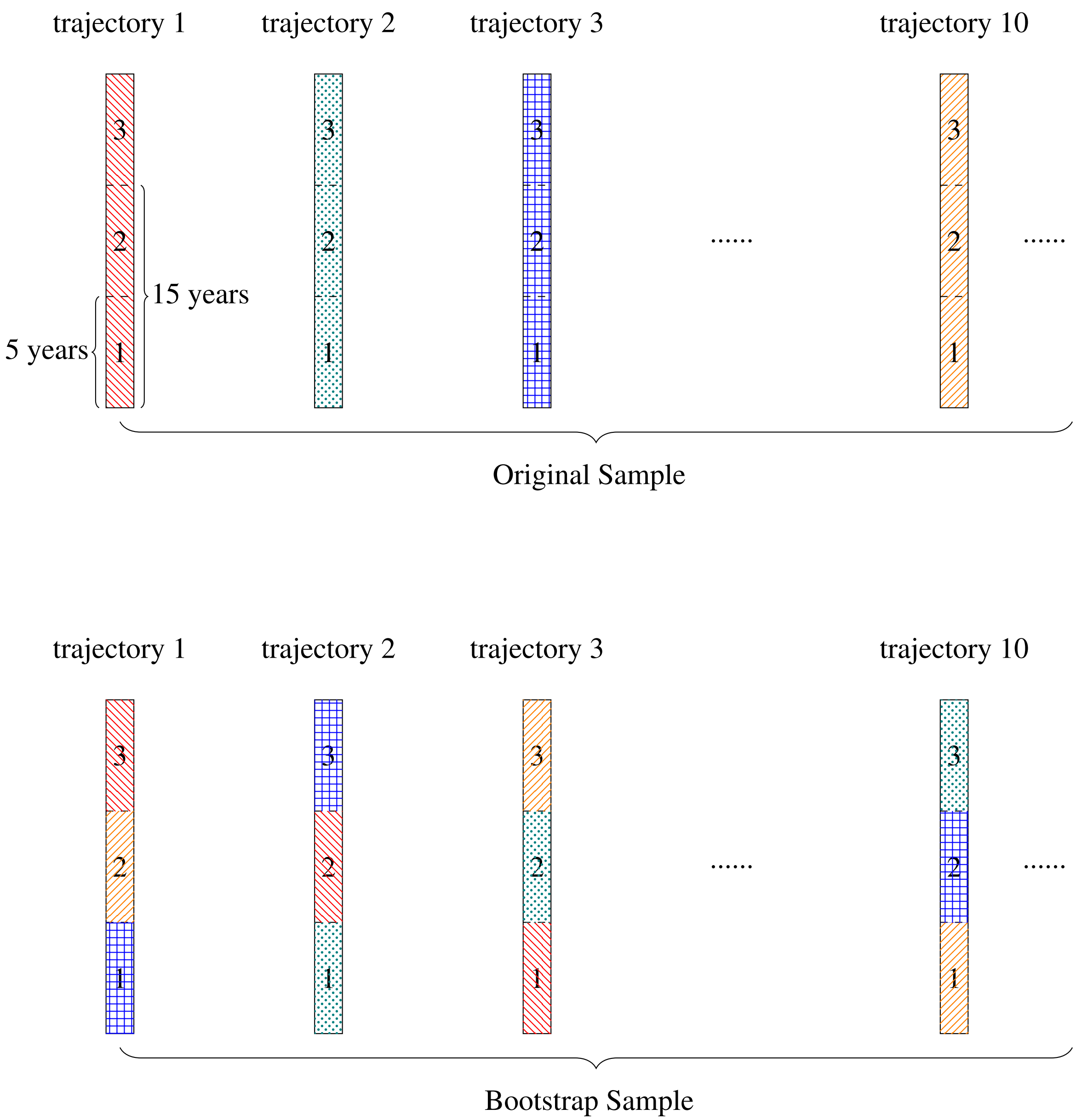
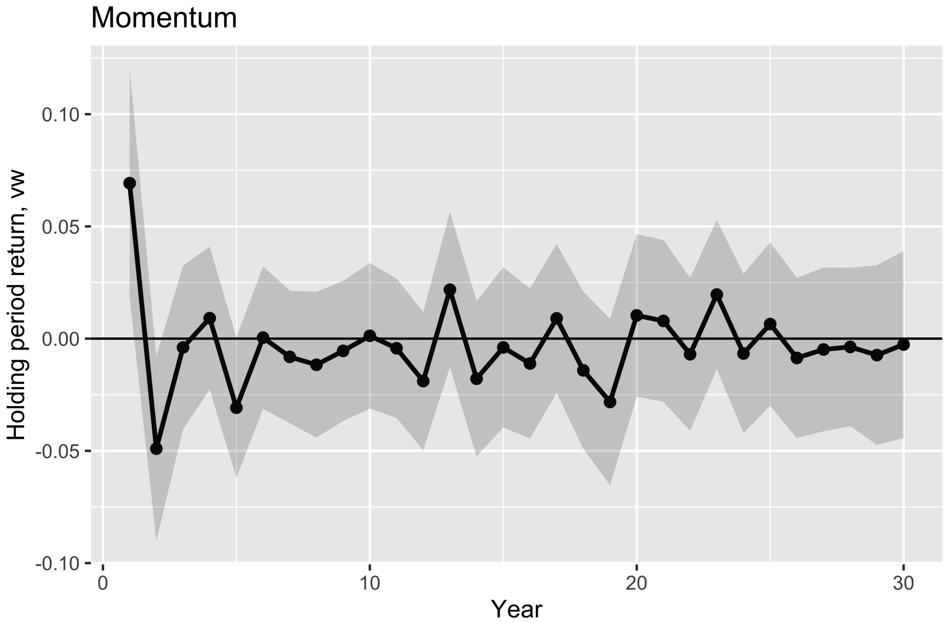
Figures

Figure 1: Block Bootstrap Illustration



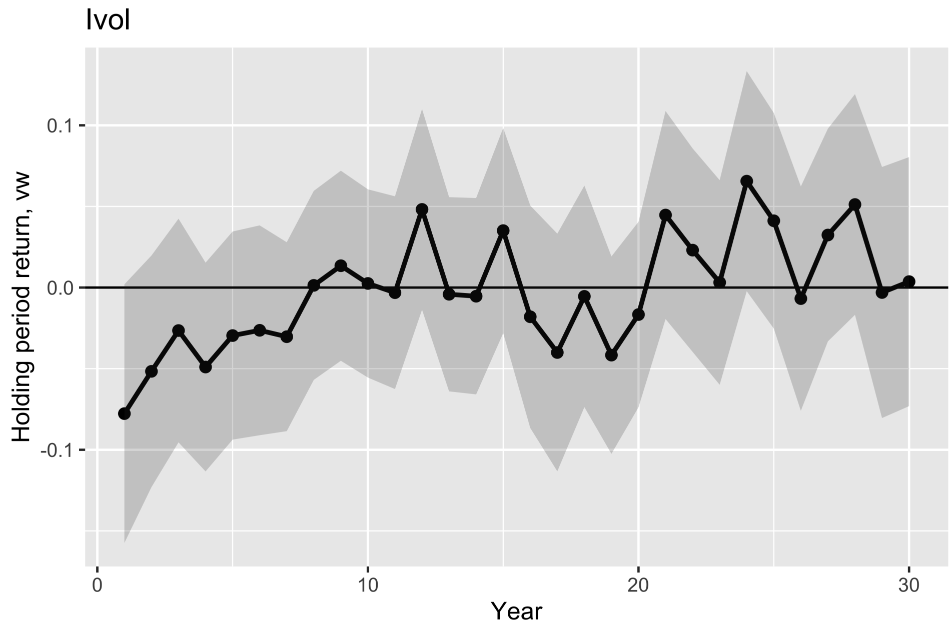
This figure demonstrates how the block bootstrap sample is generated from the original sample. A trajectory tracks the cash flows of a portfolio formed at a particular point in time for 15 years since formation. Each trajectory is then split into three 5-year blocks. The blocks are indexed by levels 1, 2 and 3, representing the first, second and third 5-year episodes since portfolio formation. The bootstrap sample has the same number of trajectories as the original sample. Each trajectory within the bootstrap sample is simulated by randomly drawing blocks with the same levels from the original sample.

Figure 2: Term Structure of the Momentum Premium



This figure plots the evolution of the average return of the momentum strategy since portfolio formation. The y-axis is the average one-year return difference between the value-weighted top decile and bottom decile. The x-axis denotes the number of years since portfolio formation. The shaded area is the 95% confidence internal.

Figure 3: Term Structure of the Idiosyncratic Volatility Premium



This figure plots the evolution of the average return of the trading strategy that longs high idiosyncratic volatility stocks and shorts low idiosyncratic volatility stocks. The y-axis is the average one-year return difference between the value-weighted top decile and bottom decile. The x-axis denotes the number of years since portfolio formation. The shaded area is the 95% confidence internal.

Tables

Table 1: List of Anomalies

Name	Literature	Sample Period	Frequency
Market Beta	Lintner (1965), Sharpe (1964), Fama and French (1992)	1970-2018	Y
Size	Banz (1981), Fama and French (1992)	1970-2018	Y
Book-to-Market	Chan and Chen (1991), Fama and French (1992)	1970-2018	Y
Gross Profitability	Novy-Marx (2013), Fama and French (2015)	1970-2018	Y
Investment	Titman, Wei, and Xie (2004), Fama and French (2015)	1970-2018	Y
Credit Rating	Avramov et al. (2009)	1985-2018	Y
Fail Probability	Campbell, Hilscher, and Szilagyi (2008)	1975-2018	M
Momentum	Jegadeesh and Titman (1993)	1970-2018	M
Long-Term Reversal	De Bondt and Thaler (1985)	1970-2018	M
Idiosyncratic Volatility	Ang et al. (2006)	1970-2018	M
Abnormal Volume	Gervais, Kaniel, and Mingelgrin (2001)	1970-2018	M
Mispricing Score	Stambaugh, Yu, and Yuan (2012)	1970-2018	M
Illiquidity	Amihud (2002)	1970-2018	Y
Liquidity Risk	Pástor and Stambaugh (2003)	1970-2018	Y
Coskewness	Harvey and Siddique (2000)	1970-2018	M
Duration	Gonçalves (2021)	1970-2018	Y

This table reports the anomalies studied in this paper. "Frequency" denotes the rebalancing frequency of the anomaly portfolios: "M" ("Y") indicates that anomaly portfolios are rebalanced monthly (Annually). For annually-rebalanced anomalies, portfolios are constructed by the end of December.

Table 2: Estimated Biases in Discount Rate Estimates (Structural Approach)

Stats \Model	FB	FB ADJ	LM	LM ADJ	BY	RD	BGN ADJ
mean	0.003	0.03	0.03	0.04	0.13	0.26	0.28
min	0.0	0.001	0.0002	0.001	0.01	0.11	0.17
25%	0.0002	0.01	0.02	0.02	0.06	0.19	0.23
50%	0.001	0.02	0.02	0.03	0.12	0.25	0.27
75%	0.003	0.04	0.04	0.06	0.16	0.31	0.32
max	0.03	0.14	0.06	0.15	0.34	0.46	0.38

This table reports the estimated biases in discount rate estimates using the structural approach. The biases for each model are estimated as the differences between the estimated discount rates and the specified discount rates across large parameter grids as detailed in Appendix A. Except for the adjusted Berk, Green, and Naik (1999) process, we simulate across a wide range of discount rates between 8% and 20%, i.e. $y \in [8\%, 20\%]$. "FB" denotes the Fama and Babiak (1968) process. "FB ADJ" denotes the adjusted Fama and Babiak (1968) process where earnings growth follows geometric random walk. "LM" denotes the Leary and Michaely (2011) process. "LM ADJ" denotes the adjusted Leary and Michaely (2011) process where earnings growth follows geometric random walk. "BY" denotes the long-run risk process in Bansal and Yaron (2004). "RD" denotes the rare disaster process in Barro (2006). "BGN ADJ" denotes the adjusted Berk, Green, and Naik (1999) process where the risk-adjusted discount rate is specified as a constant. The reported statistics are from large grids of parameter specifications. The biases are recorded in absolute values. All units are in percentage points.

Table 3: Estimated Biases in Discount Rate Estimates (Bootstrap Approach)

Anomaly	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	EMM
Market Beta	0.13	0.23	0.33	0.23	0.29	0.21	0.18	0.29	0.12	0.44	0.30	-0.02
Book-to-Market	-0.03	0.18	0.12	0.18	0.20	0.22	0.25	0.18	0.28	-0.01	0.02	-0.11
Size	0.22	0.20	0.22	0.29	0.29	0.24	0.17	0.29	0.18	0.21	-0.01	-0.07
Gross Profitability	0.46	0.21	0.10	0.38	0.21	0.17	0.28	0.39	0.47	0.29	-0.17	0.17
Investment	0.06	0.29	0.33	0.26	0.22	0.32	0.27	0.05	0.06	0.00	-0.06	-0.17
Credit Rating	0.38	0.12	-0.54	-0.04	0.11	-0.03	0.29	0.25	0.12	0.01	-0.36	0.12
Fail Probability	-0.22	0.11	0.36	0.19	0.15	0.16	0.10	0.10	-0.11	-0.26	-0.04	-0.28
Momentum	-0.02	-0.13	0.05	0.09	0.15	0.17	0.20	0.32	0.48	0.39	0.41	0.02
Idiosyncratic Volatility	0.16	0.27	0.24	0.06	0.01	0.12	0.19	0.20	0.23	-0.15	-0.31	0.06
Long-Term Reversal	0.05	0.10	0.15	0.10	0.12	0.19	0.17	0.19	0.23	-0.18	-0.23	-0.11
Abnormal Volume	0.31	0.24	0.23	0.28	0.27	0.32	0.27	0.29	0.28	0.35	0.04	-0.01
Mispricing Score	0.25	0.18	0.29	0.34	0.26	0.20	0.11	0.18	0.16	-0.15	-0.41	-0.12
Illiquidity	0.21	0.20	0.20	0.19	0.42	0.43	0.31	0.38	-0.03	0.22	0.01	-0.27
Liquidity Risk	0.23	0.14	0.42	0.17	0.26	0.16	0.38	0.40	0.44	0.22	-0.01	0.05
Coskewness	0.51	0.33	0.34	0.31	0.30	0.18	0.07	0.17	-0.08	0.04	-0.47	-0.04
Duration	0.13	0.16	0.29	0.05	0.25	0.25	0.17	0.12	-0.07	-0.45	-0.58	-0.31

This table reports the estimated biases in discount rate estimates with the block bootstrap approach. The bias for each portfolio is estimated in block bootstrap simulations as the difference between the estimated discount rate in the simulations and the specified discount rate. The specified discount rates in the simulations adopt the estimates in the true data. All units are in percentage points.

Table 4: Anomalies Based on Funding Cost

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Market Beta														
$y-r_f^{15}$	5.76	6.19	6.44	6.67	6.66	6.51	5.98	6.37	7.06	8.01	2.25	1.59	0.16	0.33
$\bar{h}_1-r_f$	7.53	8.07	8.37	9.38	9.72	9.98	10.1	9.87	9.71	11.28	3.75	1.25	4.22***	3.23
Panel (b): Size														
$y-r_f^{15}$	9.33	9.67	9.73	9.71	9.0	8.2	7.62	7.27	7.27	5.33	-4.0***	-3.22	-0.7**	-2.32
$\bar{h}_1-r_f$	15.24	13.82	12.82	12.6	11.57	11.58	10.92	10.55	9.65	7.84	-7.41***	-2.76	5.85***	4.45
Panel (c): Book-to-Market														
$y-r_f^{15}$	5.37	5.3	6.32	6.51	7.22	8.36	7.87	8.28	9.95	12.5	7.13***	4.6	0.49	1.2
$\bar{h}_1-r_f$	5.43	4.9	6.57	6.26	6.2	8.43	8.2	9.26	9.55	11.67	6.24**	2.13	4.23***	2.75
Panel (d): Gross Profitability														
$y-r_f^{15}$	10.4	8.88	4.81	5.45	6.01	6.06	6.14	6.63	6.94	8.81	-1.59**	-1.96	2.73***	4.92
$\bar{h}_1-r_f$	3.32	5.47	5.47	4.43	5.97	6.79	6.7	7.12	8.1	8.94	5.62***	2.9	3.27***	2.61
Panel (e): Investment														
$y-r_f^{15}$	8.57	7.88	7.47	6.7	6.54	6.45	6.16	6.35	7.28	7.39	-1.18	-0.98	1.28**	2.51
$\bar{h}_1-r_f$	7.6	9.82	9.02	6.9	6.14	6.92	6.08	8.06	4.56	1.02	-6.59***	-3.48	2.49	1.42
Panel (f): Credit Rating														
$y-r_f^{15}$	11.71	8.21	7.74	7.99	6.5	5.84	6.07	5.73	7.19	5.32	-6.39***	-4.23	1.94***	3.27
$\bar{h}_1-r_f$	5.41	6.18	8.87	6.79	7.81	7.55	7.7	7.81	9.37	7.63	2.22	0.57	3.31	1.53
Panel (g): Fail Probability														
$y-r_f^{15}$	6.29	6.53	6.16	5.59	5.63	5.79	5.7	5.81	6.17	6.2	-0.09	-0.21	0.58***	3.55
$\bar{h}_1-r_f$	12.69	9.52	8.53	7.23	8.82	7.48	7.75	8.31	6.65	3.11	-9.58**	-2.38	3.92*	1.71

This table documents the anomalies motivated by models of funding cost. Stocks in the cross section are sorted into 10 deciles based on the corresponding characteristics. The index of the portfolio increases with the sorting characteristic. For annual (monthly) anomalies, portfolios are rebalanced by the end of every December (month). "$y-r_f^{15}$" is the difference between the estimated long-term discount rate and the 15-year zero-coupon rate. "$\bar{h}_1-r_f$" is the annualized average short-term return of the trading strategy over the 3-month T-bill rate. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-statistics for "$y-r_f^{15}$" are based on the standard errors estimated from block bootstrap simulations detailed in Section 2.4. The t-statistics for "$\bar{h}_1-r_f$" are the standard OLS t-statistics.

Table 5: Anomalies Based on Trading Prices and Volume

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Momentum														
$y-r_f^{15}$	7.6	7.45	7.2	6.69	6.34	6.41	6.48	6.36	6.67	6.82	-0.78**	-2.12	0.76***	5.13
$\bar{h}_1-r_f$	-1.44	3.59	4.07	7.34	7.47	7.77	9.19	10.34	11.18	15.31	16.76***	5.21	3.35**	2.04
Panel (b): Long-Term Reversal														
$y-r_f^{15}$	9.1	8.36	8.06	7.52	7.17	6.63	6.34	5.85	5.37	4.87	-4.22***	-11.55	0.03	0.21
$\bar{h}_1-r_f$	12.76	11.64	10.66	9.63	9.03	9.42	9.2	8.57	7.45	7.47	-5.29**	-2.11	5.22***	3.43
Panel (c): Idiosyncratic Volatility														
$y-r_f^{15}$	5.27	6.58	7.24	7.24	7.91	8.07	7.91	7.29	6.85	7.32	2.06***	3.98	-1.49***	-8.96
$\bar{h}_1-r_f$	8.06	8.32	8.91	9.26	9.59	9.84	7.88	5.76	3.86	-2.84	-10.9***	-3.48	-0.51	-0.4
Panel (d): Abnormal Volume														
$y-r_f^{15}$	6.42	6.15	6.09	6.3	6.07	6.23	6.21	6.31	6.41	6.88	0.46***	2.72	0.31***	3.74
$\bar{h}_1-r_f$	4.47	4.9	3.92	7.21	6.51	7.2	8.1	7.77	10.44	9.57	5.1***	3.47	3.92***	3.08

This table documents the anomalies based on historical trading prices and volume. Stocks in the cross section are sorted into 10 deciles based on the corresponding characteristics. The index of the portfolio increases with the sorting characteristic. For annual (monthly) anomalies, portfolios are rebalanced by the end of every December (month). "$y-r_f^{15}$" is the difference between the estimated long-term discount rate and the 15-year zero-coupon rate. "$\bar{h}_1-r_f$" is the annualized average short-term return of the trading strategy over the 3-month T-bill rate. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-statistics for "$y-r_f^{15}$" are based on the standard errors estimated from block bootstrap simulations detailed in Section 2.4. The t-statistics for "$\bar{h}_1-r_f$" are the standard OLS t-statistics.

Table 6: Additional Anomalies

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Mispricing Score														
$y - r_f^{15}$	5.84	6.28	6.1	5.93	5.99	6.0	6.3	6.86	7.84	7.63	1.79***	7.38	0.91***	8.89
$\bar{h}_1 - r_f$	10.03	8.09	6.9	7.98	7.46	5.42	4.9	4.49	1.13	-3.18	-13.22***	-4.97	0.8	0.49
Panel (b): Illiquidity														
$y - r_f^{15}$	5.67	7.03	7.96	9.14	9.62	9.52	9.49	9.79	9.83	10.56	4.89***	3.33	-1.3***	-2.95
$\bar{h}_1 - r_f$	5.73	7.46	7.72	8.82	8.89	11.45	10.97	12.15	11.81	11.84	6.11	1.58	4.13**	2.56
Panel (c): Liquidity Risk														
$y - r_f^{15}$	7.18	7.41	6.93	5.84	5.57	5.79	6.44	6.66	6.29	6.68	-0.5	-0.59	1.21***	3.59
$\bar{h}_1 - r_f$	5.2	6.32	6.09	6.7	6.04	7.46	7.57	7.81	6.79	11.1	5.9***	2.95	3.98**	2.02
Panel (d): Coskewness														
$y - r_f^{15}$	6.46	6.09	6.18	6.39	6.7	6.9	6.7	6.05	6.12	6.08	-0.38**	-2.11	-0.61***	-8.2
$\bar{h}_1 - r_f$	9.04	8.01	6.72	7.35	7.0	6.95	6.92	5.06	5.28	3.87	-5.18**	-2.45	3.06**	2.07
Panel (e): Duration														
$y - r_f^{15}$	10.1	8.73	8.47	7.66	7.01	6.98	6.19	5.69	5.54	4.59	-5.51***	-4.19	0.25	0.68
$\bar{h}_1 - r_f$	14.22	14.13	13.85	14.96	11.06	9.6	8.36	4.88	3.4	0.61	-13.61***	-4.35	2.92	1.35

This table documents several additional anomalies. Stocks in the cross section are sorted into 10 deciles based on the corresponding characteristics. The index of the portfolio increases with the sorting characteristic. For annual (monthly) anomalies, portfolios are rebalanced by the end of every December (month). "$y - r_f^{15}$" is the difference between the estimated long-term discount rate and the 15-year zero-coupon rate. "$\bar{h}_1 - r_f$" is the annualized average short-term return of the trading strategy over the 3-month T-bill rate. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-statistics for "$y - r_f^{15}$" are based on the standard errors estimated from block bootstrap simulations detailed in Section 2.4. The t-statistics for "$\bar{h}_1 - r_f$" are the standard OLS t-statistics.

Table 7: Out-of-Sample Performances

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Momentum														
Early Sample: 1926-1970														
$y - r_f^{15}$	11.72	10.69	10.28	10.54	10.06	10.35	10.42	10.53	10.69	10.9	-0.82**	-2.17	0.79***	4.62
$\bar{h}_1 - r_f$	2.71	5.63	4.21	7.54	8.41	8.84	10.46	11.79	12.79	16.33	13.62***	2.97	5.05**	2.07
Main Sample: 1970-2018														
$y - r_f^{15}$	7.6	7.45	7.2	6.69	6.34	6.41	6.48	6.36	6.67	6.82	-0.78**	-2.12	0.76***	5.13
$\bar{h}_1 - r_f$	-1.44	3.59	4.07	7.34	7.47	7.77	9.19	10.34	11.18	15.31	16.76***	5.21	3.35**	2.04

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (b): Long-Term Reversal														
Early Sample: 1926-1970														
$y - r_f^{15}$	12.36	11.13	11.16	11.25	11.0	10.68	10.61	9.98	9.64	9.84	-2.52***	-6.12	-0.1	-0.67
$\bar{h}_1 - r_f$	14.01	12.86	11.32	10.27	10.47	9.44	9.73	9.12	7.81	5.6	-8.41**	-2.14	5.1**	2.1
Main Sample: 1970-2018														
$y - r_f^{15}$	9.1	8.36	8.06	7.52	7.17	6.63	6.34	5.85	5.37	4.87	-4.22***	-11.55	0.03	0.21
$\bar{h}_1 - r_f$	12.76	11.64	10.66	9.63	9.03	9.42	9.2	8.57	7.45	7.47	-5.29**	-2.11	5.22***	3.43

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (c): Idiosyncratic Volatility														
Early Sample: 1926-1970														
$y - r_f^{15}$	9.81	9.69	10.56	11.44	11.67	12.26	13.15	10.6	11.49	11.84	2.03***	4.97	-1.26***	-6.72
$\bar{h}_1 - r_f$	8.43	10.17	10.32	10.62	12.35	11.18	10.51	12.69	13.39	14.2	5.78	1.09	5.66**	2.54
Main Sample: 1970-2018														
$y - r_f^{15}$	5.27	6.58	7.24	7.24	7.91	8.07	7.91	7.29	6.85	7.32	2.06***	3.98	-1.49***	-8.96
$\bar{h}_1 - r_f$	8.06	8.32	8.91	9.26	9.59	9.84	7.88	5.76	3.86	-2.84	-10.9***	-3.48	-0.51	-0.4

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
							Panel (d): Abnormal Volume							
							Early Sample: 1926-1970							
$y - r_f^{15}$	10.19	10.27	10.6	10.29	10.35	10.41	10.45	10.24	10.5	10.49	0.3	1.42	-0.02	-0.12
$\bar{h}_1 - r_f$	3.78	7.78	7.06	7.85	9.59	8.75	12.67	10.64	12.36	14.3	10.52***	6.24	4.97***	2.76
							Main Sample: 1970-2018							
$y - r_f^{15}$	6.42	6.15	6.09	6.3	6.07	6.23	6.21	6.31	6.41	6.88	0.46***	2.72	0.31***	3.74
$\bar{h}_1 - r_f$	4.47	4.9	3.92	7.21	6.51	7.2	8.1	7.77	10.44	9.57	5.1***	3.47	3.92***	3.08
							Panel (e): Market Beta							
							Early Sample: 1926-1970							
$y - r_f^{15}$	8.06	9.61	10.54	10.81	11.34	11.08	10.79	10.66	10.24	11.1	3.05**	2.58	-1.46***	-3.41
$\bar{h}_1 - r_f$	6.81	7.71	6.88	9.98	10.58	10.82	10.68	12.72	10.92	14.18	7.36	1.54	4.55**	2.07
							Main Sample: 1970-2018							
$y - r_f^{15}$	5.76	6.19	6.44	6.67	6.66	6.51	5.98	6.37	7.06	8.01	2.25	1.59	0.16	0.33
$\bar{h}_1 - r_f$	7.53	8.07	8.37	9.38	9.72	9.98	10.1	9.87	9.71	11.28	3.75	1.25	4.22***	3.23
							Panel (f): Size							
							Early Sample: 1926-1970							
$y - r_f^{15}$	11.44	12.02	13.43	13.63	12.29	11.87	11.02	11.11	10.39	9.66	-1.78	-1.11	-1.2**	-2.31
$\bar{h}_1 - r_f$	18.88	15.82	12.32	12.64	11.9	12.34	11.07	9.71	9.94	7.91	-10.96**	-2.2	7.08***	2.84
							Main Sample: 1970-2018							
$y - r_f^{15}$	9.33	9.67	9.73	9.71	9.0	8.2	7.62	7.27	7.27	5.33	-4.0***	-3.22	-0.7**	-2.32
$\bar{h}_1 - r_f$	15.24	13.82	12.82	12.6	11.57	11.58	10.92	10.55	9.65	7.84	-7.41***	-2.76	5.85***	4.45

This table compares the long-term discount rates of anomalies that do not require accounting data for an early sample (1926-1970) versus the main sample (1970-2018). The main sample results are reiterated from previous tables to facilitate comparison. "$y - r_f^{15}$" is the the difference between the estimated long-term discount rate and the 15-year zero-coupon rate. "$\bar{h}_1 - r_f$" is the annualized average short-term return of the trading strategy over the 3-month T-bill rate. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-statistics for "$y - r_f^{15}$" are based on the standard errors estimated from block bootstrap simulations. The t-statistics for "$\bar{h}_1 - r_f$" are the standard OLS t-statistics.

Table 8: Case Study with Kogan and Papanikolaou (2013)

Panel (a): Idiosyncratic Volatility

Data

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
$\bar{h}_1 - r_f$	8.06	8.32	8.91	9.26	9.59	9.84	7.88	5.76	3.86	-2.84	-10.9***	-3.48	-0.51	-0.4
$y - r_f^{15}$	5.27	6.58	7.24	7.24	7.91	8.07	7.91	7.29	6.85	7.32	2.06***	3.98	-1.49***	-8.96

Model

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
$\bar{h}_1 - r_f$	14.01	12.93	12.08	11.25	10.45	9.67	8.96	8.33	7.89	7.88	-6.13		0.61	
$y - r_f$	13.56	12.56	11.75	11.01	10.3	9.66	9.12	8.75	8.67	9.48	-4.08		1.09	

Panel (b): Investment

Data

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
$\bar{h}_1 - r_f$	7.6	9.82	9.02	6.9	6.14	6.92	6.08	8.06	4.56	1.02	-6.59***	-3.48	2.49	1.42
$y - r_f^{15}$	8.57	7.88	7.47	6.7	6.54	6.45	6.16	6.35	7.28	7.39	-1.18	-0.98	1.28**	2.51

Model

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
$\bar{h}_1 - r_f$	13.47	13.09	12.83	12.24	11.45	10.61	9.79	9.33	8.72	7.57	-5.9		-0.32	
$y - r_f$	12.72	12.95	13.08	12.67	12.08	11.49	11.05	10.87	10.73	10.57	-2.15		-0.04	

Panel (c): Gross Profitability

Data

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
$\bar{h}_1 - r_f$	3.32	5.47	5.47	4.43	5.97	6.79	6.7	7.12	8.1	8.94	5.62***	2.9	3.27***	2.61
$y - r_f^{15}$	10.4	8.88	4.81	5.45	6.01	6.06	6.14	6.63	6.94	8.81	-1.59**	-1.96	2.73***	4.92

Model

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
$\bar{h}_1 - r_f$	7.67	10.15	11.2	11.7	11.83	11.82	11.53	11.21	10.88	10.54	2.87		-2.02	
$y - r_f$	8.7	10.58	11.47	11.81	11.85	11.68	11.36	11.0	10.59	10.27	1.56		-1.73	

This table compares the estimates of the short-term expected returns and the long-term discount rates between the data and the Kogan and Papanikolaou (2013) model for idiosyncratic volatility, investment and gross profitability anomalies. The results of the real data are reiterated from previous tables. All units are in percentage.

Appendix A. Specifications of the Structural Cash Flow Processes

Section 2.3.1 evaluates the biases of discount rate estimates using 7 structural cash flow processes across large parameter grids. The details of the cash flow specifications and parameter ranges are described below.

A1. The Fama and Babiak (1968) Process

Fama and Babiak (1968) estimated a payout model where a firm's dividend process is co-integrated with its earnings process. We adopt this model in our simulations to understand how the auto-correlation in dividends affects the bootstrap exercise.

According to Fama and Babiak (1968), the firm's earnings follow:

$$E_{t+1} = (1+\lambda)E_t + e_{t+1},$$

and its dividends are co-integrated with the earnings:

$$\Delta D_{t+1} = \beta_1 D_t + \beta_2 E_t + u_{t+1}$$

where

$$e_t \sim N\left(0, \sigma_e^2\right),\ u_t \sim N\left(0, \sigma_u^2\right),\ Corr\left(e_t, u_t\right) = 0$$

We adopt the following parameter ranges in our simulations: $\beta_1 \in [-0.45 - 2\times 0.15, -0.45 + 2\times 0.15]$, $\beta_2 = [0.15 - 2\times 0.05, 0.15 + 2\times 0.05]$, $\lambda = 5\%$, $\sigma_u = \frac{1}{0.6745}$, $\sigma_e = \frac{0.21}{0.6745}$, $E_0 = 100$, $D_0 = 33$, $y \in [8\%, 20\%]$. The ranges for β_1 and β_2 are 95% confidence interval reported by Table 2 of the paper. The values of σ_e, σ_u and the ratio between D_0 and E_0 are from page 1143 of the paper.

A2. The Adjusted Fama and Babiak (1968) Process

Fama and Babiak (1968) aimed to study the properties of corporate payouts. The model assumes random shocks to the level of the earnings process, or an arithmetic random walk. To better suit our purposes, we adjusted the earnings process to a geometric random walk so that the shocks affect the growth rate of the earnings:

$$E_{t+1} = \exp\left(g_{t+1}\right)E_t$$

$$g_{t+1} = \lambda + \sigma \cdot e_{t+1}$$

The co-integration between the dividend process and the earnings process remains the same. For the volatility of earnings growth, we adopt the range: $\sigma \in [7.5\%, 15.5\%]$, which is the 95% confidence interval of dividend growth volatility documented in Table II of Bansal, Kiku, and Yaron (2009). λ is assumed to be 5%.

A3. The Leary and Michaely (2011) Process

Similar to Fama and Babiak (1968), Leary and Michaely (2011) also proposed a model of corporate payout where earnings and dividends are co-integrated. The processes are specified as:

$$\Delta E_{t+1} = \delta + \gamma \times \Delta E_t + \omega_{t+1}$$
$$\Delta D_{t+1} = \beta \times (TPR \times E_{t+1} - D_t) + \varepsilon_{t+1},$$

where

$$\omega_t \sim N\left(0, \sigma_\omega^2\right),\ \varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right),\ Corr(\omega_t, \varepsilon_t) = 0$$

We adopt the following parameter ranges in our simulations: $\beta \in [0.1, 0.5]$, $TPR = 0.3$, $\delta = 0.1$, $\gamma = -0.2$, $\sigma_\omega = 0.7$, $\sigma_\varepsilon = 0.1$, $E_0 = 10$, $D_0 = 3$, $y \in [8\%, 20\%]$. The range of β and the values for TPR, δ, γ, σ_ω are from page 3245 of Leary and Michaely (2011); the value of σ_ε is from Figure 2 of the working paper version of the paper; and the ratio between D_0 and E_0 are determined by the steady state of the dividend equation, i.e. $D_0/E_0 = TPR$.

A4. The Adjusted Leary and Michaely (2011) Process

Similar to the adjusted Fama and Babiak (1968) process, we consider a variant of the Leary and Michaely (2011) process where the earnings growth follows a geometric random walk:

$$E_{t+1} = \exp(g_{t+1}) E_t$$

$$g_{t+1} = \lambda + \sigma \cdot e_{t+1}$$

with $\lambda = 5\%$ and $\sigma \in [7.5\%, 15.5\%]$.

A5. The Bansal and Yaron (2004) Process

We consider the long-run risk process as specified in Bansal and Yaron (2004):

$$\begin{aligned} D_{t+1} &= D_t \cdot \exp\left(g_{d,t+1}\right) \\ x_{t+1} &= \rho x_t + \phi_e \sigma e_{t+1} \\ g_{d,t+1} &= \mu_d + \phi x_t + \phi_d \sigma u_{t+1} \\ e_{t+1}, u_{t+1} &\sim N \; i.i.d \; (0,1). \end{aligned}$$

[24]

We adopt the following parameter ranges in our simulations: $\rho = 0.979$, $\phi_e = 0.044$, $\sigma = 0.0078$, $\mu_d = 0.004$, $\phi = \{0,3\}$, $\phi_d = [3.5, 5.5]$, $y \in [8\%, 20\%]$. The values of ρ, ϕ_e and σ adopt the calibration in Bansal and Yaron (2004). The specification of the model is of monthly frequency. We adopt $\mu_d = 0.004$ so that the annualized average dividend growth rate is close to 5%. We adopt $\phi_d = [3.5, 5.5]$ so that the annualized dividend growth volatility is in the range of $[7.5\%, 15.5\%]$, which is the 95% confidence interval estimated by Bansal, Kiku, and Yaron (2009). We consider two values of ϕ. When $\phi = 0$, the long-run risk component is switched off, and the dividend process follows a geometric random walk. When $\phi = 3$, the long-run risk component is present in the dividend growth rate and the parameter value is provided by Bansal and Yaron (2004).

A6. The Barro (2006) Process

We consider the rare disaster risk process as specified in Barro (2006):

$$\begin{aligned} D_{t+1} &= D_t \cdot \exp\left(g_{t+1}\right) \\ g_{t+1} &= \mu + \sigma u_{t+1} + v_{t+1} \\ u_{t+1} &\sim N \; i.i.d \; (0,1) \\ v_{t+1} &\sim \begin{cases} e^{-p} & 0 \\ 1 - e^{-p} & \log\left(1-b\right) \end{cases}, \end{aligned}$$

where b is the random disaster loss following the empirical distribution provided by Barro (2006) page 832 Panel B.

We adopt the following parameter ranges in our simulations: $\mu = 5\%$, $\sigma \in [7.5\%, 15.5\%]$,

[24] We consider the version of the Bansal and Yaron (2004) model without stochastic volatility in our baseline simulations. We then evaluate the influence of such an effect in Appendix B.2.

$p \in [0.01, 0.05]$, $y \in [8\%, 20\%]$. The range of dividend growth volatility, $\sigma \in [7.5\%, 15.5\%]$, adopts the 95% confidence interval estimated by Bansal, Kiku, and Yaron (2009). Our range of disaster probability, $p \in [0.01, 0.05]$, covers the parameter choices in Barro (2006) ($p = 0.017$) and Gabaix (2012) ($p = 0.0363$).

A7. The Adjusted Berk, Green, and Naik (1999) Process

In order to investigate how firms' growth options might generate skewness in their cash flows and affect the estimation of long-term discount rate, we modify the Berk, Green, and Naik (1999) model and evaluate the biases in discount rate estimates with a setting similar to theirs.

Berk, Green, and Naik (1999) is a seminal paper which studies how growth options can affect firms' cash flows and induce size and value effects in the cross section of equity risk premium. We follow their model and parameter specifications as closely as possible meanwhile further simplify the mechanics of the model for our purposes. In particular, instead of explicitly modeling the stochastic discount factor, we take a more reduced-form approach and specify a constant risk-adjusted discount rate for risky cash flows directly.

Following Berk, Green, and Naik (1999), each firm has a collection of existing projects and an option to adopt a new project in each period. Therefore, the value of the firm comes from two sources – the cash flows generated by existing assets (value of asset in place) and the potential cash flows generated by future investments (value of growth options).

A7.1 Value of Asset in Place

At date t, the cash flows from a project that was undertaken at date $j < t$ are given by the process:

$$C_j(t) = I \exp\left(\bar{C} - \beta_j - \frac{1}{2}\sigma_j^2 + \sigma_j \varepsilon_j(t)\right),$$

where I is a constant that represents the scale of the project; $\bar{C}$ is a constant; σ_j governs the variance of the cash flows; $\varepsilon_j(t)$ is a standard Gaussian shock; β_j is a project specific term. Different from Berk, Green, and Naik (1999) where β_j reflects the covariance between the cash flows and the stochastic discount factor, we use a constant risk-adjusted rate to discount expected risky cash flows and model β_j as the heterogeneity in the levels of cash flows across different projects. We draw β_j from the same distribution as in Berk, Green, and Naik (1999) (more on this later). Our specification of β_j, though being different from Berk, Green, and Naik (1999), has a similar effect on the valuation of the cash flows, as β_j also "biases" the level of cash flows in the risk-neutral measure in Berk, Green, and Naik (1999) (Equation (8)).

Following Berk, Green, and Naik (1999), the project is hit by a Poisson shock every period and faces a probability $1-\pi$ of becoming obsolete. $\chi_j(t)$ is an indicator variable that takes 1 if project j is alive at date t, and 0 if the project is obsolete. Once the project becomes obsolete, then it's discarded by the firm.

To facilitate our simulation exercise, we model risk adjustment in a reduced-form way. We specify an exogenous discount rate y such that all risky cash flows generated by the project are discounted with this rate.

Therefore, the present value of an existing project, by the end of date t, is

$$\sum_{s=t+1}^{\infty} \frac{\mathbb{E}_t\left[\chi_j(t)\cdot C_j(t)\right]}{(1+y)^t} = \sum_{s=t+1}^{\infty} \frac{\chi_j(t)\cdot I\exp\left(\bar{C}-\beta_j\right)}{(1+y)^t};$$

$$= I\exp\left(\bar{C}-\beta_j\right)\cdot\frac{\pi/(1+y)}{1-\pi/(1+y)}$$

the value of asset in place of the firm is the sum of the present value of all existing projects

$$\sum_{j=0}^{t}\sum_{s=t+1}^{\infty} \frac{\chi_j(t)\cdot I\exp\left(\bar{C}-\beta_j\right)}{(1+y)^{s-t}}$$

A7.2 Value of Growth Options

At the beginning of date t, the firm will draw a project with a pair of parameters $\{\beta_j,\sigma_j\}$. The investment of the project requires $I>0$. So the firm will choose to adopt the project if $I\exp\left(\bar{C}-\beta_j\right)\cdot\frac{\pi/(1+y)}{1-\pi/(1+y)} > I$. The value of growth options of the firm at date t is thus

$$\sum_{j=t+1}^{\infty} \frac{\mathbb{E}_t\left\{\max\left[I\exp\left(\bar{C}-\beta_j\right)\cdot\frac{\pi/(1+y)}{1-\pi/(1+y)}-I,0\right]\right\}}{(1+y)^{j-t}}.$$

A7.3 Parameter Specifications

We adopt the same parameters as in Berk, Green, and Naik (1999).

β_j is drawn from a truncated exponential distribution with support $(-\infty,\beta^*]$ and density

$$f(\beta) = \frac{\exp\left(\frac{\beta-\beta^*}{\beta^*-\bar{\beta}}\right)}{\beta^*-\bar{\beta}},$$

for $\bar{\beta}<\beta^*$. The specification of $\bar{\beta}$ and β^* are from Berk, Green, and Naik (1999), with $\bar{\beta}=0.591$,

$\beta^* = 0.728.$[25]

Following Berk, Green, and Naik (1999), once β_j is determined, σ_j is then drawn from a uniform distribution with support $\left[\left|\beta_j\right|/\sigma_z, \left|\beta_j\right|/\sigma_z + \sigma\right]$, with $\sigma_z = 0.4$ and $\sigma = 0.3 \times 3.7$.

The rest of the parameters are also adopted from Berk, Green, and Naik (1999): $I = 1$, $\bar{C} = -3.7$, $\pi = 0.99$.

For our purposes, the model is modified such that all cash flows are discounted with a risk-adjusted rate y, which is the IRR of the firm. The choice of y determines the value of new projects and thus the investment rate of the firm. Berk, Green, and Naik (1999) calibrate their model such that the probability of adopting a new project mostly falls between 5% to 10%. We take a parallel and consider $y \in [7\%, 10\%]$ such that the investment probability in our setting also falls in that range.

Lastly, following Berk, Green, and Naik (1999), we simulate a panel of 50 firms and average their cash flows and prices as the input of our evaluation of the accuracy of long-term discount rate estimates.

Appendix B. Robustness of the Block Bootstrap Procedure

B.1 Evaluation with Structural Models

Section 2.3.2 explains that one concern regarding the block bootstrap procedure might arise due to the possible wedge between the simulated trajectories and the true data generating process. To alleviate such a concern, we reuse the structural cash flow processes presented in Appendix A to show that our block bootstrap procedure is reliable across these models with parameters iterated over large grids.

Specifically, for each model, we iterate parameters across large grids. For each set of parameters, we generate two samples of cash flows: one from the true data generating process and the other from the block bootstrap simulations. We then compare the biases and standard errors estimated from these two samples, respectively, and record their differences. Finally, we adopt a different set of parameters from the grid and repeat the process.

Table 9 reports the differences between the estimated biases and standard errors from the model generated samples and the bootstrap simulated samples. In the most extreme case, bootstrap bias differs from the true value by 39 bps, and the bootstrap standard error differs from the true

[25] Berk, Green, and Naik (1999) does not report the specification of $\bar{\beta}$ and β^* directly. These two parameters are calibrated such that the probability of adopting a new project is 10% (5%) when the risk-free rate equals 0 ($\bar{r}$) in their model. We infer $\bar{\beta}$ and β^* by simulating their model.

value by 26 bps. The median differences are even much smaller. The table shows that the block bootstrap procedure successfully mimics the true data generating processes and produces bias and standard error estimates that are close to the true values.

Table 9: Accuracy of the Block Bootstrap Simulations

Stats \Model	FB	FB ADJ	LM	LM ADJ	BY	RD	BGN ADJ
mean	0.001	0.04	0.01	0.06	0.08	0.11	0.28
min	0.0	0.0001	0.001	0.001	0.002	0.02	0.17
25%	0.0002	0.01	0.01	0.01	0.02	0.06	0.23
50%	0.0005	0.02	0.01	0.03	0.08	0.11	0.29
75%	0.001	0.05	0.01	0.07	0.1	0.16	0.33
max	0.01	0.26	0.03	0.24	0.22	0.38	0.39

(a) Differences in Biases

Stats \Model	FB	FB ADJ	LM	LM ADJ	BY	RD	BGN ADJ
mean	0.004	0.04	0.05	0.06	0.09	0.09	0.02
min	0.0	0.0003	0.002	0.001	0.02	0.02	0.0001
25%	0.001	0.02	0.01	0.02	0.05	0.04	0.01
50%	0.002	0.03	0.03	0.03	0.07	0.08	0.01
75%	0.004	0.04	0.08	0.08	0.12	0.12	0.02
max	0.03	0.22	0.12	0.18	0.26	0.26	0.13

(b) Differences in Standard Errors

This table reports the absolute differences in discount rate estimates biases and standard errors between the sample generated by the block bootstrap procedure and the true data generating process across several dividend processes. "FB" denotes the Fama and Babiak (1968) process. "FB ADJ" denotes the adjusted Fama and Babiak (1968) process where earnings growth follows geometric random walk. "LM" denotes the Leary and Michaely (2011) process. "LM ADJ" denotes the adjusted Leary and Michaely (2011) process where earnings growth follows geometric random walk. "BY" denotes the long-run risk process in Bansal and Yaron (2004). "RD" denotes the rare disaster process in Barro (2006). "BGN ADJ" denotes the adjusted Berk, Green, and Naik (1999) process where the risk-adjusted discount rate is specified as a constant. The reported statistics are from large grids of parameter specifications. All units are in percentage points.

B.2 Standard Error with Persistent Risk Premium

Since the focus of our paper is on the cross-sectional heterogeneity in long-term discount rates across different anomaly portfolios rather than the time variation of discount rates, in our simulations so far, we specify the discount rates to be constant in our structural models and evaluate the accuracy of our discount rate estimates with different cash flow properties.

A natural questions arises regarding the accuracy of our estimates in an environment with time-varying risk premium, especially when the risk premium process is persistent. Intuitively, if the risk premium process contains a persistent component, then observations from a short sample would understate the true variability of the discount rate and thus misguide us to overstate the accuracy of the estimates. Moreover, it is hard to identify the persistent component in the first place if the sample is short.

To understand how such an effect would bias our standard error estimates from block bootstrap simulations, we adopt the version of the Bansal and Yaron (2004) model with stochastic volatility which induces time-varying persistent risk premium.

The cash flow process is similar to the base-line model introduced in Section 5 except that we now cash flow growth volatility to be random following Bansal and Yaron (2004):

$$
\begin{aligned}
D_{t+1} &= D_t \cdot \exp\left(g_{d,t+1}\right) \\
x_{t+1} &= \rho x_t + \phi_e \sigma_t e_{t+1} \\
g_{d,t+1} &= \mu_d + \phi x_t + \phi_d \sigma_t u_{t+1} \\
\sigma_{t+1}^2 &= \sigma^2 + \nu_1 \left(\sigma_t^2 - \sigma^2\right) + \sigma_w w_{t+1} \\
&\quad e_{t+1}, u_{t+1}, w_{t+1} \sim N\ i.i.d\ \left(0,1\right).
\end{aligned}
$$

In addition, instead of specifying constant discount rate exogenously as in Appendix A, we allow discount rate to be endogenous by specifying the Epstein and Zin (1989) preference as in Bansal and Yaron (2004). According to Equation (12) in Appendix A of Bansal et al. (2012), the log price-dividend ratio for the market claim to dividends is approximately: $z_{m,t} = A_{0,m} + A_{1,m} x_t + A_{2,m} \sigma_t^2$, where

$$A_{0,m} = \frac{1}{1-\kappa_{1,m}} \left[\Gamma_0 + \kappa_{0,m} + \mu_d + \kappa_{1,m} A_{2,m} (1-\nu_1) \sigma^2 \right.$$
$$\left. + \frac{1}{2} (\kappa_{1,m} A_{2,m} - \lambda_w)^2 \sigma_w^2 \right]$$
$$A_{1,m} = \frac{\phi_m - \frac{1}{\psi}}{1-\kappa_{1,m}\rho}$$
$$A_{2,m} = \frac{1}{1-\kappa_{1,m}\nu_1} \left[\Gamma_2 + \frac{1}{2} \left((\pi - \lambda_\eta)^2 + (\kappa_{1,m} A_{1,m} \phi_e - \lambda_e)^2 \right) \right]$$

where

$$\Gamma_0 = \log\delta - \frac{1}{\psi}\mu_d - (\theta-1) \left[A_2 (1-\nu_1) \sigma^2 + \frac{\theta}{2} (\kappa_1 A_2 \sigma_w)^2 \right]$$
$$\Gamma_2 = (\theta-1)(\kappa_1\nu_1 - 1) A_2$$
$$A_1 = \frac{1-\frac{1}{\psi}}{1-\kappa_1\rho}$$
$$A_2 = -\frac{(\gamma-1)\left(1-\frac{1}{\psi}\right)}{2(1-\kappa_1\nu_1)} \left[1 + \left(\frac{\kappa_1\phi_e}{1-\kappa_1\rho} \right)^2 \right]$$
$$\lambda_\eta = \gamma$$
$$\lambda_e = (1-\theta)\kappa_1 A_1 \phi_e$$
$$\lambda_w = (1-\theta)\kappa_1 A_2.$$

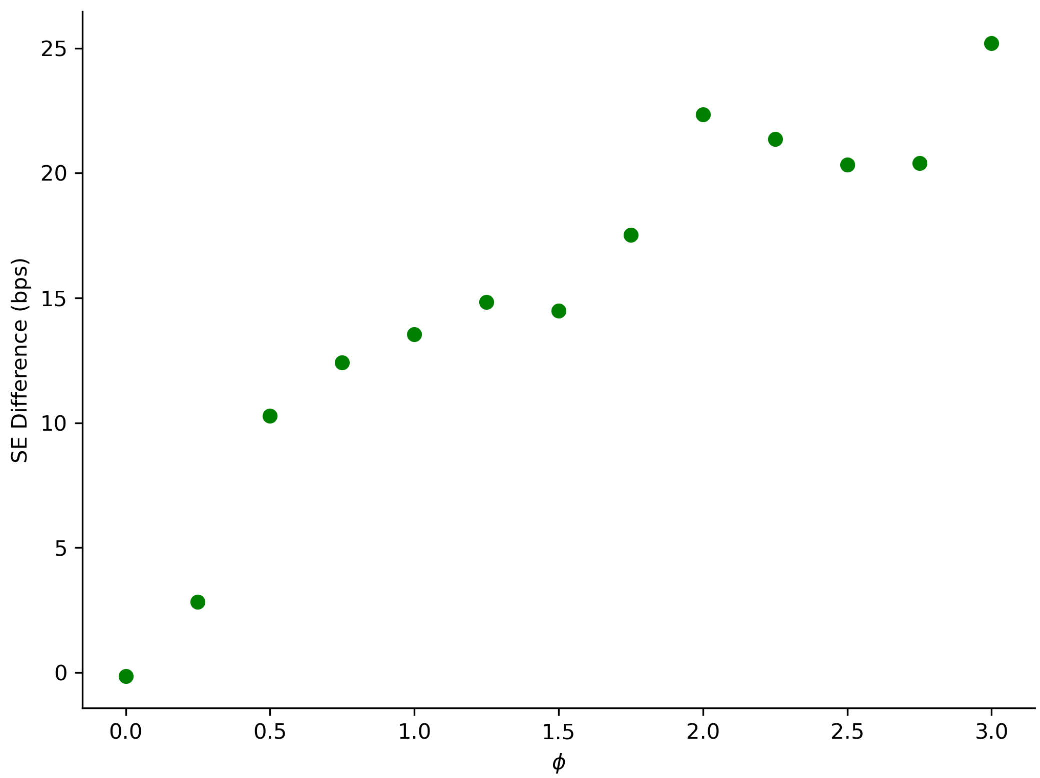
Following Bansal et al. (2012), we specify the parameters as: $\mu_d = 0.0015$, $\rho = 0.979$, $\sigma = 0.0078$, $\phi_e = 0.044$, $\sigma_w = 0.23 \times 10^{-5}$, $\gamma = 10$, $\psi = 1.5$, $\delta = 0.998$, $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, $\pi = 0$, $\nu_1 = 0.987$, $\phi_d = 4.5$, $\kappa_1 = 0.997$, $\kappa_{m,1} = 0.985$, $\kappa_{m,0} = 0.0937$.

To evaluate the influence of persistent risk premium on the accuracy of block bootstrap standard errors, we adopt $\phi \in [0,3]$. When $\phi = 0$, the long-run risk component is switched off; when $\phi = 3$, which is the specification in Bansal and Yaron (2004), long-run risk is fully turned on. We then repeat the exercise in Subsection B.1 and compare the standard error of the discount rate evaluated from the simulated paths of the true model versus the standard error derived from block bootstrap simulations.

Figure 4 shows that when ϕ is small, the standard error derived from block bootstrap simulations is almost identical to the standard error from the true model simulations, and when the long-run risk component is fully switched on, the block bootstrap simulations understate the standard error by about 25 bps.

The exercise thus show that, although block bootstrap simulations would indeed under-evaluate the standard error of discount rate estimates with the presence of significant long-run risk component, the magnitude of the understatement is small compared to discount rate estimates. Therefore, the presence of long-run risk and persistent discount rate variations is unlikely to materially affect our inference.

Figure 4: Standard Error Comparison



This figure plots the difference between the standard error of the discount rate in the simulations of the model versus the standard error from the block bootstrap simulation across different ϕ specifications.

Appendix C. Key Results for 10-Year and 5-Year Horizons

C.1 Key Results for the 10-Year Horizon

Table 10: Anomalies Based on Funding Cost

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Market Beta														
$y-r_f^{10}$	6.07	6.96	6.77	7.13	7.04	7.37	6.96	7.5	8.57	10.15	4.07**	2.22	0.74	1.15
$\bar{h}_1-r_f$	7.53	8.07	8.37	9.38	9.72	9.98	10.1	9.87	9.71	11.28	3.75	1.25	4.22***	3.23
Panel (b): Size														
$y-r_f^{10}$	10.8	11.05	10.84	10.84	10.1	8.82	8.42	8.09	8.18	6.02	-4.78***	-3.02	-0.45	-1.16
$\bar{h}_1-r_f$	15.24	13.82	12.82	12.6	11.57	11.58	10.92	10.55	9.65	7.84	-7.41***	-2.76	5.85***	4.45
Panel (c): Book-to-Market														
$y-r_f^{10}$	6.39	5.89	7.0	7.33	8.16	9.38	8.64	8.98	11.35	14.71	8.32***	4.42	0.81*	1.81
$\bar{h}_1-r_f$	5.43	4.9	6.57	6.26	6.2	8.43	8.2	9.26	9.55	11.67	6.24**	2.13	4.23***	2.75
Panel (d): Gross Profitability														
$y-r_f^{10}$	10.82	9.99	5.52	5.95	6.85	6.81	6.84	8.11	7.89	10.16	-0.66	-0.72	2.88***	4.21
$\bar{h}_1-r_f$	3.32	5.47	5.47	4.43	5.97	6.79	6.7	7.12	8.1	8.94	5.62***	2.9	3.27***	2.61
Panel (e): Investment														
$y-r_f^{10}$	8.53	9.07	8.42	7.38	6.87	7.39	6.99	6.99	8.52	9.06	0.53	0.36	1.66***	2.64
$\bar{h}_1-r_f$	7.6	9.82	9.02	6.9	6.14	6.92	6.08	8.06	4.56	1.02	-6.59***	-3.48	2.49	1.42
Panel (f): Credit Rating														
$y-r_f^{10}$	14.08	8.3	10.49	10.24	7.25	6.93	7.36	6.6	9.45	7.59	-6.48***	-3.58	2.76***	3.8
$\bar{h}_1-r_f$	5.41	6.18	8.87	6.79	7.81	7.55	7.7	7.81	9.37	7.63	2.22	0.57	3.31	1.53
Panel (g): Fail Probability														
$y-r_f^{10}$	8.01	8.72	7.67	6.79	6.98	6.96	6.69	6.62	7.18	6.96	-1.05**	-2.29	0.75***	3.92
$\bar{h}_1-r_f$	12.69	9.52	8.53	7.23	8.82	7.48	7.75	8.31	6.65	3.11	-9.58**	-2.38	3.92*	1.71

This table documents the anomalies motivated by models of funding cost. Stocks in the cross section are sorted into 10 deciles based on the corresponding characteristics. The index of the portfolio increases with the sorting characteristic. For annual (monthly) anomalies, portfolios are rebalanced by the end of every December (month). "$y-r_f^{10}$" is the difference between the estimated long-term discount rate and the 10-year zero-coupon rate. "$\bar{h}_1-r_f$" is the annualized average short-term return of the trading strategy over the 3-month T-bill rate. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-statistics for "$y-r_f^{10}$" are based on the standard errors estimated from block bootstrap simulations detailed in Section 2.4. The t-statistics for "$\bar{h}_1-r_f$" are the standard OLS t-statistics.

Table 11: Anomalies Based on Trading Prices and Volume

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Momentum														
$y-r_f^{10}$	8.63	8.15	7.68	7.36	6.94	6.99	7.22	7.26	7.96	8.25	-0.38	-0.84	1.28***	7.22
$\bar{h}_1-r_f$	-1.44	3.59	4.07	7.34	7.47	7.77	9.19	10.34	11.18	15.31	16.76***	5.21	3.35**	2.04
Panel (b): Long-Term Reversal														
$y-r_f^{10}$	10.79	9.59	9.12	8.34	7.92	7.28	6.96	6.53	6.32	5.8	-4.99***	-11.02	0.53***	3.25
$\bar{h}_1-r_f$	12.76	11.64	10.66	9.63	9.03	9.42	9.2	8.57	7.45	7.47	-5.29**	-2.11	5.22***	3.43
Panel (c): Idiosyncratic Volatility														
$y-r_f^{10}$	5.81	7.31	8.17	8.54	8.98	9.53	9.33	8.66	7.73	8.19	2.38***	3.88	-2.0***	-10.55
$\bar{h}_1-r_f$	8.06	8.32	8.91	9.26	9.59	9.84	7.88	5.76	3.86	-2.84	-10.9***	-3.48	-0.51	-0.4
Panel (d): Abnormal Volume														
$y-r_f^{10}$	7.39	7.22	7.01	7.34	7.04	7.15	7.24	7.45	7.41	7.93	0.54***	2.7	0.39***	4.0
$\bar{h}_1-r_f$	4.47	4.9	3.92	7.21	6.51	7.2	8.1	7.77	10.44	9.57	5.1***	3.47	3.92***	3.08

This table documents the anomalies based on historical trading prices and volume. Stocks in the cross section are sorted into 10 deciles based on the corresponding characteristics. The index of the portfolio increases with the sorting characteristic. For annual (monthly) anomalies, portfolios are rebalanced by the end of every December (month). "$y-r_f^{10}$" is the difference between the estimated long-term discount rate and the 10-year zero-coupon rate. "$\bar{h}_1-r_f$" is the annualized average short-term return of the trading strategy over the 3-month T-bill rate. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-statistics for "$y-r_f^{10}$" are based on the standard errors estimated from block bootstrap simulations detailed in Section 2.4. The t-statistics for "$\bar{h}_1-r_f$" are the standard OLS t-statistics.

Table 12: Additional Anomalies

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Mispricing Score														
$y - r_f^{10}$	6.8	7.21	6.89	6.64	6.85	6.92	7.3	7.49	8.64	8.33	1.54***	5.11	0.86***	7.17
$\bar{h}_1 - r_f$	10.03	8.09	6.9	7.98	7.46	5.42	4.9	4.49	1.13	-3.18	-13.22***	-4.97	0.8	0.49
Panel (b): Illiquidity														
$y - r_f^{10}$	6.36	7.71	9.13	10.18	10.67	10.86	10.54	11.56	12.08	12.26	5.9***	3.51	-1.16**	-2.58
$\bar{h}_1 - r_f$	5.73	7.46	7.72	8.82	8.89	11.45	10.97	12.15	11.81	11.84	6.11	1.58	4.13**	2.56
Panel (c): Liquidity Risk														
$y - r_f^{10}$	8.47	7.72	7.69	6.38	6.3	6.47	7.33	7.65	6.83	7.71	-0.76	-0.74	1.3***	3.25
$\bar{h}_1 - r_f$	5.2	6.32	6.09	6.7	6.04	7.46	7.57	7.81	6.79	11.1	5.9***	2.95	3.98**	2.02
Panel (d): Coskewness														
$y - r_f^{10}$	7.09	6.77	6.85	7.42	7.66	7.76	7.87	7.0	7.09	6.77	-0.31	-1.5	-0.78***	-9.39
$\bar{h}_1 - r_f$	9.04	8.01	6.72	7.35	7.0	6.95	6.92	5.06	5.28	3.87	-5.18**	-2.45	3.06**	2.07
Panel (e): Duration														
$y - r_f^{10}$	12.3	10.12	9.88	8.74	8.12	7.98	7.08	6.3	5.68	5.99	-6.32***	-4.09	0.47	1.29
$\bar{h}_1 - r_f$	14.22	14.13	13.85	14.96	11.06	9.6	8.36	4.88	3.4	0.61	-13.61***	-4.35	2.92	1.35

This table documents several additional anomalies. Stocks in the cross section are sorted into 10 deciles based on the corresponding characteristics. The index of the portfolio increases with the sorting characteristic. For annual (monthly) anomalies, portfolios are rebalanced by the end of every December (month). "$y - r_f^{10}$" is the difference between the estimated long-term discount rate and the 10-year zero-coupon rate. "$\bar{h}_1 - r_f$" is the annualized average short-term return of the trading strategy over the 3-month T-bill rate. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-statistics for "$y - r_f^{10}$" are based on the standard errors estimated from block bootstrap simulations detailed in Section 2.4. The t-statistics for "$\bar{h}_1 - r_f$" are the standard OLS t-statistics.

C.2 Key Results for the 5-Year Horizon

Table 13: Anomalies Based on Funding Cost

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Market Beta														
$y-r_f^5$	6.16	7.42	7.36	7.67	7.68	7.86	7.39	7.85	9.76	11.62	5.46**	2.12	0.97	1.19
$\bar{h}_1-r_f$	7.53	8.07	8.37	9.38	9.72	9.98	10.1	9.87	9.71	11.28	3.75	1.25	4.22***	3.23
Panel (b): Size														
$y-r_f^5$	14.18	13.12	11.94	12.26	10.69	9.84	9.57	9.23	8.64	6.34	-7.84***	-4.43	0.31	0.66
$\bar{h}_1-r_f$	15.24	13.82	12.82	12.6	11.57	11.58	10.92	10.55	9.65	7.84	-7.41***	-2.76	5.85***	4.45
Panel (c): Book-to-Market														
$y-r_f^5$	6.16	6.48	8.3	8.46	8.85	10.54	9.56	10.2	13.34	17.91	11.75***	5.06	1.27*	1.95
$\bar{h}_1-r_f$	5.43	4.9	6.57	6.26	6.2	8.43	8.2	9.26	9.55	11.67	6.24**	2.13	4.23***	2.75
Panel (d): Gross Profitability														
$y-r_f^5$	10.54	10.74	6.51	6.63	7.2	7.23	6.71	8.83	9.09	10.85	0.31	0.22	3.09***	3.76
$\bar{h}_1-r_f$	3.32	5.47	5.47	4.43	5.97	6.79	6.7	7.12	8.1	8.94	5.62***	2.9	3.27***	2.61
Panel (e): Investment														
$y-r_f^5$	10.36	9.67	9.48	8.32	7.16	7.59	7.89	7.9	8.76	9.34	-1.02	-0.59	2.16***	2.6
$\bar{h}_1-r_f$	7.6	9.82	9.02	6.9	6.14	6.92	6.08	8.06	4.56	1.02	-6.59***	-3.48	2.49	1.42
Panel (f): Credit Rating														
$y-r_f^5$	16.25	8.73	14.51	11.32	8.94	7.73	8.19	8.03	10.73	9.14	-7.12*	-1.85	2.88**	1.99
$\bar{h}_1-r_f$	5.41	6.18	8.87	6.79	7.81	7.55	7.7	7.81	9.37	7.63	2.22	0.57	3.31	1.53
Panel (g): Fail Probability														
$y-r_f^5$	9.7	11.17	9.13	8.13	8.22	8.21	7.52	7.95	8.04	8.15	-1.55**	-2.24	1.05***	3.13
$\bar{h}_1-r_f$	12.69	9.52	8.53	7.23	8.82	7.48	7.75	8.31	6.65	3.11	-9.58**	-2.38	3.92*	1.71

This table documents the anomalies motivated by models of funding cost. Stocks in the cross section are sorted into 10 deciles based on the corresponding characteristics. The index of the portfolio increases with the sorting characteristic. For annual (monthly) anomalies, portfolios are rebalanced by the end of every December (month). "$y-r_f^5$" is the difference between the estimated long-term discount rate and the 5-year zero-coupon rate. "$\bar{h}_1-r_f$" is the annualized average short-term return of the trading strategy over the 3-month T-bill rate. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-statistics for "$y-r_f^5$" are based on the standard errors estimated from block bootstrap simulations detailed in Section 2.4. The t-statistics for "$\bar{h}_1-r_f$" are the standard OLS t-statistics.

Table 14: Anomalies Based on Trading Prices and Volume

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Momentum														
$y-r_f^5$	9.36	8.5	8.11	7.71	7.33	7.54	8.08	8.4	9.55	9.81	0.45	0.67	1.87***	6.25
$\bar{h}_1-r_f$	-1.44	3.59	4.07	7.34	7.47	7.77	9.19	10.34	11.18	15.31	16.76***	5.21	3.35**	2.04
Panel (b): Long-Term Reversal														
$y-r_f^5$	12.65	11.95	11.08	9.69	8.69	8.26	7.86	7.35	7.22	7.1	-5.56***	-8.99	1.26***	4.86
$\bar{h}_1-r_f$	12.76	11.64	10.66	9.63	9.03	9.42	9.2	8.57	7.45	7.47	-5.29**	-2.11	5.22***	3.43
Panel (c): Idiosyncratic Volatility														
$y-r_f^5$	6.21	8.08	9.08	9.39	10.21	11.27	11.55	9.56	8.71	8.04	1.83***	2.75	-2.98***	-9.85
$\bar{h}_1-r_f$	8.06	8.32	8.91	9.26	9.59	9.84	7.88	5.76	3.86	-2.84	-10.9***	-3.48	-0.51	-0.4
Panel (d): Abnormal Volume														
$y-r_f^5$	8.01	7.87	7.67	7.96	7.92	8.04	7.79	8.12	8.22	8.74	0.73**	2.57	0.23*	1.68
$\bar{h}_1-r_f$	4.47	4.9	3.92	7.21	6.51	7.2	8.1	7.77	10.44	9.57	5.1***	3.47	3.92***	3.08

This table documents the anomalies based on historical trading prices and volume. Stocks in the cross section are sorted into 10 deciles based on the corresponding characteristics. The index of the portfolio increases with the sorting characteristic. For annual (monthly) anomalies, portfolios are rebalanced by the end of every December (month). "$y-r_f^5$" is the difference between the estimated long-term discount rate and the 5-year zero-coupon rate. "$\bar{h}_1-r_f$" is the annualized average short-term return of the trading strategy over the 3-month T-bill rate. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-statistics for "$y-r_f^5$" are based on the standard errors estimated from block bootstrap simulations detailed in Section 2.4. The t-statistics for "$\bar{h}_1-r_f$" are the standard OLS t-statistics.

Table 15: Additional Anomalies

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Mispricing Score														
$y - r_f^5$	7.83	7.87	7.29	7.3	7.77	7.46	7.62	8.07	8.83	8.74	0.92**	2.17	0.7***	4.45
$\bar{h}_1 - r_f$	10.03	8.09	6.9	7.98	7.46	5.42	4.9	4.49	1.13	-3.18	-13.22***	-4.97	0.8	0.49
Panel (b): Illiquidity														
$y - r_f^5$	6.77	8.49	9.8	10.97	12.4	12.73	13.71	14.32	15.12	16.14	9.37***	5.24	-0.93	-1.47
$\bar{h}_1 - r_f$	5.73	7.46	7.72	8.82	8.89	11.45	10.97	12.15	11.81	11.84	6.11	1.58	4.13**	2.56
Panel (c): Liquidity Risk														
$y - r_f^5$	8.32	8.63	8.45	6.75	6.91	7.57	8.17	7.61	7.55	9.8	1.48	0.94	1.33**	2.28
$\bar{h}_1 - r_f$	5.2	6.32	6.09	6.7	6.04	7.46	7.57	7.81	6.79	11.1	5.9***	2.95	3.98**	2.02
Panel (d): Coskewness														
$y - r_f^5$	6.99	7.02	7.33	7.87	8.43	8.41	8.24	7.64	8.12	7.98	0.99***	3.33	-0.89***	-5.55
$\bar{h}_1 - r_f$	9.04	8.01	6.72	7.35	7.0	6.95	6.92	5.06	5.28	3.87	-5.18**	-2.45	3.06**	2.07
Panel (e): Duration														
$y - r_f^5$	15.3	13.41	11.9	11.3	9.7	9.75	7.44	7.26	6.3	5.53	-9.77***	-5.53	0.41	0.57
$\bar{h}_1 - r_f$	14.22	14.13	13.85	14.96	11.06	9.6	8.36	4.88	3.4	0.61	-13.61***	-4.35	2.92	1.35

This table documents several additional anomalies. Stocks in the cross section are sorted into 10 deciles based on the corresponding characteristics. The index of the portfolio increases with the sorting characteristic. For annual (monthly) anomalies, portfolios are rebalanced by the end of every December (month). "$y - r_f^5$" is the difference between the estimated long-term discount rate and the 5-year zero-coupon rate. "$\bar{h}_1 - r_f$" is the annualized average short-term return of the trading strategy over the 3-month T-bill rate. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-statistics for "$y - r_f^5$" are based on the standard errors estimated from block bootstrap simulations detailed in Section 2.4. The t-statistics for "$\bar{h}_1 - r_f$" are the standard OLS t-statistics.

Appendix D. Comparison Between Long-Term Discount Rate and Relative Discount Factor

Table 16: Anomalies Based on Funding Cost

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Market Beta														
$y-r_f^{15}$	5.76	6.19	6.44	6.67	6.66	6.51	5.98	6.37	7.06	8.01	2.25	1.59	0.16	0.33
RDF	59.95	58.72	57.56	55.28	55.79	55.85	58.35	56.98	51.36	47.44	-12.52**	-2.15	-1.45	-0.67
Panel (b): Size														
$y-r_f^{15}$	9.33	9.67	9.73	9.71	9.0	8.2	7.62	7.27	7.27	5.33	-4.0***	-3.22	-0.7**	-2.32
RDF	41.41	42.08	42.35	42.74	45.43	47.4	49.98	51.57	51.3	63.35	21.94***	4.41	3.12*	1.85
Panel (c): Book-to-Market														
$y-r_f^{15}$	5.37	5.3	6.32	6.51	7.22	8.36	7.87	8.28	9.95	12.5	7.13***	4.6	0.49	1.2
RDF	59.78	60.97	55.46	55.29	52.31	46.88	50.51	48.2	43.88	35.75	-24.02***	-3.04	0.5	0.27
Panel (d): Gross Profitability														
$y-r_f^{15}$	10.4	8.88	4.81	5.45	6.01	6.06	6.14	6.63	6.94	8.81	-1.59**	-1.96	2.73***	4.92
RDF	40.9	47.38	64.6	62.09	57.02	57.33	57.5	56.56	53.71	45.71	4.81	1.48	-10.25***	-4.35
Panel (e): Investment														
$y-r_f^{15}$	8.57	7.88	7.47	6.7	6.54	6.45	6.16	6.35	7.28	7.39	-1.18	-0.98	1.28**	2.51
RDF	45.71	50.7	53.06	55.52	55.89	56.7	58.03	55.51	51.46	49.42	3.71	0.62	-6.97***	-2.85
Panel (f): Credit Rating														
$y-r_f^{15}$	11.71	8.21	7.74	7.99	6.5	5.84	6.07	5.73	7.19	5.32	-6.39***	-4.23	1.94***	3.27
RDF	36.66	45.41	48.78	49.9	59.76	59.57	61.54	62.5	54.18	62.92	26.25***	3.71	-9.87***	-4.0
Panel (g): Fail Probability														
$y-r_f^{15}$	6.29	6.53	6.16	5.59	5.63	5.79	5.7	5.81	6.17	6.2	-0.09	-0.21	0.58***	3.55
RDF	56.15	57.25	59.99	61.46	61.25	60.1	60.48	60.28	57.03	54.75	-1.4	-0.58	-4.37***	-4.54

This table documents the anomalies motivated by models of funding cost. Stocks in the cross section are sorted into 10 deciles based on the corresponding characteristics. The index of the portfolio increases with the sorting characteristic. For annual (monthly) anomalies, portfolios are rebalanced by the end of every December (month). "$y-r_f^{15}$" is the difference between the estimated long-term discount rate and the 15-year zero-coupon rate. "RDF" is the relative discount factor. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-statistics for "$y-r_f^{15}$" and "RDF" are based on the standard errors estimated from block bootstrap simulations detailed in Section 2.4. The t-statistics for "$\bar{h}_1 - r_f$" are the standard OLS t-statistics.

Table 17: Anomalies Based on Trading Prices and Volume

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Momentum														
$y-r_f^{15}$	7.6	7.45	7.2	6.69	6.34	6.41	6.48	6.36	6.67	6.82	-0.78**	-2.12	0.76***	5.13
RDF	47.78	48.83	51.46	55.13	56.99	56.55	56.7	57.93	56.43	55.68	7.9***	4.11	-4.59***	-6.38
Panel (b): Long-Term Reversal														
$y-r_f^{15}$	9.1	8.36	8.06	7.52	7.17	6.63	6.34	5.85	5.37	4.87	-4.22***	-11.55	0.03	0.21
RDF	40.71	45.18	47.59	50.48	52.69	55.52	56.6	59.85	62.56	65.02	24.32***	10.85	-0.74	-1.02
Panel (c): Idiosyncratic Volatility														
$y-r_f^{15}$	5.27	6.58	7.24	7.24	7.91	8.07	7.91	7.29	6.85	7.32	2.06***	3.98	-1.49***	-8.96
RDF	62.91	56.68	53.19	52.27	49.77	47.82	47.98	50.24	51.84	48.04	-14.87***	-6.47	6.07***	7.52
Panel (d): Abnormal Volume														
$y-r_f^{15}$	6.42	6.15	6.09	6.3	6.07	6.23	6.21	6.31	6.41	6.88	0.46***	2.72	0.31***	3.74
RDF	58.17	59.08	59.73	58.77	59.56	58.79	59.04	58.51	58.33	56.34	-1.83**	-2.22	-1.2***	-2.81

This table documents the anomalies based on historical trading prices and volume. Stocks in the cross section are sorted into 10 deciles based on the corresponding characteristics. The index of the portfolio increases with the sorting characteristic. For annual (monthly) anomalies, portfolios are rebalanced by the end of every December (month). "$y - r_f^{15}$" is the difference between the estimated long-term discount rate and the 15-year zero-coupon rate. "RDF" is the relative discount factor. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-statistics for "$y - r_f^{15}$" and "RDF" are based on the standard errors estimated from block bootstrap simulations detailed in Section 2.4. The t-statistics for "$\bar{h}_1 - r_f$" are the standard OLS t-statistics.

Table 18: Additional Anomalies

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	HML	t-stat	EMM	t-stat
Panel (a): Mispricing Score														
$y-r_f^{15}$	5.84	6.28	6.1	5.93	5.99	6.0	6.3	6.86	7.84	7.63	1.79***	7.38	0.91***	8.89
RDF	60.25	57.95	58.87	60.21	58.95	58.58	56.55	54.1	50.25	48.51	-11.74***	-10.38	-4.52***	-8.84
Panel (b): Illiquidity														
$y-r_f^{15}$	5.67	7.03	7.96	9.14	9.62	9.52	9.49	9.79	9.83	10.56	4.89***	3.33	-1.3***	-2.95
RDF	60.67	52.63	49.2	43.79	42.01	43.86	43.41	43.96	42.14	30.64	-30.03***	-5.41	3.58*	1.81
Panel (c): Liquidity Risk														
$y-r_f^{15}$	7.18	7.41	6.93	5.84	5.57	5.79	6.44	6.66	6.29	6.68	-0.5	-0.59	1.21***	3.59
RDF	53.16	51.06	55.62	59.52	61.12	59.65	57.57	57.27	58.53	54.16	1.0	0.23	-6.16***	-3.5
Panel (d): Coskewness														
$y-r_f^{15}$	6.46	6.09	6.18	6.39	6.7	6.9	6.7	6.05	6.12	6.08	-0.38**	-2.11	-0.61***	-8.2
RDF	58.76	59.51	58.74	57.22	55.89	53.8	54.14	58.59	56.72	56.55	-2.21**	-2.21	3.04***	7.9
Panel (e): Duration														
$y-r_f^{15}$	10.1	8.73	8.47	7.66	7.01	6.98	6.19	5.69	5.54	4.59	-5.51***	-4.19	0.25	0.68
RDF	41.73	48.96	46.44	50.65	53.77	53.86	57.62	60.06	58.69	64.05	22.33***	2.78	-0.46	-0.22

This table documents several additional anomalies. Stocks in the cross section are sorted into 10 deciles based on the corresponding characteristics. The index of the portfolio increases with the sorting characteristic. For annual (monthly) anomalies, portfolios are rebalanced by the end of every December (month). "$y-r_f^{15}$" is the difference between the estimated long-term discount rate and the 15-year zero-coupon rate. "RDF" is the relative discount factor. "HML" is the difference between decile 10 and decile 1. "EMM" is the difference between the average of deciles 1, 2, 9, 10 and the average of deciles 5, 6. All units are in percentage points. The t-statistics for "$y-r_f^{15}$" and "RDF" are based on the standard errors estimated from block bootstrap simulations detailed in Section 2.4. The t-statistics for "$\bar{h}_1 - r_f$" are the standard OLS t-statistics.