The Maturity Premium *

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Abstract

We document a maturity premium - a portfolio going long in long-maturity financed firms and short in short-maturity financed firms earns a 0.21% monthly premium, above what is predicted by the standard factor models. Moreover, we find that the beta of this portfolio is countercyclical. We argue that this is due to the weaker incentives of firms with long debt maturities to delever after negative shocks. Therefore, they tend to exhibit high leverage and high betas during downturns when the market price of risk is high, and investors require compensation for this risk. In a calibrated model we demonstrate that the difference in cyclical leverage dynamics between short and long-maturity financed firms can both qualitatively and quantitatively account for the observed maturity premium.

JEL Classifications: G12, G32, G33.

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1 Introduction

In the aftermath of the financial crisis the short-term debt financing has been shown to expose firms to rollover risk, which arises from many factors including liquidity shocks to investors.[1] We aim to see if short-term financed firms are indeed more risky and exhibit higher equity returns.[2]

Building on the recent contributions Dangl and Zechner (2016) and DeMarzo and He (2018), we show that long-term financing makes firms' leverage more counter-cyclical. Firms financed with long-term debt are more highly levered in downturns, when market price of risk is high, resulting in higher, not lower, expected returns in cross-section. The key deviation from a seminal work of Leland (1994*b*) is the absence of pre-commitment to leverage reductions. The trade-off theory would prescribe shareholders to actively buy back debt in downturns. However, it is not incentive-compatible with shareholders' objective, as injecting equity would imply a transfer of wealth to existing debtors. This conflict of interest, as described in Admati et al. (2018), is similar in spirit to the debt overhang problem of Myers (1977), except the distortion is with respect to the liability side of the balance sheet, not assets.

We begin by investigating the empirical relation between the maturity of debt and the cross-section of equity returns. We use the fraction of debt that matures in more than 3 years as a measure of firm's debt maturity (as in Custódio et al. (2013)), which we construct using data from COMPUSTAT. We sort firms into portfolios based on the debt maturity, controlling for their size, as in Fama and French (1992). Analysing the CRSP firms from January 1976 to December 2017, we find that long-term financed firms earn 0.21% return per month more than short-term financed firms, controlling for their exposure to the systemic risk. We call this return difference 'Maturity Premium' and emphasize that it is not due to mispricing. While it can not be attributed to the differences in systemic exposures of firms, we argue that it is due to counter-cyclical nature of the systemic risk exposure of the portfolio that is long firms financed with long-term debt and short firms financed with short-term debt. The maturity premium arises rationally in our conditional CAPM when returns are measured relative to the unconditional CAPM.

To ensure that the maturity premium is not driven by known firm characteristics that can be related to the differences in maturities between firms, we examine the returns of our long-short portfolio relative to

[1] Pastor and Stambaugh (2003); Acharya et al. (2011); He and Xiong (2012); Eisenbach (2017).
[2] See Friewald et al. (2018) for evidence that refinancing risk is indeed priced.

the Fama and French (1993) three-factor model or Fama and French (2016) five-factor model. Our portfolio loads negatively on the market risk, consistent with the observation that short-term financed firms have higher market betas on average. The portfolio also loads positively on the value factor (HML), confirming the intuition that value firms, besides having higher operating leverage and therefore more counter-cyclical asset betas (Zhang, 2005), are also financed with long-term debt (Choi, 2013). However, the maturity premium is not subsumed by the known factors and remains statistically significant after controlling for the differences the factor exposures of long- and short-maturity financed firms. The magnitude of the maturity premium is economically significant, comparable to the size of the value premium (0.37% per month in our sample).

Numerous robustness tests confirm our results. Our main results hold not only for value-weighted portfolio sorts, but also for equally-weighted.

To explain the empirical findings, we propose a conditional CAPM with a dynamic corporate finance foundation. Extending the framework of Dangl and Zechner (2016) and DeMarzo and He (2018), we develop a partial equilibrium dynamic trade-off model and study its implications for the cyclicality of leverage and stock returns. For tractability, we do not model the optimal maturity as a firm's decision and assume an exogenous maturity structure. However, were we to incorporate endogenous maturity choice into our framework, as in Dangl and Zechner (2016), the results would have been qualitatively the same. We also take an exogenous pricing kernel and the corresponding counter-cyclical market price of risk. In our model, firms optimally make financing and rollover decisions in response to both idiosyncratic and aggregate productivity shocks.

Following a negative productivity shock, firms in our model face a trade-off. On one hand, the higher expected default costs incentivise shareholders to reduce leverage. On the other hand, if shareholders go and buy back outstanding bonds at market prices, they would transfer wealth to remaining debtors, who benefit from lower default probability without participating in the leverage reduction. This creates a conflict of interest between shareholders and bondholders. In classical models like Fischer et al. (1989) shareholders pre-commit before issuing debt to optimally reduce leverage following drops in profitability. However, if we rule out pre-commitment to deleveraging through, say, covenants, shareholders would be hesitant to actively repurchase debt, as was pointed out by Admati et al. (2018). Dangl and Zechner (2016) and DeMarzo and

He (2018) argue that the issue is mitigated due to rollover decisions. Because part of the debt matures, shareholders' incentives are aligned with those of potential new debtors. Following profitability drops, shareholders optimally chose not to roll-over part of the maturing debt, thereby reducing leverage.

Short maturity implies higher rollover rate. Shareholders of firms financed with short-term credit suffer less from the mis-alignment of incentives with debtors. If profitability falls, they do not roll over debt and their leverage goes down quickly. This is in contrast to firms financed with long-term debt. Following a profitability decline, shareholders also chose not to roll over part of their debt. But because only a small fraction of their debt matures each period, the leverage reduces very slowly and stays elevated for a substantial period of time.

Firms rationally anticipate that short-term debt offers less conflicts and more flexibility and therefore, chose higher level of leverage than long-term financed firms. We control for the differences in average leverage levels between short and long-maturity financed firms by introducing heterogeneity in their idiosyncratic volatility. Consistent with empirical evidence, we compare firms financed with long-term debt and low idiosyncratic volatility to firms financed with short-term debt and high idiosyncratic volatility. While their average levels of leverage are the same, the dynamics of their leverage in the business cycle is starkly different.

When aggregate productivity falls sharply, short-maturity financed firms experience a higher spike in market leverage than long-maturity financed firms, a manifestation of the rollover risk. However, they quickly reduce leverage by not rolling over maturing debt. In contrast, long-maturity financed firms experience a more modest spike in market leverage, but due to low rollover rate their leverage stays elevated for a long period of time.

This matters to shareholders, who assign a higher price to the systemic risk in downturns. Long-term financed firms expose shareholders to more systemic risk during downturns than short-term financed firms. And shareholders require a compensation for this. Conditional CAPM holds in our setting. However, if we analyze equity returns of long-term and short-term financed firms controlling only for the average levels of systemic exposure, i.e. if we apply unconditional CAPM, we observe a maturity premium. A portfolio that is long long-maturity financed firms and short short-maturity financed firms earns an alpha relative to CAPM. However, it is not a mispricing. Long-term financed firms have higher levels of leverage, and therefore, higher levels of equity beta exactly when market price of risk is high. This translates into an unconditional alpha. Short-term financed firms exhibit lower covariance between their market beta and market price of risk, and therefore command a smaller unconditional alpha.

Our model replicates both the direction and the magnitude of the relation between the debt maturity and equity returns. In the simulated panel of firms, the alpha in the unconditional CAPM increases monotonously in maturity. Moreover, in the calibrated version of the model where firms characteristics (maturity, idiosyncratic risk and marginal tax rates) of short- and long-maturity financed firms resemble those we see in LMS buckets in data, we find that a long-short portfolio earns a 0.19% monthly alpha. This is close to the maturity premium of 0.21% we observe in the data. Moreover, we show that the alpha is higher for firms with higher marginal tax rate and lower idiosyncratic volatility across all maturities.

We strengthen the link between the theoretical predictions of our model and provide supporting empirical evidence for the link between maturity and equity returns through counter-cyclical leverage dynamics. In particular, we estimate a conditional CAPM model. We use information on macro-variables to predict the market price of risk, as in Choi (2013). We find that our portfolio that is long long-maturity financed firms and short short-maturity financed firms has higher exposure to systemic risk when market price of risk is high, i.e. in downturns. Moreover, the alpha estimate goes down and is no longer statistically significant. This is consistent with our model prediction that the maturity premium is a compensation for risk of higher systemic exposure during downturns, and once time-varying factor loading is accounted for, we observe no mispricing. In addition, we see that it is the long lag of the portfolio that is responsible for the increase of the systemic exposure in downturns, while short leg shows no evidence of cyclicality. We also find that the maturity premium is most pronounced among higher-levered firms.

In our model the risk of increases in financial leverage generates a maturity premium. This finding relates to papers that have analyzed operating leverage as a possible source of risk and as an explanation for the value premium. Since value firms have exercised their growth options they tend to exhibit higher operating leverage, whereas growth firms tend to have low overhead costs and operating leverage (Zhang, 2005; Cooper, 2006). If operating leverage is sticky, then decreasing revenues drive the equity of value firms closer to zero than that of growth firms due to the difference in their operating leverage. Consequently, the beta of growth firms is mostly constant in time, while the beta of value firms increases substantially in crises (Lettau and Ludvigson, 2001). Due to the fact that value firms are riskier in crises, they command an unconditionally higher required rate of return on their assets. While being plausible, the operating leverage alone cannot account for the entire size of value premium observed empirically (Clementi and Palazzo, 2015). To match the magnitude of the value premium, an extreme assumption of investment irreversibility is required, which contradicts empirical evidence on the sales of assets in the secondary market by at least 15% of firms in every given year.

In our paper, we demonstrate how long-maturity financial leverage contributes to the value premium. Financial leverage makes firms more sensitive to cash flow fluctuations in bad times, but only if the firm does not optimally delever. Short-maturity financed firms delever quickly, while long-maturity financed firms delever slowly or not at all. Therefore, the extent to which financial leverage can give rise to a value premium, depends on the difference in maturity choices of value and growth firms. Empirically, growth firms borrow with shorter maturities than value firms (Barclay and Smith, 1995; Barclay et al., 2003; Custódio et al., 2013). This can be attributed to lower cash-flow risk of value firms who have implemented their growth options.[3] The maturity choice is arguably driven by a trade-off between smaller investment debt-overhang (Myers, 1977) or financial debt overhang (Dangl and Zechner, 2016; DeMarzo and He, 2018) of short-term debt and higher transaction costs and higher default probability of long-term debt (Leland and Toft, 1996). Higher risk tilts the choice towards shorter maturities, that is why risky growth firms tend to borrow with short-maturity debt. Therefore, book-to-market acts as a noisy proxy of firms' maturity choices.

Thus, long-maturity debt of value firms creates a convex shape of equity's beta as a function of the aggregate state. It is precisely this time variation in beta, which is not captured by the standard unconditional CAPM equation, that creates a value premium through a maturity and leverage dynamics channel. Consistent with the idea that financial leverage contributes to the value premium, Doshi et al. (2016) find that unlevered equity returns exhibit no value premium.

More broadly, our paper contributes to the literature exploring asset pricing implications of corporate decisions. For example, Choi (2013) shows that a higher level of financial leverage of value firms contributes

[3] The presence of growth options increases the risk of firm's assets (Meckling and Jensen, 1976; Berk et al., 1999). The effect of options on equity risk is partially offset by the endogenously higher financial leverage of mature firms (Barclay et al., 2006), but not entirely — equity of growth firms is still riskier than equity of value firms, both in systematic (Shin and Stulz, 2000) and idiosyncratic (Cao et al., 2008) dimensions.

to the value premium. We argue that beyond the current level of debt the debt maturity plays a crucial role in generating an equity premium. Friewald et al. (2018) document an equity premium for rollover risk of firms with a larger fraction of their debt maturing within one year. While this result might appear to contradict our findings, in fact it is fully consistent with our hypothesis that short-maturity financed firms are risky over short holding horizons. Friewald et al. (2018) isolate the effect of rollover risk on firms over short horizons, considering leverage as fixed. Our analysis focuses on the combination of debt maturity and the dynamic adjustments of leverage. Cao (2018) argues that firms that borrow from bond market are more risky than firms that borrow predominantly from banks because they have more difficulty re-negotiating their debt, and this risk is priced by equityholders. Berk, Green and Naik (1999), Gomes and Schmid (2010), Kuehn and Schmid (2014), and Babenko, Boguth and Tserlukevich (2016), Gu, Hackbarth and Johnson (2017) among others, explore the implications of investment decisions and exercised growth options on equity returns.

Chen, Hackbarth and Strebulaev (2018) analyze the distress risk puzzle based on a dynamic capital structure model. In their model firms are exposed to time-varying indirect distress costs, which drive the apparent under-performance of distressed firms. In contrast to their paper, we focus on the role of finite debt maturity, while their firms issue perpetual debt. Our setup provides a complementary rational explanation for the distress risk puzzle. In our model, short-maturity financed firms have higher leverage and default probabilities, but their betas co-vary less with the market price of risk. Relative to the unconditional CAPM, short-maturity financed firms seem to under-perform long-maturity financed firms, consistent with the return pattern that gave rise to the distress risk puzzle.

Our paper also contributes to the literature on leverage adjustments. In particular, differences in maturity in our model explain differences in the speed of leverage adjustments between firms. In that sense, our paper is related to the literature on sticky leverage (Gomes et al., 2016) and transitory deviations of debt from the long-term target (DeAngelo et al., 2011; Ippolito et al., 2018). Mao and Tserlukevich (2014) explore leverage adjustments in a model where firms can use some of their assets, such as cash or other liquid assets, to repurchase debt. In this case a debt repurchase increases the riskiness of the firm, since some of the low-risk assets are de-facto transferred to tendering bondholders. The authors show that this may create incentives for equityholders to repurchase debt since bondholders accept lower tender offer prices when the debt repurchase is funded via the sale of low-risk corporate assets, such as cash. In our model we instead focus on the role of debt maturity and assume that equityholders cannot sell corporate assets to fund debt repurchases.

Chen et al. (2016) document that maturity is pro-cyclical. They argue that this is due to liquidity shocks to bond holders, which become more pronounced in crises and affect long-term bond holders more severely. In our setting, we abstract from optimal maturity adjustments.[4] However, it is likely that our results would be even strengthened by a potential shortening of maturities during downturns. This is so since, ceteris paribus, firms' betas increase with shorter maturities. In addition, firms' optimal leverage increases with shorter debt maturities in our framework. Thus, shorter maturities plus higher debt face values imply that firms would experience even sharper increases in betas during crises than fixed long-maturity firms do in our model.

Finally, other aspects of corporate policy decisions, such as the fraction of secured and convertible debt (Valta, 2016), cash holdings (Simutin, 2010), debt capacity (Hahn and Lee, 2009), and competition in the production chain (Gofman et al., 2018) have been shown to be related to equity risk premia. We contribute to this literature by demonstrating that the maturity choices by firms influence future leverage dynamics and therefore command an equity premium. Capital structure adjustments in our model vary over the business cycle. Hackbarth et al. (2006) also derive a model where firms capital structures vary with the business cycle. In contrast to our paper, they do not model the effects of debt maturity or the resulting equity return dynamics.

We also contribute to the dynamic corporate finance literature by extending the framework of Dangl and Zechner (2016) and DeMarzo and He (2018) by explicitly modeling time-varying market risk premia and analyzing the asset-pricing implications of leverage dynamics in such a setting.

2 Empirical Results

First, we document a positive relation between length of debt maturity and equity returns. We establish this result using a portfolio that is long long-maturity financed firms and short short-maturity financed firms, controlling for various risk factors including the Fama and French (1993) three factors and Fama and French

[4] Recent papers that tackle the question of maturity dynamics include Brunnermeier and Oehmke (2013); He and Milbradt (2016); Chaderina (2018) among others.

(2015) five factors.

2.1 Data

In our empirical analysis, we use monthly stock market data from the Center for Research in Security Prices (CRSP) and firm accounting data from COMPUSTAT's North America Fundamentals Annual file. We use CRSP's monthly returns on common equity of US-based enterprises from NYSE, AMEX, and NASDAQ. Firms are included when all items for computing a firm's debt maturity are available.[5] This restriction limits our sample, as COMPUSTAT does not provide all items required for the debt maturity proxy for fiscal years ending before 1974. To ensure consistency, we truncate the matched sample by excluding observations before January 1976.[6]

2.2 Debt Maturity and Firm Characteristics

The key variable in our analysis is debt maturity (*DM*). We compute it following Barclay and Smith (1995) and Custódio et al. (2013) as the relative amount of long-term debt maturing in more than 3 years.[7] Given recent empirical evidence on the use of credit lines by firms (Korteweg et al., 2018), we think that the cut-off of 3 years is justified as it allows us to exclude from long-term debt classification most of the debt from credit lines, which is arguably short-term in nature.

We compute several other metrics for each firm.[8] For every firm we compute leverage (*L*) as the ratio of book debt to the sum of book debt and market equity (see Danis et al., 2014). Moreover, we compute market capitalization (*ME*) as the price per share times the number of shares outstanding. Following Fama and French (1992, 1993) we compute book equity and the book-to-market ratio (*BM*). For the *BM*-ratio book equity for the fiscal year ending in year t is related to market equity as of December of year t. To be included in our sample, we require observations to have positive values for book equity, debt maturity and

[5]In the current analysis we include financial and utility companies. However, the main results are robust to excluding them from the sample.

[6] This truncation has to be interpreted under consideration of the procedure for matching accounting data and returns. Consequently, to ensure that necessary items are available for all firms, we have to drop two additional years from the matched return series.

[7]While we could have used alternative measures of debt maturity (for example, using information from Capital IQ), we chose to follow the previous literature. Incorporating information on the dispersion between different maturity buckets by calculating a weighted average maturity in years from COMPUSTAT did not change our main results.

[8] For detailed definitions, including the exact items used, we refer to Appendix D.

leverage.

The final sample of consists of 1,840,640 firm-month observations for a total of 18,392 unique firms over a time horizon from January 1976 until December 2017. Table 1 presents summary statistics for the final sample. In our sample, long-maturity debt plays a big role in financing of firms. On average, about half of the outstanding debt for our representative firm is maturing in more than 3 years. Yet there is quite a lot of heterogeneity. While for the firms in the largest quantile the fraction is more than 80%, in the smallest quantile more than two thirds of debt is maturing within the next three years.

Table 1: Summary Statistics. We compute mean, standard deviation, as well as the 25%-, 50%-, and 75%-quantiles of several firm characteristics monthly for the cross-section. The table presents time series averages of the monthly statistics. Excess returns, leverage and debt maturity are displayed in % and market equity in million USD. The underlying data set comprises matched observations from CRSP and COMPUSTAT over the time horizon January 1976 until December 2017. In total, the panel consists of 1,840,640 firm-month observations of 18,392 unique firms.

	Mean	SD	Q_{25}	Median	Q_{75}
Excess Returns	0.92	15.64	−6.08	0.01	6.45
Market Equity (ME)	2365.89	9903.90	59.10	260.15	1111.80
Book-to-Market Ratio (BM)	0.93	0.94	0.43	0.74	1.16
Leverage (L)	31.33	23.92	10.71	27.08	48.55
Debt Maturity (DM)	53.15	33.87	21.55	58.87	83.32

We ensure that accounting information on debt maturity, leverage, and book equity is publicly available upon portfolio assignment by following the procedures by Fama and French (1992, 1993). Thus, we consider information from year t for portfolio assignments at the end of June of year $t+1$ onwards until the following June.

The Table 2 summarizes the characteristics of firms across five debt maturity buckets. The first thing to notice is that there is a substantial heterogeneity in the average maturity profiles. Firms with the shortest maturity have just above 5% of debt maturing in the next three years, while firms with the longest maturity have more than 95% of debt maturing in more than three years. Moreover, firms in the shortest maturity bucket are substantially smaller than firms in the longest maturity bucket. However, their book-to-market ratios, a proxy for the exposure of assets to systemic risk, are almost the same. Also, shorter-maturity financed firms have lower leverage and higher idiosyncratic volatility than longer-maturity financed firms.

Table 2: Firm Characteristics Across Debt Maturity. We compute average characteristics across five debt maturity buckets. The table presents time-series averages of the annual characteristics within each bucket. Idiosyncratic volatility is estimated from rolling CAPM regressions with a five year window of monthly returns. Market equity in million USD. The marginal tax rates are based on Blouin et al. (2010).The underlying data set comprises matched observations from CRSP and COMPUSTAT over the time horizon January 1976 until December 2017.

Debt Maturity	Short	···	Medium	···	Long
Debt Maturity (%)	5.18	32.86	60.23	79.59	95.25
Market Equity	520.59	2,815.75	3,368.51	2,529.03	1,492.49
Book-To-Market	0.92	0.98	0.96	0.96	0.93
Leverage (%)	23.89	32.97	34.23	35.11	32.08
IVOL (%)	14.93	12.29	10.93	10.40	11.25
Tax Rate (%)	19.71	23.50	26.10	27.68	27.67

2.3 Portfolio Sorts

We construct debt maturity-sorted portfolios by sorting stocks into 5 size buckets and 5 conditional debt maturity buckets. At the end of each month, we rank stocks into quintiles by their size (ME) and then into conditional quintiles by their debt maturity DM. This procedure is analogous to the construction of the HML factor in Fama and French (1993). We have to consider conditional sorts within each size group due to the variation in debt maturity across size portfolios. In our sample smaller firms tend to borrow with shorter-maturity debt than larger firms, as can be seen from the Table 2. Not considering this heterogeneity would result in picking up a size effect in the maturity sorts.

For each quintile portfolio, we obtain monthly time series of returns. Table 3 summarizes the returns, alphas, and betas for each size quintile for the portfolio that is long the quintile with longest-maturity financed firms and short the quintile with the shortest-maturity financed firms. To control for differences in risk across deciles, we present estimation results and corresponding alphas from CAPM, Fama and French (1993) three-factor model and Fama and French (2015) five-factor model. The last column presents a weighted-average over all size quintiles.

Both raw and risk-adjusted returns in Table 3 indicate a positive relation between debt maturity and future stock performance. Long-minus-short portfolio has statistically significant CAPM alpha in the smallest three size quintiles, as well as on average in the entire sample. Firms financed with long-term debt outperform firms financed with short-term debt on average by 0.21% monthly, controlling for differences in systemic risk. Moreover, smaller firms financed with long-term debt outperform smaller firms financed with short-term debt by 0.44% on a monthly basis.

The factor loadings from Fama and French (2015) indicate that the long-minus-short portfolio on debt maturity behaves similar to the value premium. β^{HML} is positive and strongly statistically significant. However, the value premium does not subsume the maturity premium. Even after controlling for the value factor, the alpha on the long-minus-short maturity portfolio remains positive and statistically significant.

The maturity portfolio has marginally negative exposure to systemic risk, indicating that the average beta of long-maturity financed firms is smaller than that of the short-maturity financed firms.

3 Model

In this section, we analyze the implications of different debt maturities for the dynamics of firm leverage. The key feature of our model is the ability of firms to choose the debt roll-over intensity. It means that firms optimally decide what fraction of their maturing debt to re-finance. We build on the models of Dangl and Zechner (2016) and DeMarzo and He (2018). Following DeMarzo and He (2018), equityholders cannot credibly commit to future leverage adjustments via contractual obligations. Thus, at every instant we allow equityholders to optimally choose the amount of new debt to be issued or repurchased. This setup allows for a tractable model of the link between debt maturity and leverage dynamics, accounting for debt overhang effects. While the model lacks features such as transactions costs or different debt seniority, it allows us to analyze the effect of debt maturity on equity risk premia.

3.1 Cash Flow

We consider a market comprised of heterogeneous firms. An individual firm's cash flow before paying interest and taxes, $Y_{i,t}$, is the product of two components, namely $Y_{i,t} = X_t \cdot I_{i,t}$. First, cash flows of all firms are driven by an aggregate productivity factor, X_t, which follows a geometric mean-reverting process with a drift $\mu(X_t,t)$ and volatility σ_X

$$dX_t = \mu(X_t,t)X_t\,dt + \sigma_X X_t\,dW^P_{X,t} \tag{1}$$

$$\mu(X_t,t) = \mu_0 - k\left[log(X_t) - \left(\mu_0 - \sigma_X^2/2\right)t\right]. \tag{2}$$

The growth rate of the aggregate process $\mu(X_t,t)$ is mean-reverting with a speed of k to its time-average of μ_0.[9] The drift's deviations from μ_0 are due to X_t diverging from its expected growth path. During periods where X_t is above (below) the expected trajectory, expected growth rates are reduced (increased). Thus, the drift component introduces cyclicality of productivity growth, i.e., a business cycle.[10]

The firm-specific cash flows are orthogonal to the aggregate state variable, and are determined by a firm-specific idiosyncratic factor $I_{i,t}$, which is independent across firms. It follows a geometric Brownian motion without a drift:

$$dI_{i,t} = \sigma_i I_{i,t}\,dW^P_{i,t}. \tag{3}$$

Given the multiplicative combination of the variables, the resulting cash flow $Y_{i,t}$ of a firm i also follows a geometric Brownian motion (under the physical measure)

$$dY_{i,t} = \mu(X_t,t)Y_{i,t}\,dt + \sigma_Y Y_{i,t}\,dW^P_{Y_i,t}, \tag{4}$$

where $\sigma_Y = \sqrt{\sigma_X^2 + \sigma_i^2}$ and $dW^P_{Y_i,t} = (\sigma_X\,dW^P_{X,t} + \sigma_i\,dW^P_{i,t})/\sigma_Y$ govern the stochastic part. Moreover, under the risk neutral measure, a firm's cash flows are given by

$$dY_{i,t} = \mu_Y Y_{i,t}\,dt + \sigma_Y Y_{i,t}\,dW^Q_{Y_i,t}, \tag{5}$$

[9] Note that while $\mathbb{E}_0[\mu(t,X_t)] = \mu_0$, it is not true that $\mathbb{E}_0[X_t] = X_0 e^{\mu_0 t}$. In fact, $\mathbb{E}_0[X_t] < X_0 e^{\mu_0 t}$, and the reason the aggregate process grows at a smaller rate than it would if the drift-process was not mean-reverting is in the negative covariance between μ_t and X_t. $\mathbb{E}_0[X_t e^{\mu_t t}] = Cov(X_t, e^{\mu_t t}) + \mathbb{E}_0[X_t]\mathbb{E}_0[e^{\mu_t t}] < \mathbb{E}_0[X_t]e^{\mu_0 t}$. It is also true that $\mathbb{E}_0[e^{\mu_t t}] < e^{\mu_0 t}$ due to Jensen's inequality.

[10] Current specification for the drift process admits low values for the growth rate of the productivity process. Some of them are low enough so that the market price of risk, which is related to the drift, in the economy can turn negative. While it might seem counter-intuitive, empirical estimates for the market price of risk do turn negative (Cochrane, 2011). Hence, our specification is consistent with the data. Moreover, for robustness we also solve and simulate the model with an alternative specification for the drift that is bounded from below, ensuring that the market price of risk is always non-negative. All the main results remain qualitatively the same, and the maturity premium is only slightly smaller quantitatively. See Internet Appendix for details. Hence, our results are not driven by the negative values of the market price of risk.

where $\mu_Y < r$. In Section 4.1, we further specify the Girsanov kernel associated with this measure change from no-arbitrage conditions for the market portfolio, and characterize μ_Y. We take the consumption process of the representative consumer in the economy as given, so under our assumptions the financing decisions of a firm do neither impact the change of measure nor the market price of risk.

3.2 Debt and Equity Valuation

Consider a firm that issues debt with face (book) value $F_{i,t}$. The bond pays a fixed coupon rate c that is tax-deductible. The marginal tax rate is denoted by τ. In the spirit of finite maturity debt models (e.g., Leland (1994*a*) and Leland (1998), among others), we consider a debt structure, where a constant fraction m_i of outstanding bonds matures every period. The average maturity of outstanding debt is $1/m_i$, which is constant even if the firm stops rolling over maturing debt. Hence, cash flows to debt holders in the absence of default are given by the coupon payments and the retirement of debt $(c+m_i)F_t\,dt$. In default we assume a zero recovery. When the firm is founded, the firm chooses a debt maturity, which is then held constant throughout the firm's life. Since it can be shown that the firm founders are indifferent between alternative debt maturities we take maturity as an exogenous parameter.

The firm can issue new debt with a face value $G_{i,t}$. Negative values of $G_{i,t}$ represent voluntary retirements. As long as $G_{i,t}$ is less than or equal to the maturing debt, $m_iF_{i,t}$, then the firm's total face value of debt is either reduced or stays constant. In contrast to Dangl and Zechner (2016) and in accordance with DeMarzo and He (2018), firms in our model are also allowed to increase debt smoothly by issuing more than the maturing fraction of debt, i.e., choosing $G_{i,t} > m_iF_{i,t}$. Consequently, the dynamics of the outstanding face value of debt are given by:

$$dF_{i,t} = (G_{i,t} - m_iF_{i,t})\,dt\,. \tag{6}$$

Next, we take a look at the distributions to equity owners. We abstract from transaction costs of issuing either debt or equity. Hence, the residual cash flow net of debt-related payments and taxes, given by

$$\Pi^i_{t,t+dt} = \left\{Y_{i,t}(1-\tau) + \tau cF_{i,t} - (c+m_i)F_{i,t} + G_{i,t}v^D_{i,t}\right\}dt \tag{7}$$

is distributed to equityholders. The first term represents the operating cash flows before interest. As the coupons are tax deductible the tax benefit of debt, expressed by the second term, is added. The third and fourth term are related to the leverage adjustments. First, the currently outstanding debt $F_{i,t}$ has to be serviced by paying coupons and retiring the maturing portion. Second, new debt is issued (or bought back if $G_{i,t}$ is negative) at market prices $v^D_{i,t}$.

The market values of equity and debt claims, $V^E_{i,t}$ and $V^D_{i,t}$, are given by the conditional expectations of their respective future cash flows under the risk-neutral measure Q:

$$V_i^E(Y_{i,t}, F_{i,t}) = \mathbb{E}_t^Q \left[\int_t^{t_b} e^{-r(s-t)} \Pi^i_{t,s} \, ds \right], \text{ and} \tag{8}$$

$$V_i^D(Y_{i,t}, F_{i,t}) = \mathbb{E}_t^Q \left[\int_t^{t_b} e^{-(r+m_i)(s-t)} (c+m_i) \, ds \right] F_{i,t}, \tag{9}$$

where t_b denotes the time when the equity owners endogenously decide to declaring default of the firm.

We restrict the solution space to policy functions $G_{i,t}$ which are continuous in the state variables, i.e., the debt issuance policy is smooth. The equity maximization problem involves solving the Hamilton-Jacobi-Bellman equation, which is homogeneous in the face value of debt $F_{i,t}$. Therefore, we scale every variable by $1/F_{i,t}$, and use lower case letters to indicate the scaled version, e.g., $y_{i,t} = Y_{i,t}/F_{i,t}$ throughout.

Using the valuation principles from DeMarzo and He (2018), we find the scaled value of equity[11]:

$$v_i^E(y_{i,t}) = \frac{1-\tau}{r-\mu_Y} y_{i,t} - \frac{c(1-\tau)+m_i}{r+m_i} \left(1 - \frac{1}{1+\gamma_i} \left(\frac{y_{i,t}}{y_{bi}} \right)^{-\gamma_i} \right), \tag{10}$$

$$\gamma_i = \frac{(\mu_Y + m_i - \sigma_Y^2/2) + \sqrt{(\mu_Y + m_i - \sigma_Y^2/2)^2 + 2\sigma_Y^2(r+m_i)}}{\sigma_Y^2} > 0,$$

$$y_{b,i} = \frac{\gamma_i}{1+\gamma_i} \frac{r-\mu_Y}{r+m_i} \left(c + \frac{m_i}{1-\tau} \right),$$

where $y_{b,i}$ denotes the endogenously chosen scaled cash flow where the equityholders default.

Moreover, from the solution to the equity-maximization problem we can derive the value of debt. Given the fact that equityholders can adjust the outstanding amount of debt freely, the equilibrium price of debt

[11] While we present only the closed-form solutions in this section, Appendix A presents the details on how to solve the model.

$v_i^D(y_{i,t})$, i.e., the marginal benefit from debt issuance, will equal the marginal cost of future obligations, given by $-\partial V^E(Y,F)/\partial F$. Hence, the price of debt per unit of face value equals

$$v_i^D(y_{i,t}) = \frac{c(1-\tau)+m_i}{r+m_i}\left(1-\left(\frac{y_{i,t}}{y_{b,i}}\right)^{-\gamma_i}\right). \tag{11}$$

3.3 Debt Issuance Policy and Leverage Dynamics

The optimal debt issuance policy function $g_{i,t}$ is a key driver of leverage dynamics. As shown in Appendix A the debt issuance policy function is given by

$$g_i(y_{i,t}) = m_i\left(\frac{y_{i,t}}{y_{m,i}}\right)^{\gamma_i}, \tag{12}$$

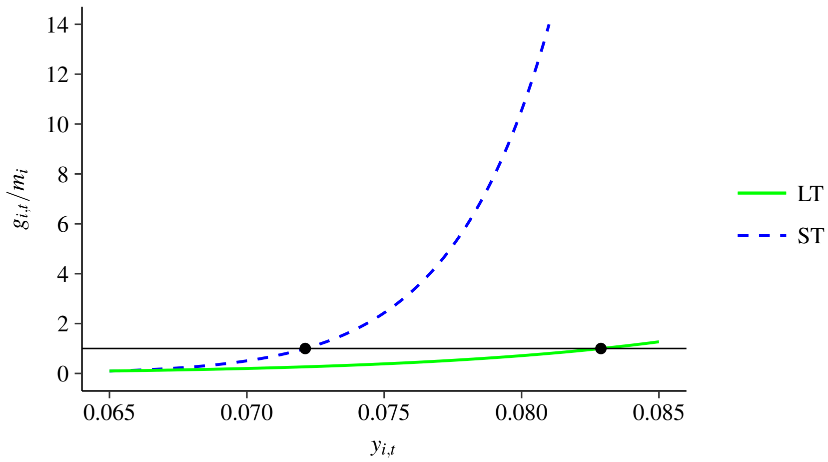
where $y_{m,i}$ denotes the scaled cash flow level at which the firm's issuance rate is exactly equal to the maturity rate m_i. It equals to:

$$y_{m,i} = y_{b,i}\left(\gamma_i\frac{c(1-\tau)+m_i}{(r+m_i)\tau c}m_i\right)^{1/\gamma_i}. \tag{13}$$

At the scaled cash flow level $y_{m,i}$ the firm keeps the outstanding amount of debt constant. Hence, for any level of cash flows $Y_{i,t}$, the face value of debt that results in the scaled level of cash flows of $y_{m,i}$, i.e., $F_{m,i,t} = Y_{i,t}/y_{m,i}$, is the target face value of debt.

Equation (12) implies that the net debt issuance is non-negative. This means that shareholders never actively repurchase debt, even though there are no associated transaction costs. This illustrates the leverage ratchet effect of Admati et al. (2018), and the debt-overhang problem that existing debt creates. Second, the roll-over rate positively depends on cash flow shocks, meaning that firms with higher cash flows per unit of face value issue more debt. Figure 1 illustrates the optimal debt issuance policy functions graphically for different levels of cash flow shocks and different maturities of debt. The long and short-term financed firms have different levels of optimal leverage. As short-term financed firms have higher target leverage levels, there are cash flow values $y_{i,t}$ for which short-term financed firms issue debt, while long-term financed firms reduce leverage through partial roll-over, everything else equal. However, the short-term financed firms respond more aggressively to changes in cash flows than long-term financed firms. They are relatively more

Figure 1: Optimal Roll-Over Rate. This graphs show the optimal roll-over rate of debt, which is given by the issuance policy $g_{i,t}$ scaled by the maturity rate m_i. This ratio equals one when the firm's net issuance is zero. The short- and long-maturity financed firms are characterized by $m_i = 0.5$ and $m_i = 0.2$, i.e., a debt maturity of 2 (ST) and 5 (LT) years, respectively. The volatility of the cash flows is $\sigma_X = 0.15$ and $\sigma_i = 0.15$. The solid (dashed) line represents the roll-over rate of the LT (ST) firm.



aggressive at both increasing the leverage after positive cash flow shocks, and decreasing leverage after negative cash flow shocks.

The market leverage in our model, given by:

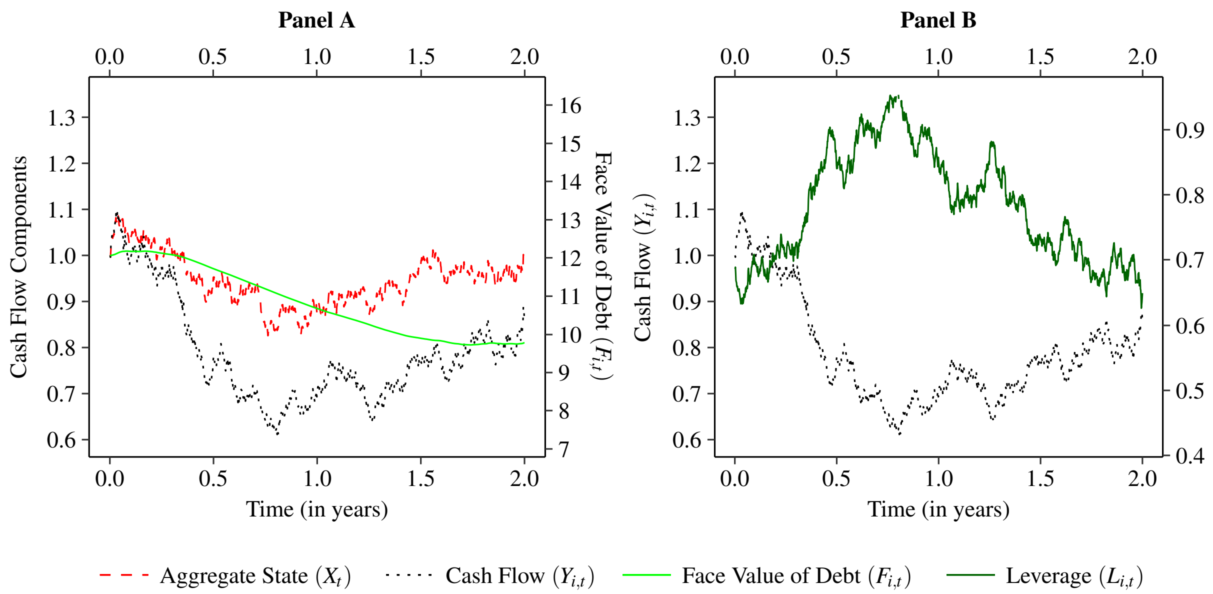
$$L_{i,t} = \frac{v_{i,t}^D}{v_{i,t}^E + v_{i,t}^D}\,, \tag{14}$$

changes over time for two reasons — the firm actively manages the face value of debt outstanding $F_{i,t}$, and the value of the firm's assets changes. The face value of debt can increase or decrease over time, as the firm sometimes decides to issue additional debt, while at other times optimally lets the debt mature and does not roll it over completely. The dynamics of $F_{i,t}$ depend on the realized path of the cash flow process in the following way

$$F_{i,t} = \left(\int_0^t \gamma_i m_i \left(\frac{Y_{i,s}}{y_{m,i}} \right)^{\gamma_i} e^{\gamma_i m_i (s-t)}\, ds \right)^{1/\gamma_i}. \tag{15}$$

Let us consider the dynamics of the face value of debt of a firm that first experiences a decrease and then an increase as illustrated in Figure 2. The graph in Panel A depicts the realizations of the aggregate

Figure 2: Evolution of Leverage. This figure illustrates the dynamics of cash flows and leverage for one firm. The graph in Panel A depicts dynamics of the aggregate state process X_t, the cash flows $Y_{i,t}$, and the face value of debt $F_{i,t}$. Panel B shows the dynamics of leverage. The parameters for this simulation are: $\mu_0 = 5\%$, $k = 0.25$, $\sigma_X = 15\%$, $\sigma_i = 15\%$, $r = 5\%$, $\delta = 4\%$, $c = r/(1-\tau)$, $\tau = 30\%$. The LT firm is has an average maturity rate of $m_i = 0.2$ (5 years), while the ST firm has $m = 0.5$ (2 years).

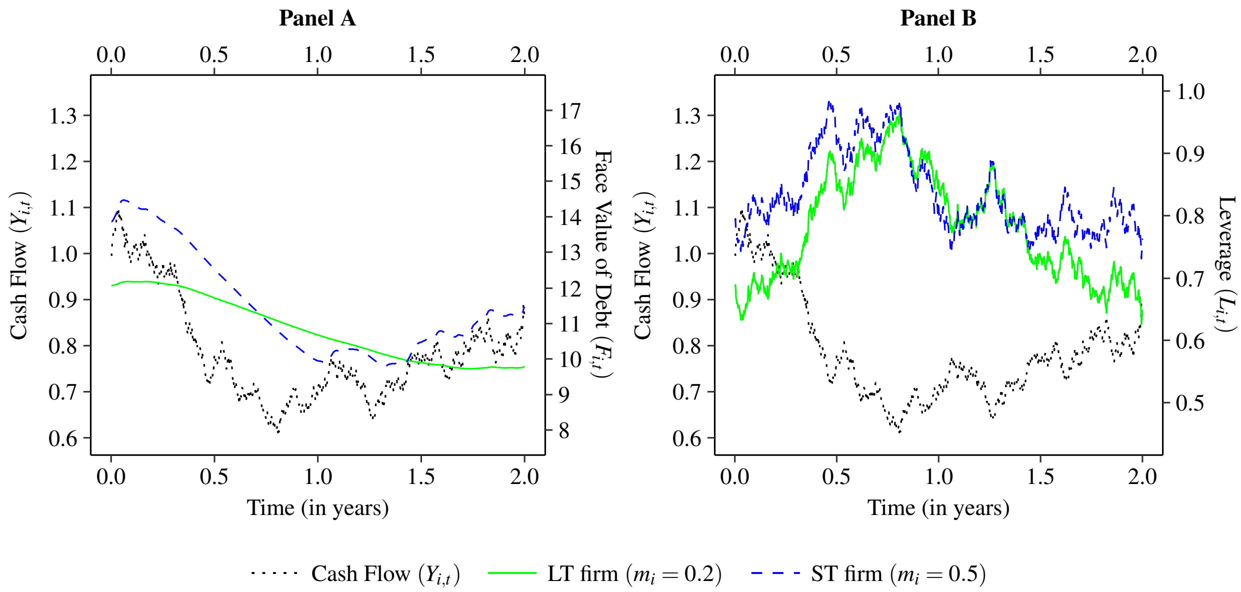


process and the cash flow process. The difference between the two lines is due to the idiosyncratic risk component. The firm is long-term financed, with an average bond maturity of 5 years. The right-hand y-axis depicts the evolution of the face value of debt. Following a decrease in cash flows, the firm starts reducing its outstanding debt. The reduction process is gradual and slow, in each period the firm is rolling over only a fraction of its maturing debt. When cash flows increase, the firm starts issuing debt. The corresponding evolution of market leverage is shown in Panel B of Figure 2. Its path follows that of the face value of debt, with fluctuations around that path reflecting changes in the market value of equity and debt due to the stochastic cash flow shocks.

3.4 The Leverage Ratchet Effect and Maturity

The goal of our theoretical model is to establish the effect of different debt maturities on the dynamics of leverage over a profitability cycle. In this subsection, we look at the evolution of market leverage of two firms — one financed with long-term debt (low m_i) and one financed with short-term debt (high m_i) —

Figure 3: Debt Maturity and the Leverage Ratchet Effect. This figure illustrates the differences in leverage dynamics for a short- (dashed lines) and long-maturity (solid lines) financed firms (referred to ST and LT, respectively). Panel A (Panel B) shows the face value of debt $F_{i,t}$ (leverage $L_{i,t}$) for two firms faxing the the same cash flow process $Y_{i,t}$. The parameters for this simulation are: $\mu_0 = 5\%$, $k = 0.25$, $\sigma_X = 15\%$, $\sigma_i = 15\%$, $r = 5\%$, $\delta = 4\%$, $c = r/(1-\tau)$, $\tau = 30\%$. The LT firm has an average maturity rate of $m_i = 0.2$ (5 years), while the ST firm has $m = 0.5$ (2 years).



that were hit with the same sequence of cash flow realizations. Our focus is on the difference in leverage responses between the two firms.

Following Admati et al. (2018), we define the ratchet effect of leverage as shareholders not willing to actively repurchase debt following a deterioration of market conditions. In the notation of our model, we see that $g_{i,t} > 0$, which means that firms never actively repurchase debt, even though it is frictionless to doing so (no transaction costs on repurchasing of debt). The reason for this lies in the debt overhang that existing debt imposes on shareholders. However, as pointed out by Dangl and Zechner (2016) and by DeMarzo and He (2018), this intuition does not apply one-to-one to the refinancing of maturing debt. Shareholders sometimes find it optimal to roll over only a fraction of maturing debt, effectively reducing their leverage. Therefore, the amount of maturing bonds is the maximum by which the firm reduces its outstanding debt. Long-term financed firm are slow to decrease debt, while short-term financed firm respond relatively fast to negative profitability shocks. We illustrate this intuition in Figure 3.

The graph in Figure 3 Panel A illustrates the different adjustments of the face value of debt between a short-term and a long-term financed firm, where both firms experience the same cash flows. The face value of debt for the short-term financed firm follows ups and downs of the cash flows process very closely. This is not the case for the long-term financed firm. Its face value responds less to cash-flow fluctuations, which is most noticeable when cash flows decrease — the face value of debt also decreases, but much slower. As a result, we see in Panel B that the leverage of the long-term financed firm increases much more than the leverage of the short-term financed firm due to deterioration of cash flows. These dynamics are due to the leverage ratchet effect, which manifests itself in the slow deleveraging process for the long-term financed firm.

Moreover, as in classical models of Leland (1994*a*), firms with shorter debt maturity chose to have a higher average leverage. Following a negative cash flow shock, the leverage of short-term financed firms goes up, then quickly down, but remains on average higher than that of the long-term financed firms. Short-term financed firms take advantage of their flexibility with leverage changes, and have a smaller default risk for a given level of leverage than long-term financed firms. They optimally lever up to a higher level than long-term financed firms, taking advantage of the extra tax shield.

Table 3: Debt Maturity-Sorted Portfolios. Panel A shows the average excess return of the individual value-weighted portfolios. The last row contains the long-maturity minus short-maturity portfolios (LMS). The portfolios are formed by double sorts on size (5 buckets at 20%, 40%, 60%, and 80%-percentile) and debt maturity (5 buckets at 20%, 40%, 60%, and 80%-percentile) conditional within each size group. Panel B examines the LMS portfolios within each size bucket (LMS1 for small to LMS5 for large firms) and average return of these five portfolios the last column (LMS itself). The long-short portfolios are represented by excess returns (r^e) as well as alpha estimates from CAPM-regressions (α^{CAPM}), the 3-factor model by Fama and French (1993) (α^{FF3}) and the 5-factor model by Fama and French (2015) (α^{FF5}). Moreover, risk factor loadings for FF5 are shown. We report t-statistics based on standard errors following Newey and West (1987, 1994) in parentheses. The underlying data set comprises matched observations from CRSP and COMPUSTAT from January 1976 until December 2017.

Panel A: Portfolio Sorts

		Size				
		Small	·	Medium	·	Large
Debt Maturity	Short	0.59	0.71	0.72	0.81	0.62
	·	0.66	0.82	0.86	0.89	0.68
	Medium	0.75	0.82	0.92	0.89	0.66
	·	0.86	0.89	0.99	0.93	0.60
	Long	0.92	0.91	0.86	0.86	0.64
LMS		0.33	0.20	0.14	0.05	0.01

Panel B: Debt Maturity (LMS)

	LMS1	**LMS2**	**LMS3**	**LMS4**	**LMS5**	**LMS**
r^e	0.33**	0.20	0.14	0.05	0.01	0.15*
	(2.01)	(1.25)	(1.21)	(0.59)	(0.09)	(1.93)
α^{CAPM}	0.44***	0.32**	0.22*	0.04	0.03	0.21***
	(2.98)	(2.03)	(1.82)	(0.41)	(0.26)	(2.93)
α^{FF3}	0.35***	0.20	0.09	0.00	0.11	0.15**
	(2.69)	(1.44)	(0.82)	(0.04)	(0.94)	(2.31)
α^{FF5}	0.30**	0.10	−0.06	−0.11	0.25**	0.10*
	(2.27)	(0.59)	(−0.60)	(−1.22)	(2.57)	(1.67)
β^{M}	−0.08**	−0.04	0.02	0.08***	−0.13***	−0.03*
	(−1.99)	(−1.17)	(0.75)	(3.62)	(−4.45)	(−1.78)
β^{SMB}	−0.17***	−0.23***	−0.12***	−0.01	0.07	−0.09***
	(−3.09)	(−3.53)	(−2.76)	(−0.28)	(1.41)	(−2.97)
β^{HML}	0.36***	0.33***	0.33***	0.00	−0.13	0.18***
	(5.44)	(3.80)	(5.25)	(0.08)	(−1.32)	(3.92)
β^{RMW}	0.20***	0.22	0.36***	0.18**	−0.26***	0.14***
	(2.73)	(1.51)	(5.18)	(2.50)	(−3.55)	(2.65)
β^{CMA}	−0.21*	0.05	0.01	0.20**	−0.16	−0.02
	(−1.89)	(0.46)	(0.07)	(2.21)	(−1.41)	(−0.45)

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

4 Asset Pricing Implications of Debt Maturity

In this section, we explore the asset-pricing implications of different maturities of debt. The focus of our analysis are the differences in leverage dynamics, their effect on the dynamics of equity betas, and the resulting perceived alphas.

4.1 Market Return and the Market Price of Risk

We consider the market to be populated by many firms, not only those that we analyze in the previous section. Individual firm's decisions and composition of surviving firms does not affect the dynamics of the market portfolio in our analysis, reminiscent of our assumptions that firms' financing decisions do not affect the market price of risk. The market portfolio $M(X_t)$ is driven by the aggregate productivity level X_t, which is defined in Equation (1). This market portfolio is traded and its return over a time increment is:

$$r^M_{t,t+dt} = (\mu(X_t,t) + \delta)\, dt + \sigma_X\, dW^P_{X,t}, \tag{16}$$

where $\delta > 0$ represents aggregate dividends. Assuming no-arbitrage and complete markets we change to the risk neutral measure. Given that the market portfolio is traded, its risk neutral drift equals the risk-free rate r. The market price of risk is therefore given by a Girsanov transformation as:

$$\lambda_t = \frac{(\mu(X_t,t) + \delta - r)}{\sigma_X}. \tag{17}$$

It is time-varying due to the variation in $\mu(X_t,t)$, as shown in Equation (C-8). Furthermore, we denote by η_t the market risk premium for bearing systematic risk, which equals $\eta_t = \sigma_X \lambda_t$.

The risk-neutral drift of a firm's cash flows, μ_Y, consistent with the no-arbitrage condition is given by $\mu_Y = r - \delta$. It follows from writing the cash flow process under the risk-neutral measure that:

$$\frac{dY_t}{Y_t} = \mu(X_t,t)\, dt + \sigma_X\, dW^P_{x,t} + \sigma_i\, dW^P_{Y_i,t} = (\mu(X_t,t) - \sigma_X \lambda_t)\, dt + \sigma_X\, dW^Q_{x,t} + \sigma_i\, dW^Q_{Y_i,t} = \mu_Y\, dt + \sigma_Y\, dW^Q_{Y_i,t}. \tag{18}$$

4.2 Dividend Growth Rate and The Market Price of Risk

In our model, the market price of risk λ_t is driven by shocks to the aggregate productivity process X_t. The higher the aggregate productivity is, the smaller is the market price of risk, reflecting the counter-cyclical nature of the representative investor's risk-aversion or the appetite for risk in the economy. The productivity process X_t affects a firm's dividends through two channels. First, the operating profit in the current period, $Y_{i,t}$ depends positively on the aggregate productivity. Second, a positive shock to operating profits is a signal for higher profitability going forward and the firm's optimal debt level goes up. Thus, the firm issues more debt than is currently maturing. Hence, net proceeds from debt issue, i.e. $G_{i,t}v^D_{i,t} - (c+m_i)F_{i,t}$, are positive, increasing dividends further. That is, dividends in our model positively depend on the aggregate productivity process. Since the aggregate productivity process is negatively related to the market price of risk, as discussed above, dividends are negatively related to the market price of risk.

This is consistent with empirical evidence. In particular, Van Binsbergen and Koijen (2010) document that the dividend growth rate is contemporaneously negatively correlated with market excess returns.[12] However, the empirical relation is not perfect. This is not surprising, given that in our model dividends are essentially a pass-through process, except for dynamic leverage adjustments. If we were to consider investments, retained earnings, and cash holdings, then the link between the productivity process and the dividend growth rate would be weaker. DeMarzo and He (2018) consider endogenous investment and no leverage commitment and find that safe firms issue debt less aggressively than in a setting when their investment is fixed. Hence, when productivity is high, firms far from default will increase investment and issue debt less aggressively and pay out smaller dividends than firms with a fixed investment strategy as in our model. The endogenous response of the debt issuance policy dampens the link between productivity shocks and dividend growth rate even further, beyond the effect of investment spending. Thus, any of these model extensions would reduce the negative relation between dividends and the market price of risk, in line with empirical estimates.

However, while certainly interesting, the focus of our study is not on the dynamics of dividends. And for tractability we abstract from investment and corporate cash holdings.

[12] Van Binsbergen and Koijen (2010) find that the expected dividend growth rate, however, is positively related to the excess market returns. This is also consistent with our model because the aggregate productivity process is mean-reverting to its trend.

4.3 Equity Returns and Equity Beta

Next, we turn our attention to the analysis of the link between leverage and systematic exposure of the firm, i.e., its beta. Instantaneous equity returns to equityholders can be computed as:

$$r^E_{t,t+dt} = \frac{dV^E_t + \Pi_{t,t+dt}}{V^E_t} \,. \tag{19}$$

Utilizing the equity-pricing equation (see details in Appendix B):

$$r^E_{t,t+dt} \;=\; r\,dt + \left(1 + \frac{v^D_{i,t}}{v^E_{i,t}}\right)\eta_t\,dt + \left(1 + \frac{v^D_{i,t}}{v^E_{i,t}}\right)\sigma_Y\,dW^P_{Y_i,t}, \tag{20}$$

we arrive at a decomposition of equity returns that consists of three components: the risk-free rate, the market price of risk times the exposure to the systematic risk, and a random component.

Under the risk-neutral measure the expected value of equity returns is just the risk-free rate r.[13] Under the physical measure it is:

$$\begin{aligned}\mathbb{E}^P_t\left[r^E_{t,t+dt}\right] &= \mathbb{E}^P_t\left[r\,dt + \left(1 + \frac{v^D_{i,t}}{v^E_{i,t}}\right)\eta_t\,dt + \left(1 + \frac{v^D_{i,t}}{v^E_{i,t}}\right)\sigma_Y\,dW^P_{Y_i,t}\right] \\ &= r\,dt + \left(1 + \frac{v^D_{i,t}}{v^E_{i,t}}\right)\eta_t\,dt\,. \end{aligned} \tag{21}$$

This expression illustrates that the conditional CAPM holds in our setting. The asset beta is normalized to one in our setting, and the equity beta is then one plus debt over equity, i.e., $\beta_{i,t} = 1 + \frac{v^D_{i,t}}{v^E_{i,t}}$, while η_t represents the time-varying market risk premium.[14]

We think about $\beta_{i,t}$ as representing a scaling of each firm's asset betas. In our model asset beta is normalized to one, but in reality firms differ substantially in the systematic exposure of their physical assets. The variations in beta that we analyze are on top of any differences in asset betas. While betas in our setting are by construction larger than one, we think of them as representing an amplifying factor relative to the asset beta of each firm. For example, a beta of 1.3 in our setting corresponds to an equity beta of a real firm

[13] $\mathbb{E}^Q_t\left[r^E_{t,t+dt}\right] = \mathbb{E}^Q_t\left[r\,dt + \left(1 + \frac{v^D_{i,t}}{v^E_{i,t}}\right)\sigma_Y\,dW^Q_{Y_i,t}\right] = r\,dt.$

[14] Naturally, we obtain the same result if we derive beta using a classical formula $\beta_{i,t} = \frac{Cov_t(r^E_{t,t+dt}, r^M_{t,t+dt})}{Var_t(r^M_{t,t+dt})}$. Details can be found in the Appendix.

that is 30% larger than its asset beta, which is due to financial leverage. Therefore, while all betas in our model are above one, our model nevertheless is consistent with real data, once heterogeneity in asset betas is taken into account.

4.4 Shocks and Beta

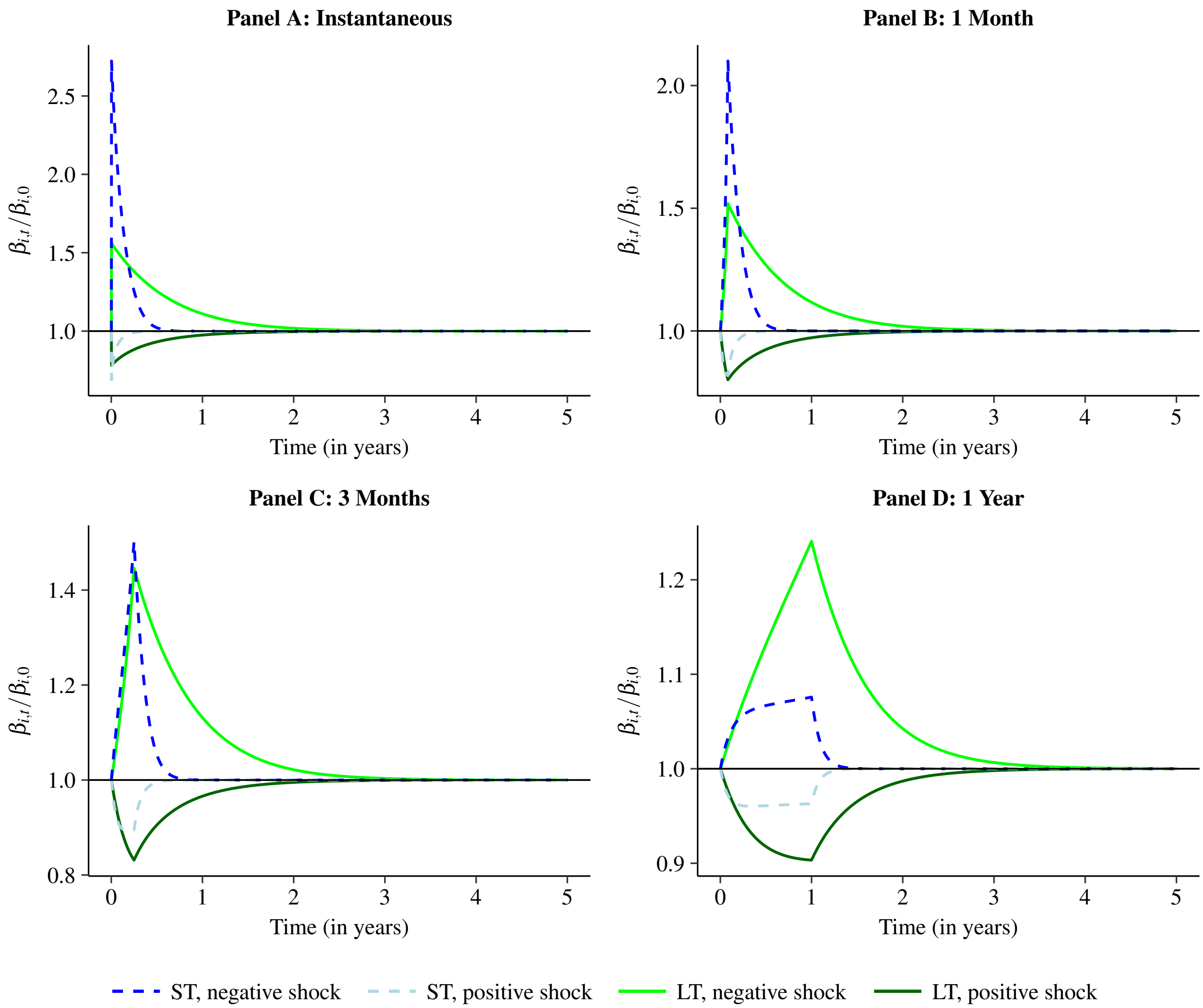
The dynamics of beta in our setting is determined by the dynamics of financial leverage. We have already established, that due to the ratchet leverage effect, long-term financed firms have larger increases in leverage following negative cash flow shocks. We therefore expect the beta of long-term financed firms to increase more in bad times.

To visualize the difference between how the beta of short and long-term financed firms responds to cash flow shocks, we analyze alternative scenarios where we consider specific cash flow paths. We start with an instantaneous increase or decrease in cash flows by 15% and hold the subsequent cash flows constant at these shocked levels.[15] The results are plotted in Figure 10 in the top left-hand subplot. Firms' initial leverage ratios are chosen so that they are at their targets, i.e., at the initial cash flow level, each firm rolls over exactly 100% of its expiring debt. For an instantaneous negative shock, the short-term financed firm experiences a larger spike in leverage and therefore beta, but it quickly reduces the face value of debt by not rolling over the entire amount. Its beta falls quickly within a year after the negative cash flow shock. The opposite is true for a long-term financed firm. It experiences a smaller initial spike in leverage, but it takes substantially longer, more than three years, to reduce its leverage back to the target level.

Next we investigate how beta responds if cash flow shocks are more gradual. We consider cash-flows where the change takes place linearly over a month, three months and a year. After that period, cash-flows are again held constant, while firms adjust their leverage by issuing or retiring maturing debt. The plots in Figure 10 demonstrate that the more gradual the shock is, the more pronounced is the difference between the impact on leverage and betas of long-term financed firms compared to short-term financed firms. With a decrease in cash flows over a year, short-term financed firms delever by not rolling over their debt, so their leverage increases much less than that of long-term financed firms. Moreover, the leverage of long-term financed firms stays elevated for more than 4 years after the shock, while the leverage of short-term financed

[15]Note that this is the cash flow path that we consider in our simulation but, of course, the firms in our simulations do not anticipate that the cash flows will remain constant as they move through time.

Figure 4: Evolution of Beta Following Cash Flow Shocks. This figure shows beta evolutions for linear cash flow increases and decreases of 15% over different time intervals. After the cash flow has completed the change it is held constant, but the firms continue rolling over debt. In the Panel A the shock is instantaneous, while Panels B and C are based on cash flow shocks over 1 and 3 months, respectively. Finally, in Panel D the shock happens over an entire year. The solid (dashed) lines represent $\beta_{i,t}/\beta_{i,0}$ for a firm with $\sigma_i = 0.15$ (while $\sigma_X = 0.15$) and $m_i = 0.2$ ($m_i = 0.5$) — i.e., a debt maturity of 5 (LT) and 2 (ST) years, respectively. The initial beta $\beta_{i,0}$ is chosen such that the firm rolls over the amount of debt that matures. The lines featuring initial spikes (drops) represent reactions to cash flow decreases (increases).



firms goes back to normal after 2 years.

To summarize, an instantaneous deterioration of cash flows initially affects short-term financed firms more severely, raising their cash flows and thus equity betas more sharply. While long-term debt financed firms' initial leverage and equity beta spike is more modest, their leverage and betas remain elevated for a long time following the initial cash flow shock. If the cash flow deterioration is more gradual, then short-term debt financed firms' leverage and equity betas never rise that much, since these firms reduce debt levels quickly in response to decreasing cash flows. By contrast, long-term financed firms' leverage and betas rise more, as their debt reductions are very slow. They exhibit elevated levels of leverage and equity betas for a long period of time.

4.5 The Expected Equity Returns Over Holding Horizons

Instantaneous expected equity returns in Equation (21) are time-varying. The dynamics of a firm's leverage together with the time-varying price of risk determine the evolution of conditional expected returns. We compare the behaviour of expected equity returns over different time horizons $\mathbb{E}_0\left[r^E_{0,\tau}\right]$ for firms with short- and long-maturity debt. Visually this is illustrated in Figure 5. The two firms, financed with long- and short-term debt, start at the same exposure to systematic risk, and therefore, have the same instantaneous expected equity returns.[16]

Leverage responds to cash flow shocks in an asymmetric way. Following good cash flow shocks, firms are more eager to increase the face value of debt than they are to decrease it following negative shocks because of debt overhang. Therefore, going forward, we on average expect the leverage of firms to go up. The expected upward trend in leverage means that shareholders require a higher return on equity, which explains the positive slopes in Figure 5.

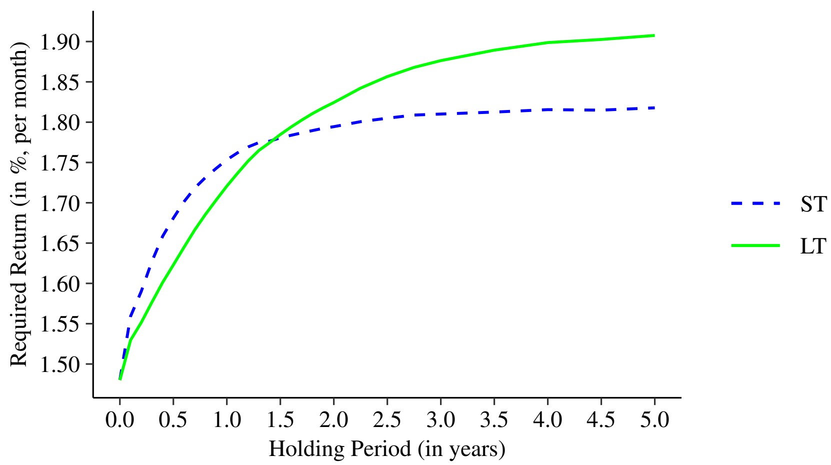
Short-term financed firms are quicker at adjusting leverage both up and down. Following a bad cash flow shock, they delever more quickly than long-term financed firms because they have a smaller fraction of debt outstanding, and hence, are subject to a smaller debt-overhang. Following good cash flow shocks, short-term financed firms do not hesitate to increase the face value of debt, as, through short maturity, they have the commitment to delever when needed. Hence, over a short horizon (up to 2 years), we expect a larger increase in leverage for short-term firms. Short-term financed firms require a higher premium on their equity than long-term financed firms in the near future.

[16] In fact, we let the firms start at the leverage level at which the firms issue exactly as much debt as matures. Moreover, the idiosyncratic volatility of the short-term financed firm is chosen such that the firms' leverage levels result in the same initial $\beta_{i,0}$ for both firms.

However, over a longer horizon, long-term financed firms are more risky. They are expected to increase their leverage more than short-term firms, and shareholders require a higher premium.

The positive co-movement between beta (leverage) and market price of risk makes the slopes of equity yield curves steeper. The more leverage increases exactly when the market price of risk is high, the higher is the required compensation for bearing this risk.

Figure 5: Expected Equity Returns Over Holding Horizons. This graphs show the expected equity returns $\mathbb{E}_0\left[r^E_{0,\tau}\right]$ over different investment horizons starting at $t=0$ and ending in period τ. The expectations are calculated as averages based on the simulated changes in $\beta_{i,t}$ and η_t. The simulated panel consists of 1,000 firms per economy and 5,000 economies. Defaulted firms are not replaced. The solid (dashed) line represents the function for a long-maturity (short-maturity) financed firm with $\sigma_i = 0.15$ ($\sigma_i = 0.2429$) and $m_i = 0.2$ ($m_i = 0.5$) — i.e., a debt maturity of 5 (LT) and 2 (ST) years, respectively. The (m_i, σ_i)-pairs produce the same $\beta_{i,0}$ initially. Other parameters are as in the benchmark case, i.e., $\mu_0 = 0.05$, $k = 0.25$, $\sigma_X = 0.15$, $\delta = 0.04$, $r = 0.05$, $\tau = 0.3$.



4.6 Unconditional CAPM and Alpha

We can re-write the expression for the conditional expected equity return stated in Equation (21) using $\beta_{i,t}$ to arrive at a notation similar to the CAPM as

$$\mathbb{E}_t\left[r^E_{t,t+dt}\right] = r\,dt + \left(1 + \frac{v^D_{i,t}}{v^E_{i,t}}\right)\sigma_X \lambda_t\,dt = r\,dt + \beta_{i,t}\eta_t\,dt, \tag{22}$$

where $\eta_t = \sigma_X \lambda_t = \mu(X_t, t) + \delta - r$ is the time-varying market risk premium.

In our model, the conditional version of the CAPM holds, period by period. However, an unconditional

CAPM does not hold because $\beta_{i,t}$ and η_t are related through the evolution of the aggregate state X_t. And unconditional alpha, according to Lewellen and Nagel (2006), can be calculated as:

$$\alpha_i = \left[1 - \frac{\eta^2}{\sigma_M^2}\right] Cov(\beta_{i,t}, \eta_t) - \frac{\eta}{\sigma_M^2} Cov(\beta_{i,t}, (\eta_t - \eta)^2), \tag{23}$$

where $\eta = \mathbb{E}[\eta_t]$ is the unconditional mean of the market risk premium, and $\sigma_M^2 = \sigma_X^2 + \sigma_\eta^2$ is the unconditional variance of the market return. Note that in our model $\sigma_{t,M} = \sigma_X$, that is, the conditional market volatility is constant in time.[17]

In our setting, $Cov(\beta_{i,t}, \eta_t)$ is non-zero because of the time-varying market price of risk λ_t. In the downturns, when market risk premium η_t is high because of low aggregate productivity X_t, the firm's leverage is high, and correspondingly its systematic risk exposure $\beta_{i,t}$ is high. Therefore, there is a positive relationship between the market risk premium η_t and the firm's exposure to risk $\beta_{i,t}$. This co-movement is not captured by the unconditional CAPM and appears as α in CAPM regressions.

As can be seen from the expression in squared brackets in Equation (23), whether the covariance between beta and the market risk premium translates into an increase or a decrease of alpha depends on the market's squared Sharpe ratio. If the Sharpe ratio is below one, then the covariance between beta and the market price of risk leads to an increase in alpha. Since empirical estimates for Sharpe ratios are normally well below one[18], this condition will hold under plausible market conditions.

The second term in Equation (23) denotes the covariance between beta and the squared deviation of the market price of risk from its mean. If η_t is distributed symmetrically around its mean, as is the case in our model, this term will be zero. This is also found in our numerical simulations below, which reveal that this second term is quantitatively very small. Summarizing, the observed α_i should be a scaled version of the beta's covariance with the market risk premium.

[17] The formula in 23 is for an annual alpha with $dt = 1$. Generally speaking, alpha over increments of time dt is

$$\alpha_{i,dt}\, dt = \left[1 - \frac{(\eta\, dt)^2}{\sigma_M^2\, dt}\right] Cov(\beta_{i,t}, \eta_t\, dt) - \frac{\eta\, dt}{\sigma_M^2\, dt} Cov(\beta_{i,t}, (\eta_t\, dt - \eta\, dt)^2),$$

and its annualized version is:

$$\alpha_{i,dt} = \left[1 - \frac{\eta^2}{\sigma_M^2}\, dt\right] Cov(\beta_{i,t}, \eta_t) - \frac{\eta}{\sigma_M^2} Cov(\beta_{i,t}, (\eta_t - \eta)^2)\, dt\,.$$

[18] Using the market excess return from Prof. French's homepage, the market's annual Sharpe ratio for our time interval equals 0.52.

4.7 The Maturity Premium

Short- and long-maturity firms have different dynamics of leverage and therefore different dynamics in their exposure to systematic risk. In particular, long-maturity firms experience larger increases in leverage and it remains elevated longer during recessions. This implies that there is more co-movement between betas and the market price of risk for long-maturity firms than for short-maturity firms.

However, it is important to control for the differences in the average leverage levels between long-term and short-term financed firms because it affects the covariance between betas and market price of risk above and beyond the leverage ratchet effect. More levered firms will have higher average levels of beta, and therefore, ceteris paribus, higher levels of $Cov(\beta_{i,t}, \eta_t)$. Since we want to isolate the effect of the leverage ratchet effect on equity returns, we remove the effect of the average level of leverage by comparing firms with different maturity but the same average leverage levels.

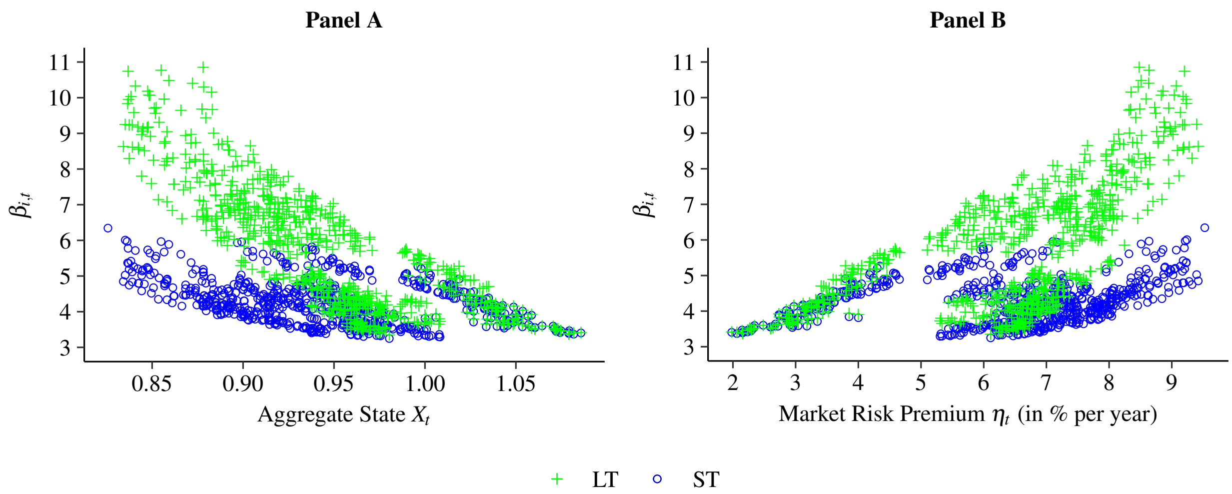
In our analysis so far we have only considered the difference between firms financed with long and short-term debt. In our model the a-priori choice of maturity is irrelevant for the firm.[19] However, in data we observe that firms with higher idiosyncratic volatility tend to be financed with shorter maturity debt (e.g., Custódio et al., 2013). This is consistent with theoretical predictions of Dangl and Zechner (2016). Firms with higher idiosyncratic volatility, everything else equal, chose lower level of leverage, as their default probability is higher. Therefore, to achieve the same level of average leverage for short- and long-term financed firms, we compare long-term financed firms with low idiosyncratic volatility to short-term financed firms with high idiosyncratic volatility.

Hence, in what follows we will compare firms that differ both in their maturity and idiosyncratic volatility, such that their average leverage level is the same.

In Panel A we see a simulated scatter-plot of beta over the aggregate state X_t for firms financed with long- and short-term debt. When the aggregate productivity process is low, long-financed firms exhibit larger betas than short-maturity firms, despite the fact that during high-productivity states betas of these firms are very similar. Panel B in Figure 6 depicts the same relation in a scatter plot of beta on market risk premium. Long-term financed firms have more co-movement between beta $\beta_{i,t}$ and the market risk premium

[19] This is because this setting ignores many important features that would be relevant for the optimal maturity choice, for example, transaction costs of issuing debt.

Figure 6: Beta, Aggregate State, and Market Risk Premium. This figure shows the endogenous development of $\beta_{i,t}$ over the aggregate state variable X_t in Panel A and over the market risk premium η_t in Panel B. The crosses (circles) depict $\beta_{i,t}$ for a firm with $\sigma_i = 0.10$ ($\sigma_i = 0.20$) and $m_i = 0.2$ ($m_i = 0.5$) — i.e., a debt maturity of 5 (LT) and 2 (ST) years, respectively. The underlying parameters are as in the benchmark case, i.e., $\mu_0 = 0.05$, $k = 0.25$, $\sigma_X = 0.15$, $\delta = 0.04$, $r = 0.05$, $\tau = 0.3$.



η_t.

Next, in order to assess if our model can account for the observed magnitude of the maturity premium, we simulate two panels of firms. See Table 4. The short- and long-maturity financed firms in our simulations resemble in their characteristics the firms in short- and long-maturity buckets that we observed in the data (see Table 2). The short-maturity financed firms have higher levels of idiosyncratic volatility and lower levels of the marginal tax rates. The longer-maturity financed firms exhibit higher alpha relative to the CAPM than short-maturity financed firms, and the resulting maturity premium is 0.19% per month, comparable to the 0.21% maturity premium that we estimated in the data (see Table 3).

4.8 Comparative Statics

Finally, we conduct a broad simulation study of the maturity effect for CAPM alphas. We simulate the capital structure model introduced in Section 3 to assess the asset pricing implications. In total, we simulate 5,000 economies of 1,000 firms for 10 years. At origination of the analysis all firms start at their target leverage levels. Then, we average the quantities of interest over firms in every economy and then over

Table 4: Maturity Premium. This tables presents the simulation results for a panel of short-maturity and long-maturity financed firms that replicate the maturity premium observed in the data. The main parameters for this simulation are as in the benchmark specification, i.e., $\mu_0 = 0.05$, $k = 0.25$, $\sigma_X = 0.15$, $\delta = 0.04$, $r = 0.05$, $\tau = 0.3$. The parameters for the short and long-maturity financed firms are chosen to reflect the characteristics of the firms in the lowest and highest maturity buckets, see Table 2.

	Short	Long	LMS
Debt Maturity (years)	1	5	
σ_i	20%	10%	
τ	20%	30%	
α	0.06%	0.25%	
Maturity Premium			0.19%

economies. All parameters are as in the benchmark specification, see Table 5.

Table 5: Benchmark Simulation Parameters. This table details the parameters of the simulation study. We group them into three categories. First, we present cash flow parameters associated with $Y_{i,t}$ under both measures. Second, we show parameters used for three rates and debt related parameters. Finally, we provide details on the simulation setting.

Cash Flow	μ_0	k	σ_X	σ_i	μ_Y
	0.05	0.25	0.15	0.15	0.01
Rates & Debt	r	δ	τ	$1/m$	c
	0.05	0.04	0.30	[1, 10]	0.07
Simulation	economies		firms	years	Δt
	10,000		2,500	10	1/1200

Idiosyncratic volatility. We apply this procedure for different specifications of (m_i, σ_i)-pairs. The relation between the perceived CAPM alpha and the maturity of debt ($1/m_i$) is illustrated in Figure 7. Moving along each line from left to right and holding idiosyncratic volatility constant, we see that as the average maturity of debt increases, the unconditional alpha also increases. However, the largest effects of debt maturity on alpha occur for expected maturity increases from one to six years. Additional maturity increases beyond six years have a relatively moderate additional effect. The reason for this result is the inverse relation between debt maturity and target leverage. Firms with very long-term debt, optimally lever less. This is so, since they rationally anticipate that they will not delever when profitability decreases, thereby creating bankruptcy risk. Thus, as we move to very long debt maturities, the additional covariance between beta and the market price of risk for given leverage tends to be offset by the lower target leverage ratios.

Figure 7: Maturity Premium. In this figure we present the resulting alpha from unconditional CAPM regression of simulations for different maturity-volatility pairs (over 10,000 economies of 2,500 firms each). Alphas are represented in % per month. All parameters underlying this simulation are detailed in Table 5.

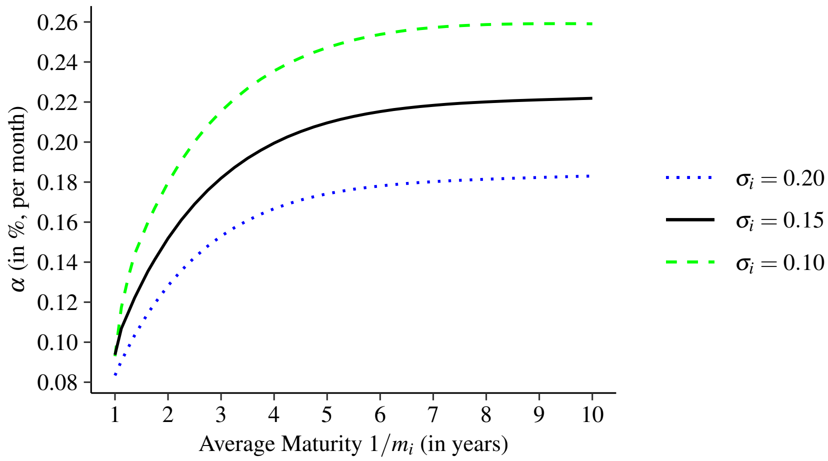
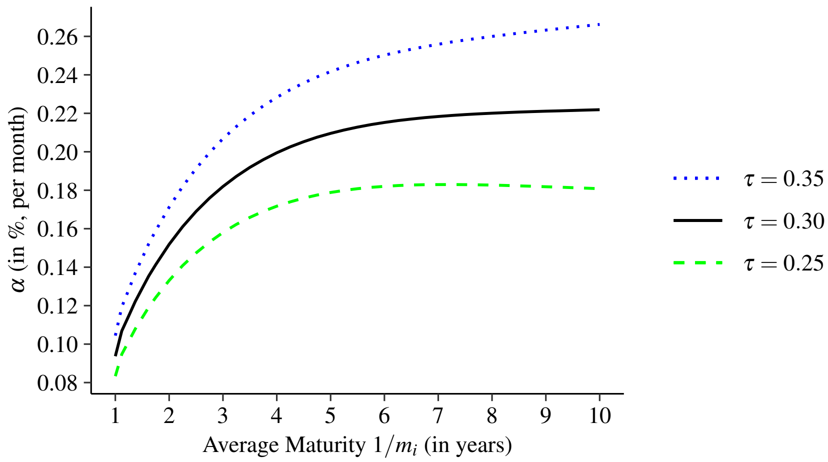


Figure 7 also demonstrates that firms with more idiosyncratic risk exhibit a smaller maturity premium. The top line corresponds to firms with smaller idiosyncratic volatility, the bottom line is for the firms with largest idiosyncratic volatility. The top line has larger change in alphas for the same change in maturities than the bottom line. This happens due to an inverse relation between idiosyncratic volatility and target leverage. Thus, for high-risk firms the maturity premium is mitigated since they choose lower target leverage ratios.

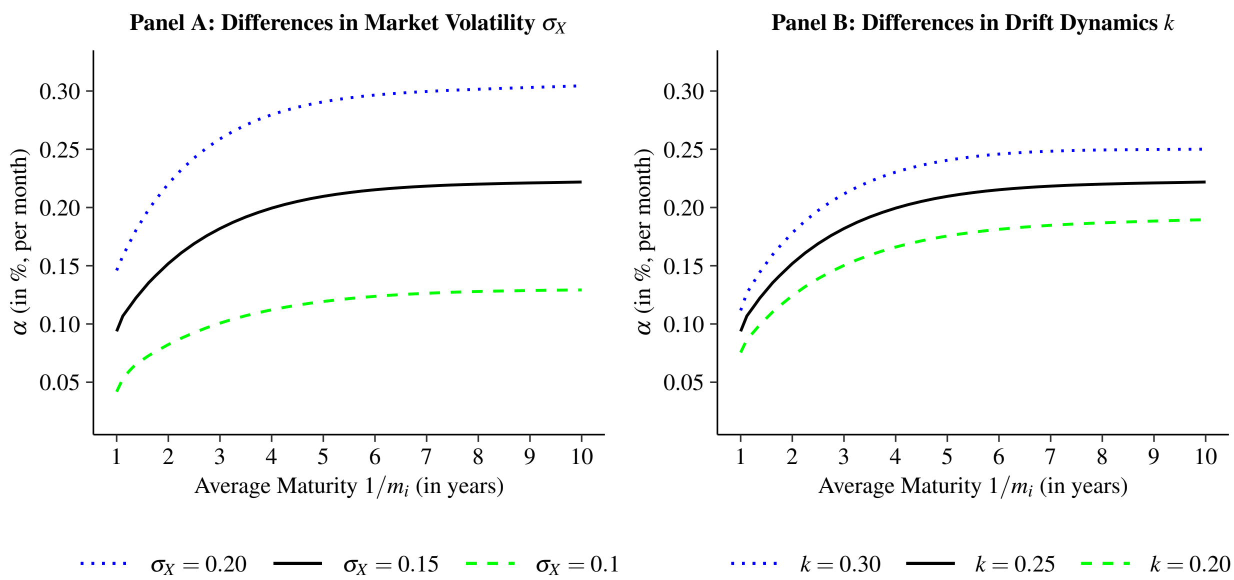
Overall, Figure 7 illustrates that the longer the maturity of debt, the larger is the CAPM alpha. This means that the ratchet effect of leverage indeed makes firms more risky in downturns and that this dominates the roll-over risk of short-term firms, which may lead to short-term spikes in leverage and betas. Of course this result depends on the underlying process of economic uncertainty. For example, if one would add the possibility of crashes in the X process, then presumably this would hurt short-term financed firms more than long-term financed firms, as argued above, thereby reducing or even eliminating the maturity premium.

Figure 8: Maturity Premium - Tax Benefits of Debt. In this figure we present the alpha from unconditional CAPM regression on simulated panels of firms (over 5,000 economies of 2,500 firms each) for different maturities and tax rates. Alphas are represented in % per month. The black solid line represents the base case scenario. We use an idiosyncratic volatility of $\sigma_i = 0.15$. The rest of the model parameters are in Table 5.



Marginal tax rate. To provide additional insights on how the maturity premium is related to firms' characteristics, we study the maturity premium for different levels of tax benefits of debt. The higher the tax benefits of debt, the higher the target debt level. We therefore vary the tax rate τ in the simulations displayed in Figure 8. First we note that the higher the tax rate, the higher the alpha. Moreover, for all tax levels we find that unconditional alphas increase substantially as we increase a firm's average debt maturity. The increase is much more pronounced for firms with larger tax rates. For example, for a tax rate of 0.35, the monthly alpha increases from less than 10 basis points per month to over 24 basis points as we move from a one year debt maturity to a six year debt maturity. If the tax benefit of debt is only 0.25, then the alpha increases to only slightly just above 18 basis points as we move to a six year debt maturity. Thus, we would expect firms with a significant tax benefit of debt, and therefore higher leverage ratios, to exhibit larger maturity premia.

Figure 9: Maturity Premium - Market Risk and Mean Reversion. In this figure we present the alpha from unconditional CAPM regression of simulations (over 5,000 economies of 2,500 firms each) for different maturities and three levels of market volatility (Panel A) and three values of mean reversion k in the productivity drift process (Panel B). Alphas are represented in % per month. The black solid lines in both panels represent the base case scenario. We use an idiosyncratic volatility $\sigma_i = 0.15$. The rest of the model parameters are in Table 5.



The productivity process. We move on to investigating the effects of the productivity process' parameters on the maturity premium levels. We run several additional simulations and show the results in Figure 9. Panel A illustrates the effect of the volatility of the productivity process σ_X. As can be seen, the maturity premium increases with σ_X. This is the opposite reaction to an increase in the idiosyncratic volatility σ_i, as shown in Figure 7. As the volatility of the productivity process increases, the market price of risk becomes more volatile. Hence, it co-moves more with betas of long-maturity financed firms, which we capture as higher unconditional alphas. In contrast, an increase in idiosyncratic volatility has no effect on the volatility of the market price of risk.

Furthermore, Panel B in Figure 9 shows the effect of the mean reversion k in the drift process. The larger k, the larger the maturity premium. This result is also due to the increased volatility of the market price of risk. To understand the intuition, consider the polar case $k = 0$, when the market price of risk does not change. Then there is no co-movement between beta and the market price of risk, and no maturity premium.

4.9 Debt Maturity Dynamics

Chen, Xu and Yang (2016) document counter-cyclical dynamics of average maturity. They argue that the liquidity premium that long-term bond-holders require goes up during downturns, making short-term debt more attractive.[20] Another reason why firms might chose to shorten maturity in crises is that they use short maturity bonds as a commitment to delever in the near future, as argued by Chaderina (2018).

In our model, as in DeMarzo and He (2018), the choice of maturity of any future debt issuances does not affect shareholder equity value. However, it might be a concern to the mechanism of our model if firms, for reasons outside of our model, were to shorten their debt maturity once they enter a crisis.

Despite the fact that our model does not have predictions for the optimal maturity choice, it is plausible that the effect of debt maturity on beta dyanamics would be even strengthened if long-term debt firms were allowed to reduce maturities during crises. Consider the following thought experiment: right after a negative shock to productivity long-term debt firms reduce their debt maturities. First, it is easy to show that, ceteris paribus, shortening debt maturities in our framework leads to a beta increase. Second, firms financed with shorter-maturity bonds have higher optimal leverage. So, while shortening maturity reduces debt overhang, it increases optimal leverage. Therefore, firms that experienced a reduction in maturity will not delever after a negative shock but not because of debt overhang but rather because of higher optimal leverage level. Therefore, allowing firms to shorten maturity in crisis is unlikely to weaken or even eliminate our prediction that firms that had longer maturity pre-crisis will have higher systematic exposure in crisis.

[20] See also Bruche and Segura (2017).

5 Empirical Evidence of the Model Mechanism

In this section we investigate further the model mechanism, and verify in data that the maturity premium at least partially can be explained by the difference in leverage cyclicality of long and short-maturity financed firms. First, we investigate the cyclicality of the systemic risk loadings of our long-short strategy.

5.1 Conditional CAPM and Beta Dynamics

The existence of a maturity premium in our model relies on the fact that long-maturity financed firms experience larger and more prolonged increases in their exposure to systematic risk in downturns than short-maturity financed firms. This premium represents a compensation for the positive covariance between betas of long-term financed firms and the market price of risk. In this subsection we provide direct empirical evidence that the beta of long-term financed firms increases in crises and that this increase is larger than that experienced by short-term financed firms.

In our evaluation of the conditional CAPM we follow the methodology of Choi (2013). While it might be most intuitive to study the evolution of short-window realized betas over the business cycle and its ability to account for the observed maturity premium, this leads to over-conditioning bias in the alpha estimation, as argued by Boguth et al. (2011). To avoid this problem, we follow the 2-step instrumental variable approach as in Choi (2013). We restrict the conditioning space to a linear combination of lagged macro variables that best predicts contemporaneous market price of risk.[21]

To estimate the dynamics of the LMS-portfolio's beta we consider the following conditional version of the CAPM:

$$r_t^{LMS} = \alpha + \beta_0 r_t^M + \beta_1 \eta_t r_t^M + \varepsilon_t, \tag{24}$$

where η_t is the market price of risk. The average exposure to systematic risk of the portfolio is captured by the value β_0, as in the classical CAPM. Moreover, the time-variation in beta is captured by the third coefficient, i.e., β_1. Therefore, the conditional beta is $\beta_0 + \beta_1 \eta_t$.

This specification is consistent with our theoretical model. Recall that X_t, the aggregate productivity

[21] Our approach is similar to that of Jagannathan and Wang (1996). They use credit spread as a conditioning variable, while we also use information in treasury bill rate and the dividend yield.

process, drives both the evolution of the firm's beta and the market price of risk. Therefore, the only time variation in firm's beta that is relevant for the maturity premium is the one that is projected on the variation in the market price of risk. Hence, using market price of risk as a conditioning variable is a natural choice in our setting.

Since we do not directly observe the market price of risk, we use the vector of lagged controls, Z_{t-1}, to estimate it. In particular, using 2-stage IV approach we estimate the following conditional CAPM:

$$r_t^{LMS} = \alpha + \beta_0 r_t^M + \beta_1 \eta(Z_{t-1}) r_t^M + \varepsilon_t, \qquad (25)$$

Our instruments Z consist of variables that are likely to drive the countercyclical market risk premium. As predictors we use the dividend yield (DY), the default spread (DS), the term spread (TS), the T-Bill rate (TB) and the consumption, wealth and income ratio (CAY) (see Lettau and Ludvigson, 2001). These variables are commonly employed in the literature (see Choi, 2013). We obtain their estimates from Amit Goyal's homepage, and the detailed description of their construction is available in Welch and Goyal (2007).[22]

The first step in the IV approach is to fit a one-month ahead predictive regression to span the observed market return by the predictors mentioned above, i.e.,

$$r_t^M = \delta_0 + \delta_1 \mathrm{DY}_{t-1} + \delta_2 \mathrm{DS}_{t-1} + \delta_3 \mathrm{TS}_{t-1} + \delta_4 \mathrm{TB}_{t-1} + \delta_5 \mathrm{CAY}_{t-1} + \varepsilon_t^M = \eta(Z_{t-1}) + \varepsilon_t^M. \qquad (26)$$

Results of the fitting estimation are presented in Table 6. In the first column, we use all four macro-variables to forecast the market return. As we see, only two out of five are significant. Hence, we re-estimate the model using only the significant dividend yield and t-bill rate as explanatory variables. Model (2) of the same table contains the estimates of this specification.

Although the two regression coefficients reported for model (2) are statistically highly significant, the overall predictive power of the regression is small. It explains only approximately 2% of the overall variation in the market risk premium. Thus, despite the statistical significance of dividend yield and t-bill rate, the predicted market returns are quite noisy. This is in line with a low power of the macro-models in explaining

[22] We use quarterly CAY estimates in our monthly predictive regressions. We lag CAY observations by one month and hold them constant for the subsequent two months.

Table 6: Predictive Variable. In this table shows the result of the predictive regression of the market's excess return on lagged predictors. The lagged explanatory variables are the dividend yield (DY), the default spread (DS), the term spread (TS), the T-Bill rate (TB) and the consumption, wealth and income ratio (CAY). In Model (1) we include all predictors, while in Model (2) only the significant variables from the first test are used. We report t-statistics based on standard errors following Newey and West (1987, 1994) in parentheses. The time horizon lasts from January 1976 until December 2017.

	Model (1)	Model (2)
DY	2.38***	1.60**
	(2.91)	(2.43)
DS	−7.10	
	(−0.08)	
TS	−34.47	
	(−1.34)	
TB	−36.15**	−18.99**
	(−2.16)	(−2.37)
CAY	16.75	
	(1.30)	
Intercept	11.77***	7.40***
	(3.05)	(2.74)

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

the dynamics of expected equity returns (Cochrane, 2011).

In the next step, we use the coefficient estimates reported in Table 6 to calculate the predictor $\hat{\eta}_t = \hat{\eta}(Z_{t-1})$ for the conditional model presented in Equation (25). We note that $\hat{\eta}_t$ is high in times when the dividend yield is high and the t-bill rate is low, consistent with a countercyclical market risk premium. The results of this conditional CAPM are reported in Table 7. The table has three sets of two columns, for long lag, sort lag of the portfolio and the difference between the long and the sort lags. The first column in each set contains estimates of a standard unconditional CAPM. The LMS portfolio exhibits a negative unconditional market beta and a positive alpha. This alpha is by construction identical to the alpha reported in Panel B of Table 3.

The second column in each set contains estimates from the conditional CAPM. We interact the market return with our estimated market risk premium to assess the time-variation in beta of the LMS portfolio. The interaction term β_1 is positive and statistically significant at the 5% level for the long lag, meaning that the long-term financed firms have higher beta in crisis than outside of crisis. For the short-financed firms we do not see such cyclical behavior in beta.

In the LMS portfolio, the interaction term β_1 is positive and statistically significant at the 1% level. This suggests that the beta of long-maturity financed firms increases in times when the market price of risk is high, i.e., in recessions or crises times, more than the beta of short-maturity financed firms. Hence, the overall exposure to market risk of the LMS portfolio increases when the market price of risk increases. This is exactly in line with our theoretical predictions. Long-term financed firms have an increased exposure to systematic risk in times when the market risk premium is particularly high.

Moreover, we also find that the estimated alpha decreases from 0.21 to 0.19 once we introduce our conditioning variable. Thus, the time-variation in beta that we capture via our estimated market price of risk explains part of the maturity premium that we observe in the unconditional CAPM. The magnitude of the reduction in alpha between the unconditional and the conditional CAPM is statistically significant, as we show in our GMM estimation.

To provide additional evidence that long-maturity financed firms have higher systemic risk exposure when market price of risk is high, we estimate the conditional beta of LMS using short-window CAPM regressions. We plot the resulting relationship between the market price of risk and the time-varying beta of

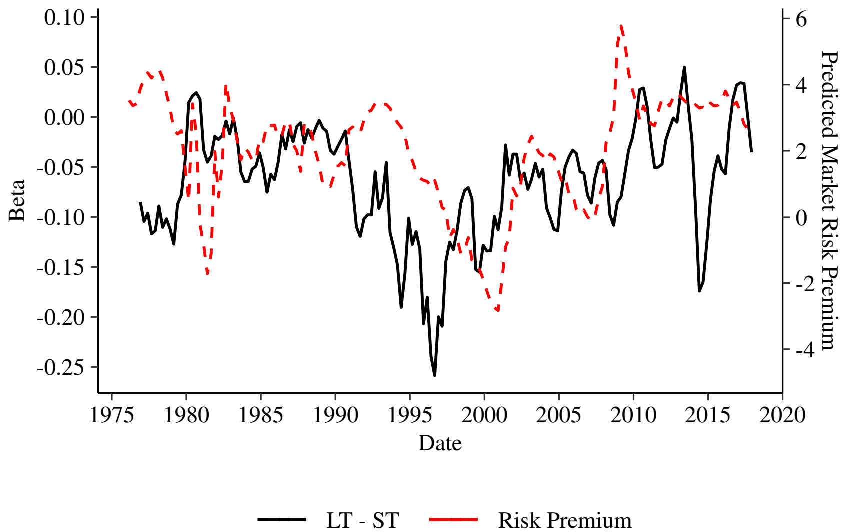
Table 7: Maturity Premium in a Conditional CAPM. This table presents the result of an unconditional and a conditional version of the CAPM, respectively. The dependent variable is the return of the long leg (columns 1 and 2), the short leg (columns 3 and 4), and the long-minus-short portfolio (columns 5 and 6) as constructed in Table 3. The conditional version includes an interaction between the predicted market risk premium and the market return. The predicted risk premium is constructed using lagged predictors and the coefficient estimates shown in Model (2) of Table 6. While β_0 corresponds to the unconditional estimate, β_1 represents the coefficient on the interaction. In the last column we report the difference between the unconditional and conditional intercept from a GMM estimation. The time horizon lasts from January 1976 until December 2017.

	Long-Term		Short-Term		Long-Minus-Short		
	Uncond.	Cond.	Uncond.	Cond.	Uncond.	Cond.	$\alpha_u - \alpha_c$
α	0.15	0.11	−0.06	−0.07	0.21***	0.19***	0.02**
	(1.24)	(0.92)	(−0.41)	(−0.52)	(3.39)	(2.77)	(2.10)
β_0	1.04***	0.98***	1.13***	1.11***	−0.09***	−0.13***	
	(33.19)	(24.12)	(36.77)	(23.73)	(−6.51)	(−5.79)	
β_1		0.10**		0.04		0.06***	
		(2.40)		(0.78)		(2.57)	

***$p < 0.01$, **$p < 0.05$, *$p < 0.1$

LMS in Figure 10. The market price of risk is from the Equation 26 estimated using monthly returns, then annualized and reporting averages for each quarter. The beta of LMS is the time-series of slope estimates from running CAPM regressions within quarters using daily returns, averaged over the rolling window of four preceding quarters. The two time-series co-move positively with a correlation coefficient of 0.22, which is statistically significant at 1% level. Note that the beta estimates in this analysis do not rely on the choice of conditioning macro-variables, reinforcing the robustness of our empirical observation that long-maturity financed firms have higher systemic exposure in downturns.

Figure 10: Time Dynamics of The Conditional Beta and The Market Price of Risk. This figure shows the estimated dynamics of the conditional beta $\hat{\beta}_0 + \hat{\beta}_1 Z_{t-1}$, see Equation (25), of a long-minus-short portfolio as constructed in Table 3, and the market risk premium, see Equation (26). The time horizon lasts from January 1976 until December 2017.



5.2 Interaction between Debt Maturity and Leverage

As we demonstrate in Section 4.7, our model predicts that a firm's leverage should be related to the maturity premium. Specifically, we find that firms with high idiosyncratic risk, and thus lower target leverage, exhibit lower debt maturity premia. Similarly, firms with low net benefits of debt also exhibit lower target leverage and lower maturity premia. Intuitively, this is easy to understand. When a firm has very little debt, then even a very long maturity will only marginally affect the covariance of its beta with the market price of risk. In this subsection we investigate this prediction empirically.

In general, any firm with long-term debt is impacted by negative shocks and the resulting slow adjustment of debt. Yet firms with high leverage will show a stronger reaction compared to firms with low levels of debt. Thus, we look at the interaction of leverage and debt maturity by conducting another set of conditional double sorts based on leverage and debt maturity.

Indeed, we find evidence that the effects of long debt maturities on equity risk are stronger among firms with high leverage. In Table 8 Panel B, the risk-adjusted returns for the portfolio that is long firms with long debt maturities and short firms with short-term debt in the high leverage bucket (i.e., LMS5) show positive alphas. Firms with long-term debt produce risk-adjusted excess returns between 0.38% and 0.41% per month in all factor models, which are statistically-significant at the 10% level. Moreover, the alpha estimate for highly levered firms exceeds the magnitude of the LMS premium reported in Table 3.

For firms with low leverage ratios, our model predicts that the risk associated with longer debt maturities is reduced. We find support for this in the data. For firms with the lowest leverage ratios (i.e., LMS1) we do not find a premium significantly different from 0.

6 Conclusion

In this paper we show theoretically and empirically that long-maturity financed firms have higher expected returns than short-maturity financed firms. We provide evidence that this is due to the risk of leverage increases in downturns, which are more severe for long-maturity financed firms. Short-maturity financed firms are more exposed to rollover risk and may therefore be more risky during a sharp and instantaneous decline, while for the more progressive declines commonly associated with recessions long-maturity firms are more risky.

While a conditional CAPM holds in our model, increases in leverage during downturns generate a co-movement between firms' betas and the market price of risk, which appears as alpha in unconditional CAPM regressions. Empirically, we document a monthly premium of 0.21% for a strategy that goes long long-term financed firms and short short-term financed firms, which we call maturity premium.

Our paper sheds light on the contribution of leverage dynamics to asset pricing patterns, that appear as anomalies relative to the unconditional CAPM. While the role of operating leverage and investment irreversibility has been shown to explain the value premium, they alone can't match the magnitude. We show that long-term maturity is what makes financial leverage hard to reverse because of debt overhang. Hence, long-maturity financial leverage gives rise to a maturity premium. As value firms tend to be financed with long-term debt, the book-to-market ratio proxies for maturity choice. We therefore demonstrate that long-term financial leverage contributes to the value premium. However, controlling for the value factor, the portfolio of long minus short maturity financed firms still generates an unconditional alpha. This means that the maturity factor is distinct from the value factor and captures the important risk of leverage increases in downturns. We believe that a fuller exploration of the effects of dynamic corporate decisions on equity pricing is a highly attractive agenda for future research, as it can shed light on patterns asset-pricing patterns that appear as anomalies today.

Table 8: Value-Weighted Returns for Dependent Sorts on Debt Maturity and Leverage. Panel A shows the average excess return of the individual value-weighted portfolios. The last row contains the long-maturity minus short-maturity portfolios (LMS). The portfolios are formed by double sorts on leverage (5 buckets at 20%, 40%, 60%, and 80%-percentile) and debt maturity (5 buckets at 20%, 40%, 60%, and 80%-percentile) conditional within each leverage group. Panel B examines the LMS portfolios within each leverage bucket (LMS1 for firm with low to LMS5 for firms with high leverage). The long-short portfolios are represented by excess returns (r^e) as well as alpha estimates from CAPM-regressions (α^{CAPM}), the 3-factor model by Fama and French (1993) (α^{FF3}) and the 5-factor model by Fama and French (2015) (α^{FF5}). Moreover, risk factor loadings for FF5 are shown. We report t-statistics based on standard errors following Newey and West (1987, 1994) in parentheses. The underlying data set comprises matched observations from CRSP and COMPUSTAT from January 1976 until December 2017.

Panel A: Portfolio Sorts

		Leverage				
		Low	·	Medium	·	High
Debt Maturity	Short	0.78	0.89	0.84	0.79	0.77
	·	0.40	0.74	0.78	0.79	0.87
	Medium	0.60	0.71	0.88	0.77	0.84
	·	0.55	0.63	0.81	0.69	0.84
	Long	0.61	0.71	0.83	0.81	0.94
LMS		−0.17	−0.18	−0.01	0.02	0.17

Panel B: Debt Maturity (LMS)

	LMS1	**LMS2**	**LMS3**	**LMS4**	**LMS5**
r^e	−0.17	−0.18	−0.01	0.02	0.17
	(−1.10)	(−1.18)	(−0.09)	(0.13)	(0.91)
α^{CAPM}	−0.08	−0.23	−0.00	0.18	0.38*
	(−0.57)	(−1.36)	(−0.00)	(0.99)	(1.81)
α^{FF3}	−0.11	−0.10	0.11	0.19	0.41**
	(−0.70)	(−0.73)	(0.77)	(1.06)	(2.20)
α^{FF5}	−0.35**	0.00	0.10	0.24	0.34*
	(−2.31)	(0.01)	(0.62)	(1.44)	(1.89)
β^{M}	0.01	0.06	0.02	−0.24***	−0.38***
	(0.16)	(1.39)	(0.47)	(−4.12)	(−6.22)
β^{SMB}	−0.19**	−0.37***	−0.37***	−0.07	0.42***
	(−2.34)	(−5.09)	(−5.88)	(−0.63)	(4.27)
β^{HML}	0.07	−0.03	−0.18**	−0.12	−0.45***
	(0.89)	(−0.32)	(−2.22)	(−0.91)	(−2.65)
β^{RMW}	0.48***	−0.10	−0.01	−0.23**	−0.06
	(5.43)	(−0.80)	(−0.10)	(−2.20)	(−0.39)
β^{CMA}	0.14	−0.31**	0.10	0.26	0.51**
	(1.42)	(−2.27)	(0.87)	(1.56)	(2.56)

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

APPENDIX

A Model Solution

For the valuation of the equity claim consider the Hamilton-Jacobi-Bellman equation (*HJB* below)[23] associated with the expected future dividends shown in Equation (8). The required return is equal to the risk-free rate r when the firm issues the optimal amount of debt at any point in time, which in turn determines the dynamics of the total face value of debt, $dF_{i,t}$, as defined in Equation (6). Hence, we need to solve the following HJB equation for the optimal $G_{i,t}$

$$\begin{aligned} rV^E(Y_{i,t},F_{i,t}) =& \max_{G_{i,t}} \left\{ Y_{i,t}(1-\tau) + \tau c F_{i,t} - (c+m)F_{i,t} + G_{i,t} v^D_{i,t} + (G_{i,t} - mF_{i,t}) V^E_F(Y_{i,t},F_{i,t}) \right\} \\ &+ \mu_Y V^E_Y(Y_{i,t},F_{i,t}) + 1/2 \sigma^2_Y V^E_{YY}(Y_{i,t},F_{i,t}). \end{aligned} \tag{A-1}$$

Issuing a marginal unit of debt generates benefits of $v^D_{i,t}$ to equityholders and costs of $V^E_F(Y_{i,t},F_{i,t})$ for future payments to debt holders. Assuming that debt is issued smoothly at the discretion of equityholders, equating marginal benefits and marginal costs results in the following first-order-condition (FOC)

$$v^D_{i,t} + V^E_F(Y_{i,t},F_{i,t}) = 0. \tag{A-2}$$

DeMarzo and He (2018) lay out optimality conditions for the debt issuance policy, which are met in our setup. Using the FOC from Equation (A-2) in the HJB shown in Equation (A-1) yields the following HJB that does not depend on $G_{i,t}$

$$\begin{aligned} rV^E(Y_{i,t},F_{i,t}) =& Y_{i,t}(1-\tau) + \tau c F_{i,t} - (c+m)F_{i,t} - mF_{i,t} V^E_F(Y_{i,t},F_{i,t}) \\ &+ \mu_Y V^E_Y(Y_{i,t},F_{i,t}) + 1/2\, \sigma^2_Y V^E_{YY}(Y_{i,t},F_{i,t}). \end{aligned} \tag{A-3}$$

Now we divide both state variables by the face value of debt $F_{i,t}$, which leaves one state variable constant. From here onwards, lower case letters refer to scaled versions of the upper case variables (e.g., the scaled

[23] Here we use subscripts Y and F for the functions of the market value, where superscripts denote that it is either the equity or debt market value, respectively, to denote partial derivatives with respect to those variables to save on notation.

cash flow level $y_{i,t} = Y_{i,t}/F_{i,t}$ and the scaled issuance policy $g_{i,t} = G_{i,t}/F_{i,t}$). Subsequently, the dynamics of the firm's scaled cash flow process under the risk-neutral measure from Equation (5) accounting for the scaling by $1/F_{i,t}$ are given by

$$dy_{i,t} = (\mu_Y + m_i - g_{i,t})y_{i,t}\,dt + \sigma_Y y_{i,t}\,dW^{Q}_{Y_i,t}\,, \tag{A-4}$$

which also changes the HJB from Equation (A-3) to

$$(r+m_i)v_i^E(y_{i,t}) = y_{i,t}(1-\tau) + c\tau - (c+m_i) + (\mu_Y + m_i)y_{i,t}v_Y^E(y_{i,t}) + 1/2\sigma_Y^2 y_{i,t}^2 v_{YY}^E(y_{i,t}). \tag{A-5}$$

To solve Equation (A-5) we impose the boundary condition for $y_{i,t} \to \infty$, where the equity value should converge to the perpetuity of the after-tax cash flows plus the coupons tax shield less the bond's perpetuity value. Furthermore, at the cash flow level where equityholders default y_b, equity is worth nothing. Finally, the optimal default boundary is determined by the smooth-pasting condition, i.e., $v_Y^E(y_b) = 0$. Then, the equity value function is given by

$$v_i^E(y_{i,t}) = \frac{1-\tau}{r-\mu_Y}y_{i,t} - \frac{c(1-\tau)+m_i}{r+m_i}\left(1 - \frac{1}{1+\gamma_i}\left(\frac{y_{i,t}}{y_{b,i}}\right)^{-\gamma_i}\right), \tag{A-6}$$

with the exponent equal to

$$\gamma_i = \frac{(\mu_Y + m_i - \sigma_Y^2/2) + \sqrt{(\mu_Y + m_i - \sigma_Y^2/2)^2 + 2\sigma_Y^2(r+m_i)}}{\sigma_Y^2} > 0, \tag{A-7}$$

and the default boundary

$$y_{b,i} = \frac{\gamma_i}{1+\gamma_i}\frac{r-\mu_Y}{r+m_i}\left(c + \frac{m_i}{1-\tau}\right). \tag{A-8}$$

The scaled value of debt, i.e., the price per unit of face value, follows from the FOC in Equation (A-2) as

$$v_i^D(y_{i,t}) = \frac{c(1-\tau)+m_i}{r+m_i}\left(1-\left(\frac{y_{i,t}}{y_{b,i}}\right)^{-\gamma_i}\right). \tag{A-9}$$

While we initially did not have to specify the debt issuance policy $G_{i,t}$, we still need to derive it. We do so by considering the HJB for the value of debt. The value of debt can also be based on the expectation of future principal payments and coupons paid to debt holders as shown in Equation (9), like

$$\begin{aligned} & rv^D(Y_{i,t},F_{i,t}) = \\ & = c+m(1-v^D(Y_{i,t},F_{i,t}))+(G_{i,t}-mF_{i,t})v_F^D(Y_{i,t},F_{i,t})+\mu_Y v_Y^D(Y_{i,t},F_{i,t})+1/2\sigma_Y^2 v_{YY}^D(Y_{i,t},F_{i,t}). \end{aligned} \tag{A-10}$$

Next, we impose the FOC from Equation (A-2) on the derivative with respect to the debt level $F_{i,t}$ of the HJB for equity in Equation (A-3) to find another HJB for the price of debt, which is equal to

$$\begin{aligned} & -rv^D(Y_{i,t},F_{i,t}) = \\ & = \tau c-(c+m)+mv^D(Y_{i,t},F_{i,t})+mF_{i,t}v_F^D(Y_{i,t},F_{i,t})-\mu_Y v_Y^D(Y_{i,t},F_{i,t})-1/2\,\sigma_Y^2 v_{YY}^D(Y_{i,t},F_{i,t}). \end{aligned} \tag{A-11}$$

Finally, adding Equations (A-10) and (A-11) results in the following expression for the optimal debt issuance policy (in its scaled version)

$$g_i(y_{i,t}) = \frac{(r+m_i)\tau c}{c(1-\tau)+m_i}\frac{1}{\gamma_i}\left(\frac{y_{i,t}}{y_{b,i}}\right)^{\gamma_i}. \tag{A-12}$$

We can see that there is a cash flow level, which we denote by $y_{m_i,i}$, at which the firm rolls over exactly the maturing amount of debt m_i, which keeps the face value of debt constant. This cash flow level equals

$$y_{m,i} = y_{b,i}\left(\gamma_i\frac{c(1-\tau)+m_i}{(r+m_i)\tau c}m_i\right)^{1/\gamma_i} \tag{A-13}$$

and can be used to restate Equation (A-12) from above to the version of the main text in Equation (12).

In the end, the evolution of the face value of debt $F_{i,t}$ is the result of debt issuance decisions and the constantly maturing portion of debt. By combining the law of motion of $F_{i,t}$ presented in Equation (6) with the optimal debt issuance function from Equation Equation (12), we find that

$$dF_{i,t} = \left(m_i \frac{Y_{i,t}^{\gamma_i}}{y_{m,i}^{\gamma_i}} F_{i,t}^{1-\gamma_i} - mF_{i,t} \right) dt \,. \tag{A-14}$$

While Equation (A-14) is not linear in $F_{i,t}$ we can substitute $H = F^{\gamma_i}$. Then we can find a solution to the differential equation for $dH = \gamma_i F_{i,t}^{\gamma_i} dF$ given that $H_0 = 0$. In the end, we can insert $F_{i,t}$ back into the general solution and find

$$F_{i,t} = \left(\int_0^t \gamma_i m_i \left(\frac{Y_{i,s}}{y_{m,i}} \right)^{\gamma_i} e^{\gamma_i m_i (s-t)} \, ds \right)^{1/\gamma_i} . \tag{A-15}$$

B Return on Equity

In this subsection we analyze equity returns in detail and demonstrate that under the risk-neutral measure the expected value of equity returns is r, consistent with FOC of equity pricing, while innovations to cash flows are amplified by a firm's financial leverage v^D/v^E.

$$r_{t,t+dt}^E = \frac{dV_t^E + \Pi_{t,t+dt}}{V_t^E} \,. \tag{B-1}$$

$$
\begin{aligned}
r^E_{t,t+dt} &= \frac{V^E_F\,dF_t + V^E_Y\,\mu_Y Y_t\,dt + V^E_Y\,\sigma_Y Y_t\,dW^Q_{Y_i,t} + \frac{1}{2}V^E_{YY}\sigma^2_Y Y^2_t\,dt + \Pi_{t,t+dt}}{V^E(Y_t,F_t)}, \\
\frac{\partial V^E(Y,F)}{\partial F} &= \frac{\partial}{\partial F}\left(V^E\left(\frac{Y}{F},1\right)F\right) = -\frac{Y}{F^2}\frac{\partial V^E\left(\frac{Y}{F},1\right)}{\partial \frac{Y}{F}}F + V^E\left(\frac{Y}{F},1\right) \\
V^E_F &= -yv^E_y + v^E \qquad \text{(B-2)} \\
rV^E(Y,F) &= \max_G \Pi_{t,t+dt} + V^E_F\,dF_t + V^E_Y\,\mu_Y Y_t\,dt + \frac{1}{2}V^E_{YY}\sigma^2_Y Y^2_t\,dt \\
r^E_{t,t+dt} &= \frac{1}{V^E(Y_t,F_t)}\left(rV^E\,dt + V^E_Y\,\sigma_Y Y_t\,dW^Q_{Y_i,t}\right) \\
&= r\,dt + \frac{V^E_Y Y_t}{V^E(Y_t,F_t)}\sigma_Y\,dW^Q_{Y_i,t};\ \text{ divide by } F \\
&= r\,dt + \frac{v^E_Y y_t}{v^E(Y_t,F_t)}\sigma_Y\,dW^Q_{Y_i,t};\ \text{and using Equation (B-2) we arrive at} \\
&= r\,dt + \frac{v^E - V^E_F}{v^E(Y_t,F_t)}\sigma_Y\,dW^Q_{Y_i,t};\ \text{using FOC from Equation (A-2)} \\
&= r\,dt + \frac{v^E + v^D}{v^E}\sigma_Y\,dW^Q_{Y_i,t}; \\
&= r\,dt + \left(1 + \frac{v^D}{v^E}\right)\sigma_Y\,dW^Q_{Y_i,t} \qquad \text{(B-3)}
\end{aligned}
$$

Under the physical measure we find:

$$
\begin{aligned}
r^E_{t,t+dt} &= r\,dt + \left(1 + \frac{v^D}{v^E}\right)\sigma_Y\,\lambda_t\,dt + \left(1 + \frac{v^D}{v^E}\right)\sigma_Y\,dW^P_{Y_i,t} \\
r^E_{t,t+dt} &= r\,dt + \left(1 + \frac{v^D}{v^E}\right)\eta_t\,dt + \left(1 + \frac{v^D}{v^E}\right)\sigma_Y\,dW^P_{Y_i,t}\,. \qquad \text{(B-4)}
\end{aligned}
$$

C Detailed Beta Derivation

The equity beta can be calculated as:

$$\beta_{i,t} = \frac{Cov_t(r^E_{t,dt}, r^M_{t,dt})}{Var_t(r^M_{t,dt})} \tag{C-1}$$

$$= \frac{1}{Var_t(r^M_{t,dt})} Cov_t\left(\left(1 + \frac{v^D_i(y_t)}{v^E_i(y_t)}\right)\sigma_Y dW^P_{Y,t}; \sigma_x dW^P_{x,t}\right) \tag{C-2}$$

$$= \frac{1}{\sigma^2_x}\sigma_Y\sigma_x\left(1 + \frac{v^D_i(y_t)}{v^E_i(y_t)}\right) Cov_t(dW^P_{Y,t}, dW^P_{x,t}) \tag{C-3}$$

$$= \frac{1}{\sigma_x}\sigma_Y\left(1 + \frac{v^D_i(y_t)}{v^E_i(y_t)}\right) Cov_t\left(\frac{1}{\sigma_Y}(\sigma_x dW^P_{x,t} + \sigma_i dW^P_{i,t}), dW^P_{x,t}\right) \tag{C-4}$$

$$= 1 + \frac{v^D_i(y_t)}{v^E_i(y_t)}. \tag{C-5}$$

D Definition of Variables

In this section we provide definitions for the metrics and proxies used in the empirical part. For each item used from either COMPUSTAT or CRSP we identify the source. The item abbreviations are matched to variable descriptions in Table C-1.

Table C-1: COMPUSTAT & CRSP Item Description. Items from COMPUSTAT are listed below in capital letters, all variables from CRSP are listed using lower case letters.

Item Name	Variable Description
CSHO	Common Shares Outstanding
*DD*1	DD1 – Long-Term Debt Due in One Year
*DD*2	DD2 – Debt Due in 2nd Year
*DD*3	DD3 – Debt Due in 3rd Year
DLC	Debt in Current Liabilities - Total
DLTT	Long-Term Debt - Total
PRCC_F	Price Close - Annual - Fiscal
PSTKRV	Preferred Stock Redemption Value
PSTKL	Preferred Stock Liquidating Value
PSTK	Preferred/Preference Stock (Capital) - Total
TXDITC	Deferred Taxes and Investment Tax Credit
altprc	Price Alternate
shrout	Number of Shares Outstanding

We define *leverage* as the ratio of book debt to book debt plus market equity, as in Danis et al. (2014).

$$L := \frac{DLC + DLTT}{DLC + DLTT + PRCC\_F * CSHO} \tag{C-1}$$

Next, we define a proxy for *debt maturity* by looking at the share of debt maturing in more than 3 years to the total amount of book debt, as proposed by Barclay and Smith (1995). To measure debt maturing in more than 3 years we subtract debt maturing in the 2^{nd} and 3^{rd} year (items $DD2$ and $DD3$, respectively) from the total of long-term debt.

$$DM := \frac{DLTT - DD2 - DD3}{DLC + DLTT} \tag{C-2}$$

Market equity is defined as the price per share times shares outstanding scaled by a factor 10^{-3}.

$$ME := \frac{|altprc| * shrout}{1000} \tag{C-3}$$

The *book value of equity* is defined in line with Fama and French (1992, 1993) as the book value of stockholder's equity adjusted for the value of tax effects of deferred taxes and investment credit and subtracting the book value of preferred stock. The value of preferred stock (abbreviated $[BVPS]$) is determined by taking redemption, liquidation, or par value (from COMPUSTAT $PSTKRV$, $PSTKL$, or PSK, respectively) depending on availability in the given order.

$$BE := SEQ + TXDITC - [BVPS] \tag{C-4}$$

Finally, the *book-to-market* ratio is calculated as proposed by Fama and French (1992, 1993). This means to relate book equity as computed by the fiscal year ending in year t to market equity as of December of year t.

$$BM := \frac{BE}{ME} \tag{C-5}$$

We use returns on the ordinary equity of individual firms (CRSP) in excess of the risk-free rate (**Excess**

Returns). The data on risk-free rates (1 month T-bill rates) is taken from the Kenneth French web-page.[24]

For the conditional version of the CAPM, we take data on the macro-economic variables that we use as predictors from Amit Goyal's homepage. The dividend yield (denoted by DY) is the log difference between dividends and lagged prices. Where the dividends are the 12-month moving sum of dividends on the S&P 500. The default spread (denoted by DS) is the difference between the yields on BAA and AAA-rated corporate bonds. The term spread (denoted by TS) is defined as the yield difference between long-term and short-term government bonds. T-bill rate is denoted as TB. CAY is estimated at the quarterly frequency:

$$c_t = \alpha + \beta_a a_t + \beta_y y_t + \sum_{i=-8}^{8} b_{a,i} \Delta a_{t-i} + \sum_{i=-8}^{8} b_{y,i} \Delta y_{t-i} + \varepsilon_t, \tag{C-6}$$

$$CAY_t := c_t - \hat{\beta}_a a_t - \hat{\beta}_y y_t, \tag{C-7}$$

where c_t is aggregate consumption, a_t is aggregate wealth, and y_t is the aggregate income. The sample used for estimating CAY is 1st quarter of 1951 to 4th quarter of 2018.

[24]See http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#Research.

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Internet Appendix to

The Maturity Premium

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E Non-negative Market Price of Risk

Here we present results for an alternative specification of the drift process that ensures non-negative market price of risk.

Define the aggregate productivity drift to be:

$$\mu(X_t,t) = \max\left\{\mu_0 - k\left[log(X_t) - \left(\mu_0 - \sigma_X^2/2\right)t\right], r - \delta\right\}, \tag{C-8}$$

where r is the risk-free rate, and δ is the aggregate dividend process. The lower bound of $r-\delta$ ensures that the market price of risk, which is

$$\lambda_t = \frac{\mu(X_t,t) + \delta - r}{\sigma_X}, \tag{C-9}$$

is non-negative.

Introducing a lower bonds on mu_t reduces time-variation in the market price of risk λ_t, and therefore, reduces the covariance between betas of firms and the market price of risk. The resulting unconditional alpha estimates as a function of maturity, are below in Figure 11.

While the general pattern of returns is exactly the same as in the benchmark model, the level of unconditional alphas is smaller for all maturity levels. And the spread between short and long-maturity financed firms is also smaller. The simulated maturity premium is smaller than in the benchmark setting. With the lower zero bound on the market price of risk, the simulated maturity premium is (???).

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Figure 11: Maturity Premium with Non-negative Market Price of Risk. In this figure we present the resulting alpha from unconditional CAPM regression of simulations for different maturity-volatility pairs (over 10,000 economies of 2,500 firms each). Alphas are represented in % per month. All parameters underlying this simulation are detailed in Table 5. The zero lower bound is imposed on the market price of risk.