Belief Polarization and Investment*

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Abstract

We study a canonical real option model where the decisions to acquire and exercise an option are made sequentially by a group of agents with heterogeneous beliefs. Applying results from the political economy literature on spatial voting, we show that when the group mediates disagreement through voting, inefficient underinvestment can occur: although each group member would acquire the option if he had post-acquisition control rights, the group votes against acquisition. We show that this inefficiency occurs when group members' beliefs are polarized, that is, divided into two opposing factions of equal size. Given the pervasive nature of group decisions in the life of organizations and institutions, our theory is particularly relevant for the behavior of venture capitalists, financing syndicates, corporate boards, and committees at large.

Keywords: group decisions, dynamic voting, real investment.

Contents

List of Tables

List of Figures

1 Introduction

The vast majority of decisions undertaken by corporations and institutions are the result of group efforts. Boards of directors, venture capital groups, financing syndicates, and committees, are obvious reminders of the ubiquitous nature of group decisions in all aspect of economic life. Despite the pervasiveness of group decisions at different levels within a corporation, the finance literature has so far devoted little attention to the possible coordination frictions that can emerge in such settings.

In this paper we aim to address this gap by considering a canonical investment problem: the decision to acquire a license to develop a project. To this problem, extensively studied by the real option literature, we add a key variant: the licensing and investment decisions are undertaken by a group of individuals rather than an "entrepreneur" or a "representative manager". We further assume that group members have heterogenous beliefs and that the group makes decision based on majority voting. To focus explicitly on the coordination frictions that this governance rule entails, we explicitly abstract away from modelling informational asymmetries and bargaining among group members.

We borrow from the political economy literature on voting and treat the dynamic investment problem as a dynamic *voting* game. We find that agents' disagreement about the value and optimal management of the investment opportunity can be an important source of coordination friction within the group. Specifically, we show that when group members' beliefs are sufficiently *polarized*,[1] the equilibrium of the voting game exhibit *inefficient underinvestment*: a project that is deemed worthy by each group member is nevertheless rejected by the group.

Our result that polarization can lead to investment inefficiency rests on the following four key assumptions. First, corporate decisions are made by a group that cannot avoid disagreement by abstaining from the decision. Second, members of the group have heterogeneous beliefs about the profitability of the project. Third, group members cannot solve their

[1] Formally, polarization occurs in our context when the beliefs of half of the group members are concentrated around the beliefs of the most optimistic agent whereas the beliefs of the other half of group members are concentrated around the most pessimistic agent.

disagreement by trading their vote. Fourth, disagreement about what the group should do is mediated by a majority vote: both the acquisition and the subsequent investment decision are undertaken when there is strict majority support.

Under these assumptions, we studied the following real option problem. At an initial time, the firm has the opportunity to purchase a license that allows it to produce and sell output in exchange for a fixed investment cost. The value of the licensed technology evolves stochastically, so that the exercise decision is a standard timing option. However, although each member of the group agrees on the volatility of the underlying process, they disagree on the expected profitability and hence each member sees a different optimal time at which to begin production.

Inefficient underinvestment results because heterogeneous views on optimal investment timing, that are mediated by majority vote, can emphasize extreme views. Each member of the group rationally anticipates the outcome of the vote that will determine the investment time. When this anticipated time reflects an extreme view, it becomes so undesirable to some agents that the initial acquisition becomes not worth undertaking. When the number of these dissatisfied agents is large enough, they form a *decisive coalition* that blocks the acquisition.

We show that the size of the group is critical for the likelihood of investment inefficiency. Specifically, inefficiency can occur only in even-numbered groups. By contrast, in odd-numbered groups, the number of agents who oppose licensing cannot form a decisive coalition and as a result the group will always license. This result, which is a direct consequence of the median voter theorem (Black, 1948), is consistent with the widely used rule of thumb in venture capital, that calls for boards with an odd numbers of seats.

In addition to characterizing investment inefficiency due to polarization, we highlight the role of polarization relative to risk. Increases in volatility in a simple timing option like ours means that investment will be delayed to take advantage of the value of waiting. But investment delays are not lost and hence not inefficient: the investment is simply delayed to a more satisfactory time. In contrast to an increase in volatility, polarization does not lead to investment delay, but to lost investment opportunities. We show that polarization within

a group is an important economic force that interacts with volatility. In particular, underinvestment due to polarisation can occur for any level of volatitliy. However, all else being equal, high volatility mitigates underinvestment. This means that, when the investment environment is more uncertain, underinvestment can occur only if the degree of polarization among group members is sufficiently high.

The voting protocol is a key determinant of this inefficiency. We focus on majority voting, where underinvestment requires sufficient disagreement between the two agents whose beliefs straddle the median belief of the group. In contrast, with unanimity, underinvestment requires that the range of beliefs in the group is sufficiently dispersed. We also explore the implication of other voting protocols such as supermajority and majority with vetoers and show that even under these more general protocols, underinvestment requires some degree of polarization within the group.

Our work contributes to a relatively new literature on group decision making in financial economics. Garlappi, Giammarino, and Lazrak (2017) also show that inefficient underinvestment can occur in group decision making. However, their mechanism relies on agents learning about the value of a project overtime and on a utilitarian governance rule that aggregates individual beliefs for the purpose of decision making. In contrast, in our setting underinvestment occurs even in the absence of learning and without a utilitarian aggregation rule. The only requirement is that decisions are made sequentially and that disagreements are mediated through voting.

Our paper is also related to a recent political economy literature on dynamic voting that rationalizes the emergence of inefficient gridlock and status quo. Dziuda and Loeper (2016) show that, in a dynamic voting game where preferences evolve over time and where the previous decision becomes the next status quo, inefficiency and deadlock car arise. Building on this intuition, Donaldson, Malenko, and Piacentino (2017) analyze the problem of gridlock within corporate board of directors.

The key mechanism for obtaining gridlock in these papers is the change of agents' preferences overtime and the endogeneity of the status quo. Agents oppose current proposals even if they are perceived as better than the status quo. If a current proposals is accepted, it

becomes the status quo for future. Agents who hope for the for the arrival of their absolute preferred proposal in the future will therefore block any current proposal. Although our result is similar in that it shows the possibility of an inefficient group choice, our mechanism differs along several dimensions. First, we provide a context where inefficient behaviour can occur even in the absence of endogenous status quo. Second, in our setting agents preference do not change overtime and everybody is fully informed. Our mechanism concerns agents who, for unmodeled reasons, disagree on a key aspect of the option. Pessimists are reluctant to acquire a license, and if the license is acquired, are the first to propose the exercise of the investment option. This occurs because to a pessimist, the opportunity cost of not exercising the option, once acquired is high, leading them to be eager to start production early. In contrast, optimists are eager to acquire the license, and, once acquired, prefer to wait longer to begin production. For optimists, the opportunity cost of waiting to exercise the option is low. If the differences between the optimists and pessimists is large enough, inefficient underinvestment can result.

Our paper also contributes to the real option literature (Dixit and Pindyck, 1994; McDonald and Siegel, 1986) which we extend to allow for the case in which real options are exercised by a group instead of an individual. To the best of our knowledge, ours is the first study to formally analyze the exercise of a real investment option by a group of agents with heterogeneous beliefs. Our analysis provide a general tractable framework to study dynamic investment decisions undertaken collectively by a group of agents.

2 An illustrative example

Consider a group of two agents who form a "Decision Making Group" (DMG hereafter). At time zero the DMG must decide, on behalf of the corporation, whether or not to acquire a license (the licensing decision) that gives the firm the right to invest in a project at a future (the investment decision).

DMG decisions. Both the licensing and the investment decisions are determined by strict majority rule (unanimity in a group with two members). The licensing vote takes place at

time zero while the investment vote can happen at *any* time afterwards. In particular, after the majority of members of the DMG have agreed to acquire the licence, any member of the DMG can, at any time, propose that the investment take place immediately. The proposal will then trigger a vote and investment will occur only if the majority, i.e.both parties, agree. We also assume that when voting for the license acquisition decision, the DMG members anticipate the outcome of the second decision and vote strategically accordingly. Figure 1 illustrates the decisions faced by the DMG.

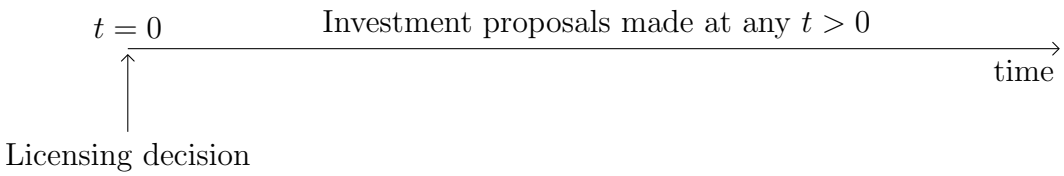

Figure 1: Model timeline

At time $t = 0$ the group decides whether to acquire a license to invest in a project. Licensing will occur only under if the strict majority of group members agree. Investment proposals are made by any group members at any time after $t = 0$ and undertaken only if the strict majority of group members agree.

Technology. Investment is irreversible and requires the payment of a cost I. The two agents, labeled P and O disagree on the underlying profitability of the project. Formally, the value of the project X follows a geometric Brownian motion with volatility σ but investors disagree on its expected drift, that is

$$dX_t = \mu_n X_t + \sigma X_t dW_{n,t}, \quad X_0 = x > 0, \quad \mu_P < \mu_O,$$

where $W_{n,t}$ is a standard Brownian motion under investor $n's$ belief, $n \in \{P, O\}$.

Agent P is the pessimist and agent O is the optimist ($\mu_P < \mu_O$). We assume that agents are risk-neutral and discount cash flow at the risk free rate r.[2] We further assume that $\mu_O < r$. If this was not the case, then O will always be better off waiting and simply holding

[2]Equivalently, one can assume that markets are complete so that changes in X can be spanned by existing assets, in which case the agents' discount rate will include a risk premium proportional to the correlation between the changes in the replicating portfolio and the source of systematic risk, e.g., the market portfolio in a CAPM world.

the option to invest. Intuitively, the difference $\delta_O \equiv r - \mu_O$ and $\delta_P \equiv r - \mu_P$ represent the *opportunity cost* the agent incurs by waiting to invest.[3] Therefore, since $\delta_O < \delta_P$, the optimist O, perceives a *lower* opportunity cost of delaying investment than the pessimist P. It is important to emphasize that although investors have different beliefs, they have the same information. In other terms investors agree on the nature of the information that they receive but they disagree on how to interpret it.

Agents payoff. We assume that all members of the DMG share equally in the investment from acquiring and exercising the option. Hence, once the DMG has acquired the license at time zero, the payoff upon investment at any time t for each investor is

$$\pi_t = \frac{1}{2}(X_t - I).$$

From the point of view of each individual group member, the decision to invest is a well-known stopping problem (Dixit and Pindyck, 1994). Acting individually, the optimal investment time for agent n, is given by the stopping time

$$\tau_n = \inf\left\{t \geq 0 \ : \ X_t \geq X_n^*\right\},$$

where the time-invariant threshold X_n^* is given by

$$X_n^* = \frac{m_n}{m_n - 1} I, \quad \text{with} \quad m_n = \frac{-(\mu_n - \frac{1}{2}\sigma^2) + \sqrt{(\mu_n - \frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2} > 1. \tag{1}$$

Note that, from (1), the pessimist agent has a strictly lower investment threshold than the optimist, that is $\mu_P < \mu_O$ implies

$$X_P^* < X_O^*.$$

Intuitively, this follows because the pessimist perceives a higher opportunity cost of delaying the exercise of the option to invest.

[3]For example, if X were the price of a stock, δ will represent its dividend rate, the opportunity cost of holding an option to acquire a stock, instead of the stock itself.

In general, the value at time zero to investor i of investing at a generic threshold $\widehat{X}$ is given by

$$J_n(x,\widehat{X}) = \frac{1}{2}\mathbb{E}_n\left[e^{-r\hat{\tau}}(X_{\hat{\tau}} - I)\right] = \begin{cases} \frac{1}{2}\left(\frac{x}{\widehat{X}}\right)^{m_n}(\widehat{X} - I), & \text{for } x \leq \widehat{X} \\ \frac{1}{2}(x - I), & \text{for } x > \widehat{X} \end{cases}, \tag{2}$$

where $\hat{\tau}$ is the stopping time $\hat{\tau} = \inf\{t : t \geq 0 \ : \ X_t \geq \widehat{X}\}$. Figure 2 illustrates the values $J_P(x,\widehat{X})$ and $J_O(x,\widehat{X})$ as a function of the threshold $\widehat{X}$, given a project value $x < \widehat{X}$. The value (2) to investor n is maximized at the optimal threshold X_n^* defined in (1). It easy to show that the values defined in (2) are uniformly ranked as illustrated in Figure 2:

$$J_P(x,\hat{X}) \leq J_O(x,\hat{X}), \text{ for all } \hat{X} > 0$$

with the inequality being strict for $x < \hat{X}$. Thus

$$J_P(x,X_O^*) < J_P(x,X_P^*) < J_O(x,X_P^*) < J_O(x,X_O^*), \text{ for all } x < X_O^* \tag{3}$$

At time $t = 0$ the cost of acquiring the license to invest is L. This cost is equally borne by both agents. Hence the NPV to agent n of acquiring the license and investing at the threshold $\widehat{X}$ is

$$V_n(x,\widehat{X}) = J_n(x,\widehat{X}) - \frac{L}{2}. \tag{4}$$

Equilibrium investment. Because both the decision to acquire the license at time zero and the decision to invest require a majority (i.e.unanimity in a group with two agents), we note that this sequential voting problem may exhibit inefficient investment due to the fact that different members form different *decisive coalitions* for the two elections.[4]

To see this, consider the optimists voting expectations. She understands that, if investment has not occurred prior to at any point X_O^* then the pessimist will agree to invest at any time thereafter that the optimist chooses. Although the pessimist would prefer to have invested sooner, if that has not happened he is better off supporting investment as soon as

[4]We define a decisive coalition as a subset of DMG members who have sufficient voting power to tilt the decision in their favor.

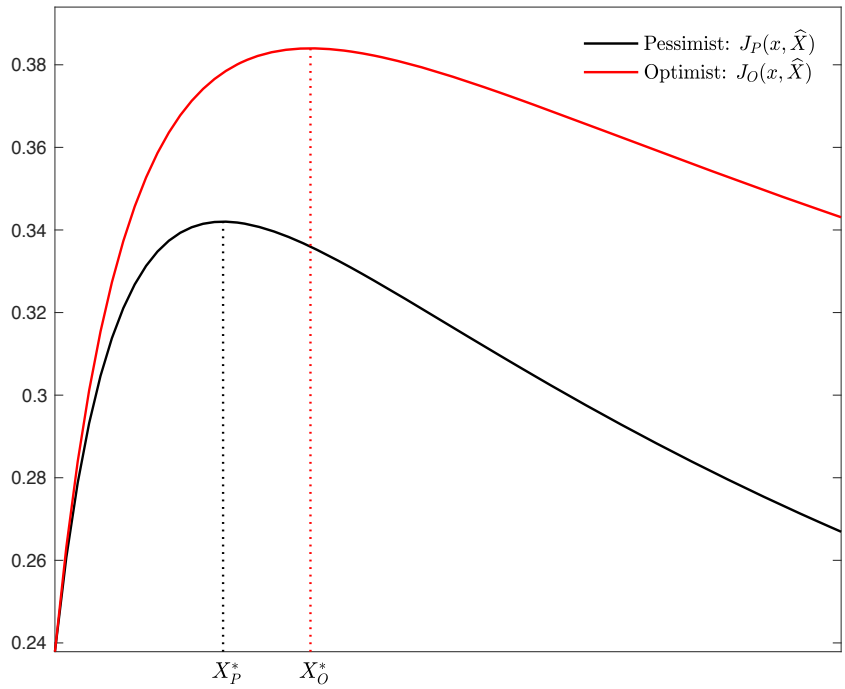

Figure 2: Value of the licensing option

The figure depicts the individual value of the licensing option to the pessimist, P, and optimist O, as a function of the exercise threshold $\widehat{X}$. Parameter values: $r = 0.05$, $\sigma = 0.3$, $I = 1$, $\mu_P = 0.01$, $\mu_O = 0.02$, $x = X_P^*/2$ where X_P^* is defined in (1)

the optimist is willing to invest. Hence, the optimist will not support investment prior to X_O^* and will propose investment and obtain majority support at $\tau_O^* = \inf\{t : X_t \geq X_O^*\}$. Thus the decisive coalition for the investment decision, denoted by $\mathcal{D}_I$, is $\mathcal{D}_I = \{P, O\}$ and the equilibrium investment policy must be $\widehat{X} = X_O^*$.

Anticipating this, the pessimist will understand that the time zero value of the subsequent exercise decision will be given by $J_P(x, X_O^*)$. From the ranking property (3), we have that

$$J_P(x, X_O^*) < J_P(x, X_P^*) < J_O(x, X_O^*) \text{ for all } x < X_O^*. \tag{5}$$

When $L/2 < J_P(x, X_O^*)$, both P and O will agree to license, and the decisive coalition for the licensing vote, denoted $\mathcal{D}_L$, is $\mathcal{D}_L = \{P, O\}$. Hence, $\mathcal{D}_I = \mathcal{D}_L$.

We can see from inequality (5), however, that there are alternative values of the initial licensing cost L satisfying $J_P(x, X_O^*) < L/2 < J_P(x, X_P^*)$. In such a case, although both

agents would agree to acquire the license if they were able to unilaterally pick the subsequent investment timing, the licensing will be rejected by the group. In fact the decision to license will be blocked by investor P resulting in a decisive coalition is $\mathcal{D}_L = \{P\}$ and we have $\mathcal{D}_L \neq \mathcal{D}_I$. We refer to this situation as *underinvestment*. Investment is blocked by an extreme (pessimistic) view. A compromise timing would facilitate investment but majority voting does not allow compromise. Any value L for which

$$J_P(x, X_O^*) < \frac{L}{2} < J_P(x, X_P^*) < J_O(x, X_O^*), \tag{6}$$

would give rise to underinvestment.

Discussion. Notice that the underinvestment that occurs under condition (6) is inefficient. Both investors would benefit from giving all the DMG decision power to investor P for the investment decision. Under the latter assumption, the DMG will invest at the threshold policy X_P^*, yielding the welfare $J_P(x, X_P^*)$ to investor P and $J_O(x, X_P^*)$ to investor O. Since

$$\frac{L}{2} < J_P(x, X_P^*) < J_O(x, X_P^*),$$

letting investor P rule the DMG for the investment decision Pareto improves the equilibrium outcome of a strict majority decision rule. To see this, note that under strict majority, underinvestment occurs and the licence is not acquired resulting to a value of $L/2$ to both P and O. Hence giving decision power to P will eliminate underinvestment and generate a payoff that is greater than $L/2$ to *both* agent, a Pareto improvement.

At a broader level, the inefficient investment that we describe in this example requires a change of the identity of the *decisive* individuals for the licensing decision and the investment decision. For the licensing decision, it is Investor P who is decisive as he wants to forgo the licence and, as a result, the DMG forgoes the license. In the off equilibrium path where the license is acquired, it is Investor P who is decisive as the DMG timing decision coincides with the preferred timing decision of Investor P. When there is no disagreement between DMG members, both investors are decisive as they are identical and, as a result, underinvestment cannot occur. Similarly, if the decision rule is dictatorial, that is, when all the voting power

is in the hand of Investor O (resp. P), the decisive individual remains Investor O (resp. P) for both the licensing and the timing decision. The change of identity of the decisive does not take place and as a result underinvestment does not occur.

Perhaps less intuitive is the fact that, as we formally prove in the next section inefficient underinvestment cannot occur with a DMG that includes *three* or, in general, an odd number of members and is governed with strict majority rule. In this case, the median belief member of the DMG must be the decisive investor for the timing decision as his preferred timing in the earlsiest investment time that can generate a majority vote. For the licensing decision two outcomes are possible. If the median member decide to licence then there is no underinvestment since the DMG will then license. In fact all investors who are more optimistic than the median belief investor will in this case vote for licensing, thereby forming a decisive coalition leading the DMG to licence. If the median belief member rejects licensing, then all investors who are more pessimistic that the median belief investor will also reject licensing, which lead the group to forgo the licence. Although the DMG does not invest, this is not an inefficient underinvestment: The median belief member rejects the license despite acting as a dictator for the timing decision. This result resembles the median voter theorem in static spatial voting models in one dimension. Our result is however different because we have a dynamic voting game. The dynamics of voting depart from the classical spatial voting framework because the licensing policy preferences change in anticipation of the future investment vote.

Finally observe that the occurrence of inefficient investment characterized by inequality (6) will be more frequent and hence more empirically relevant when the gap in the beliefs of the two pivotal members is larger. This suggest that the underinvestment inefficiency will be more important empirically for DMGs with small numbers of members and when beliefs are polarized. This statement will be made quantitatively more precise in the next section.

3 A general model of investment decision by groups

3.1 The framework

Consider a filtered probability space $(\Omega, (\mathcal{F}_t)_{t\in\mathbb{R}}, \mathbb{F}, \mathbb{P}_1)$ where $\mathbb{F}$ is the natural filtration of a Brownian motion $W_{1,t}$. We denote by $\mathbb{E}_1$ the expectation operator under $\mathbb{P}_1$. A group $\mathcal{N} = \{1, \ldots, n, \ldots, N\}$ of N infinitively lived investors, a *decision making group* or DMG, collectively decides sequentially whether and when to invest in a project.

Technology and information. Investment is irreversible and the value of the project X follows a geometric Brownian motion. Investors agree on the volatility of the value process but disagree on its expected growth. Specifically, investor 1 perceives the value process as

$$dX_t = \mu_1 X_t + \sigma X_t dW_{1,t}, \quad X_0 = x > 0$$

where $\mu_1 \geq 0$ and $\sigma > 0$. Investor 1's belief is captured by the probability measure $\mathbb{P}_1$ under which the value process is a geometric Brownian motion with drift μ_1.

Investors $n = 2, \ldots, N$ disagree with investor 1 on the value of the drift of the value process. Their perceived drift is μ_n, where, without loss of generality, we assume that

$$\mu_1 \leq \mu_2 \leq \ldots \leq \mu_N, \tag{7}$$

that is, investors are ranked based on how optimistic they are with regard to the expected growth of the value of the process X_t. Investor's n belief, for each $n = 2, \ldots, N$, is given by a probability measure $\mathbb{P}_n$, equivalent to $\mathbb{P}_1$ under which the value process satisfies[5]

$$dX_t = \mu_n X_t + \sigma X_t dW_{n,t}, \quad X_0 = x > 0$$

[5]By the Girsanov Theorem (Karatzas and Shreve, 2012), the probability $\mathbb{P}_n$ is formally defined as $\left(\frac{d\mathbb{P}_n}{d\mathbb{P}_1}\right)_{\mathcal{F}_t} = \xi_{n,t}$, where he process $\xi_{n,t}$ is the non-negative $(\mathbb{P}_1, \mathbb{F})$-martingale defined by $d\xi_{n,t} = -\eta_n \xi_{n,t} dW_{1,t}, \;\; \xi_{n,0} = 1$, with $\eta_n = \frac{\mu_n - \mu_1}{\sigma}$.

where W^n is a standard Brownian motion under the probability $\mathbb{P}_n$ and given by

$$W_{n,t} = W_{1,t} - \eta_n \, t,$$

where $\eta_n = \frac{\mu_n - \mu_1}{\sigma}$.

It is important to emphasize that although investors have different beliefs, they have the same information. This is mathematically captured by the fact that the Brownian filtration generated by the Brownian motion $W_{n,t}$ for each $n = 1, \ldots, N$ is identical to the natural filtration of the commonly observed process X_t. In other terms investors agree on the nature of the information that they receive but they disagree on how to interpret it. Furthermore, note that disagreement about the drift is only possible if $\sigma > 0$. If $\sigma = 0$ then $X_t = xe^{\mu t}$ and therefore all agents can learn the drift μ by observing X_t and x.

Investors' preferences, payoffs and strategies . Investors are risk neutral and discount future cash flows at the risk free rate r. We assume that

Assumption 1. *Investors are sufficiently impatient, that is,*

$$r > \mu_N.$$

Given condition (7), this assumption implies that for each investor, there are states of the world where it is optimal to exercise the investment option. Without this assumption, the patient investors will always oppose exercise of the investment option.

The DMG make two consecutive decisions. The first decision is the *licensing* decision, taken at time 0. This decision consists of two alternatives. The first alternative (license) is to collectively pay a cost L to acquire exclusive ownership of the right to exercise the investment opportunity at a future date. The second alternative is to reject the acquisition of the licence.

If the members of the DMG agree to licence, at any time in the future, any DMG member can propose to the group the opportunity to collectively pay an investment cost I that grants

access to a completed project generating an individual lump sum payoff

$$\frac{1}{N}(X_t - I)$$

to each group member. If no proposal is offered at a given date or if a proposal is rejected by the DMG, the option remains unexercised and available for exercise in future dates.[6]

We assume that the investment strategies investor follow to propose an investment decision to the rest of the group are of the threshold type, that is,

Assumption 2. *Investors submit a proposal to the group when the observed value of the project is larger than a threshold value* $\widehat{X} \in [0, \infty]$. *The proposal date is the stopping time* $\tau_{\widehat{X}}$ *at which the threshold* $\widehat{X}$ *is first reached, that is,*

$$\tau_{\widehat{X}} = \inf\{t : X_t \geq \widehat{X}\}.$$

This class of timing decisions is standard in the real option literature and natural, given the stationary structure of our game.This formulation greatly simplifies our equilibrium voting outcome and allows us to discuss our economic insight in a transparent way.[7]

If a proposal $\widehat{X} > 0$ is accepted by the DMG the value to investor $n = 1, \ldots, N$ is given by

$$J_n(x, \widehat{X}) = \frac{1}{N}\mathbb{E}_n\left[e^{-r\hat{\tau}}(X_{\hat{\tau}} - I)\right] = \begin{cases} \frac{1}{N}\left(\frac{x}{\widehat{X}}\right)^{m_n}(\widehat{X} - I), & \text{for } x \leq \widehat{X} \\ \frac{1}{N}(x - I), & \text{for } x > \widehat{X} \end{cases}, \tag{8}$$

[6]Note that it is possible to engage in "take-it-or-leave it options.", that is, an agent that propose a vote to invest, could potentially threat the group to withdraw his support in the future if the proposal does not go through. Those threat will however not be credible because this will amount to sub-optimally give up an option to invest at future dates.

[7]Note that under Assumption 2, it is possible to express the second stage DMG decision as a canonical *spatial voting model* in one dimension (see Black (1948, 1958) and Miller (2014)) with single-peaked preferences.

with m_n defined in equation (1).[8] It can be shown that the values J_n are ordered, that is,

$$J_n(x, \widehat{X}) \leq J_{n+1}(x, \widehat{X}) \text{ for all } \widehat{X} \in (0, \infty), \; n = 1, .., N-1, \tag{9}$$

and therefore their graph do not intersect, as illustrated in Figure 2. It is also well known that the value J_n is single-peaked as described in Figure 2 and maximized at

$$J_n(x, X_n^*) = \sup_{\widehat{X}} J_n(x, \widehat{X})$$

where X_n^* is the bliss point for investor n defined in equation (1) and satisfying

$$X_1^* < \ldots < X_N^*. \tag{10}$$

In our baseline analysis we assume that at the current level of the project value x, no group member finds it optimal to propose an immediate exercise of the investment option. This amounts to insuring that each agent has faces a non-trivial investment decision.

Assumption 3. *We assume that at the current level $X_0 = x$ of the project value the investment option is out of the money for all investors, that is,*

$$x < X_1^*.$$

Lastly, we assume that investors' preferences and the DMG decision rules (described below) are common knowledge and that investors anticipate the future investment decisions

[8] Notice that after some straightforward calculations, and using Doob's optional sampling theorem (see, e.g., Karatzas and Shreve (2012), Theorem 3.22, p. 19) it can be shown that the payoff to investor n can also be represented under the probability $\mathbb{E}_1$ as

$$J_n(x, \widehat{X}) = \mathbb{E}_1 \left[\frac{1}{2} e^{-\hat{r}\tau_{\widehat{X}}} (X_{\tau_{\widehat{X}}} - I) \left(\frac{X_{\tau_{\widehat{X}}}}{x} \right)^{\frac{-\eta_n}{\sigma}} \right],$$

where

$$\hat{r} = r - \frac{\eta_n}{2} \left(-\eta_n - \sigma + 2\frac{\mu_n}{\sigma} \right).$$

Therefore, the collective decision of a group of investors with heterogeneous beliefs identical discount rate and identical payoff can be analyzed as a group decision of investors who hold identical beliefs but heterogeneous discount rate and heterogeneous payoff.

of the DMG when expressing their preferences to the group at the licensing stage. This implies that, if the proposal $\widehat{X}$ is adopted by the group, the value of the licensing option at time zero is given by

$$V_n(x, \widehat{X}) = J_n(x, \widehat{X}) - \frac{L}{N}. \tag{11}$$

At time zero each DMG member n decides whether to support the acquisition of a license, if $V_n(x, \widehat{X}) > 0$ or reject it, otherwise.

3.2 Decisions rules, equilibrium and inefficient underinvestment

The DMG makes decisions with simple majority voting in both the licensing and investment stage. Formally,

Assumption 4. *The DMG make decisions based on a simple majority rule, that is, a decision is undertaken by the group if it is accepted by at least k members, with k the smallest integer larger than $N/2$.*

This assumption is motivated by the fact that majority voting is a pervasive collective decision process in small committees of board of directors and venture capital syndicates.

An equilibrium outcome for the DMG consists of a pair $s = (\lambda, \widehat{X}) \in \{0, 1\} \times [0, \infty]$, where $\lambda = 1$ ($\lambda = 0$) denotes that the DMG acquires (reject) the licensing option, and the variable $\widehat{X}$ denotes the threshold at which the DMG undertakes the investment. Equivalently, investment takes place at the stopping time $\tau_{\widehat{X}}$. We adopt the convention that $\widehat{X} = \infty$ means that the DMG never invest. The strategy of declining the acquisition of the license is denoted by $s_{\mathcal{N}} \equiv (0, \infty)$. We denote by $\mathcal{S} \equiv \{0, 1\} \times [0, \infty]$ the set of possible strategies.

In order to state the equilibrium conditions, it is convenient to define the set of *feasible* thresholds $\mathcal{X}_m$, as the set of threshold $\widehat{X}$ such that a proposal to invest at time $\tau_{\widehat{X}}$ generates a majority vote.

The following definition describes the equilibrium in our dynamic voting game:

Definition 1. *The strategy $s = (1, \widehat{X})$ is an equilibrium outcome of the dynamic voting game if*

1. *The threshold* $\widehat{X}$ *is the smallest feasible threshold:* $\widehat{X} = \inf_X \{X : X \in \mathcal{X}_m\}$.

2. *Conditional on* $\widehat{X} \in \mathcal{X}_m$ *being implemented in the second stage vote, the licensing decision* $\lambda = 1$ *generates a majority vote of the DMG against* $s_{\mathcal{N}}$.

The strategy $s = s_{\mathcal{N}} \equiv (0, \infty)$ *is an equilibrium outcome of the dynamic voting game if none of the strategies* $(1, \widehat{X})$ *with* $\widehat{X} \in \mathcal{X}_m$ *generates majority support of the DMG in an election against* $s_{\mathcal{N}}$.

As stated in our definition of the equilibrium we assume that the equilibrium investment strategy is in the set of feasible strategies $\mathcal{X}_m$. In this complete information voting game, we therefore assume that no proposal is made unless it is expected to generate a majority vote. We additionally assume that the equilibrium investment threshold is the smallest element of the set of feasible strategies. This means that whoever moves first with a proposal that can generates majority will determine the investing policy of the DMG.[9]

At the licensing stage, investors internalize their expectation on the investment policy in their decision of how to vote. Although the voting game is a complete information game, for strategic motives, investors may not vote sincerely in the licensing vote. They could vote against the alternative they would favour if they were acting individually.

3.3 Analysis of the voting game

We now solve for the equilibrium of the dynamic voting game and the resulting behaviour of the DMG. To state our results, it is convenient to identify two particular group members: the *left pivot*, n_L, and the *right pivot*, n_R defined as follows.

[9]The investment voting game could also be modeled as a static game where investors vote over alternative threshold policies at time $t = 0$ as opposed to making proposals to invest at time $t > 0$. Since the set of alternative is one dimensional (indexed by the thresholds $\widehat{X}$) and the individual preferences are single peaked, we can apply the result of existence of equilibria from the canonical spatial voting model in political economy (see Black (1948, 1958) and Miller (2014)). This static interpretation of the investment voting game would give rise to multiple equilibria but our dynamic interpretation allows to select only one of them because whoever moves first in time with a proposal determine the equilibrium.

Definition 2. *When the number of group members N is odd, the left pivot and the right pivot are the same individual*

$$n_L = n_R = \frac{N+1}{2}$$

When the number of group members is even, the left and right pivot are defined by

$$n_L = \frac{N}{2}, \quad n_R = \frac{N}{2} + 1.$$

The next lemma characterizes the set of feasible investment thresholds

Lemma 1. *The set of feasible investment is given by*

$$\mathcal{X}_m = [X^*_{n_R}, \infty).$$

Proof: Because investment occurs only if a strict majority of members supports it, when n_R propose to invest, all agents $n < n_R$ will support the investment proposal because their optimal investment threshold is smaller than that of agent n_R. ∎

The lemma implies that the bliss point threshold for the right pivot n_R, given by $X^*_{n_R}$ is the first threshold that generates a majority vote. The *decisive coalition* that is, the coalition that generate the outcome of the vote when the threshold $\widehat{X} = X^*_{n_R}$ is proposed is given by $\{1, .., n_R\}$. The lemma also implies that a candidate equilibrium should be of the form $s = (1, X^*_{n_R})$ or of the form $s = s_{\mathcal{N}}$. The next proposition characterize the equilibrium outcome of the dynamic voting game.

Proposition 1. *The equilibrium outcome of the voting game is unique and consists of*

1. *Not licensing, i.e., $s = s_{\mathcal{N}}$, if*

$$J_{n_L}(x, X^*_{n_R}) < \frac{L}{N} \tag{12}$$

2. *Licensing and invest at the threshold $X^*_{n_R}$, i.e., $s = (1, X^*_{n_R})$, when*

$$J_{n_L}(x, X^*_{n_R}) > \frac{L}{N}.$$

The proposition implies that the preference of the left pivot is binding for the decision to license. When the equilibrium outcome is to not license, the decisive coalition for the licensing decision is the set $\{1, \ldots, n_L\}$. When the equilibrium outcome is to license, the decisive coalition is $\{n_L, n_R, \ldots, N\}$. The proposition also shows that the equilibrium outcome for a DMG with N investors is identical to the equilibrium outcome for a DMG formed with only members n_L and n_R.

3.4 Underinvestment

In this section, we show that the voting equilibrium may generate an inefficient underinvestment. Formally,

Definition 3. *Inefficient underinvestment occurs when the equilibrium outcome is not to license ($s_{\mathcal{N}}$) and yet all investors prefer to invest if they were acting individually, that is,*

$$\frac{L}{N} < J_n(x, X_n^*) \quad \textit{for } n = 1, \ldots, N. \tag{13}$$

The decision to not license is inefficient under condition (13). If the investors could commit to the threshold policy $\widehat{X} = X_1^*$, licensing would generate a unanimous vote. The inequality (13) and condition (9) imply

$$0 < J_1(x, X_1^*) - \frac{L}{N} < J_2(x, X_1^*) - \frac{L}{N} < \ldots < J_N(x, X_1^*) - \frac{L}{N}.$$

Therefore, each investor get a positive payoff from licensing and they will all vote for licensing provided they can commit to the threshold X_1^*.

Notice that inefficient underinvestment cannot occur if the all the voting power is attributed to a single investor (a *dictator*) for both elections. In that case, the group will follow the dictator's preferred investment timing policy and condition (13) implies that the dictator will accept the licensing. Thus inefficient underinvestment cannot occur when the DMG is ruled by a self interested dictator.

Similarly, inefficient underinvestment cannot occur with a utilitarian planner who perceive a value process following a geometric Brownian motion with the average drift

$$\mu_{\text{pool}} = \frac{\sum_{n=1}^{N} \lambda_n \mu_n}{N}$$

with weights satisfying $\lambda_n \geq 0$ and

$$\sum_{n=1}^{N} \lambda_n = 1.$$

Such a planner will adopt the investment threshold X^*_{pool} defined by equation (1) with μ_{pool} replacing μ_n. Because $\mu_1 \leq \mu_{\text{pool}}$, we must have

$$J_1(x, X_1^*) \leq J_{\text{pool}}(x, X^*_{\text{pool}}).$$

This inequality implies that if all individual investors would license if they were the sole owner of the license, the planner also decides to license. Hence underinvestment does not occur when such a utilitarian planner is in charge of the DMG decisions at both round. This result is to be contrasted with the result in Garlappi, Giammarino, and Lazrak (2017) where the DMG is using a utilitarian criteria to make decisions.

The next proposition provides a characterization of inefficient underinvestment.

Proposition 2. *If the number of group member is odd, inefficient underinvestment cannot occur in equilibrium. If the number of group members is even, inefficient underinvestment occurs when*

$$J_{n_L}(x, X^*_{n_R}) < \frac{L}{N} < J_1(x, X_1^*). \tag{14}$$

The right inequality of (14) is in fact equivalent to (13).[10] Since the DMG will adopt the investment policy $X^*_{n_R}$ if the license is acquired, the left inequality of (14) shows that the left pivot n_L rejects the licensing. As a result, all investors who are more pessimistic

[10] To see this recall that condition (9) implies

$$J_1(x, X_1^*) < J_2(x, X_1^*) < J_2(x, X_2^*) < J_3(x, X_2^*) < J_3(x, X_3^*) < \ldots < J_N(x, X_N^*).$$

Therefore, $J_1(x, X_1^*) = \min_n J_n(x, X_n^*)$ and the equivalence of the right inequality in (14) with condition (13) obtains.

than the left pivot, that is investors $\{1, \ldots, n_R - 1\}$, will also reject the licensing. Therefore, condition (14) implies that inefficient underinvestment as described in Definition 3 occurs.

Underinvestment takes place because the investors $\{1, \ldots, n_L\}$ dislike the threshold policy $X^*_{n_R}$. Inefficient underinvestment is an implication of strategic voting in the licensing decision: investors anticipate the investment threshold taken by the DMG and when some of them dislike the anticipated threshold, they vote against licensing. When the group of investors who dislike the anticipated threshold form half of the group, they preclude those who favour the licensing decision , that is the investors $\{n_R, \ldots, N\}$, to form a majority leading the DMG to reject licensing.

Proposition 2 also shows that the underinvestment problem we highlight does not occur with an odd number of voters. The reason is that the left pivot coincides then with the right pivot giving rise to a single median voter $n_m = n_R = n_L$. If the license is acquired, the DMG follows the median voter investment threshold $X^*_{n_m}$. Even though many pessimistic investors may dislike the anticipated investment strategy $X^*_{n_m}$, they will not be able to block licensing since those in favor of licensing, the investors $\{n_m, \ldots, N\}$, form a majority.

Another useful intuition to understand the impossibility of inefficient underinvestment with an odd number of investors is to realize that the median voter n_m acts as a dictator of the DMG. This observation is reminiscent of the median voter theorem from the canonical spatial voting models. In fact, for both elections, if an alternative is taken by the DMG it must be that the median voter adhere to that alternative: With ranked single peaked preferences and an odd number of voters, any majority should include the median voter. Therefore, the DMG choice will not change if the median voter was the dictator of the DMG. We know however that inefficient underinvestment cannot take place if the DMG is ruled by a dictator. The reason is that if all group members would invest if they act individually, the dictator will not reject licensing because the dictator is by definition acting individually.

4 Quantitative implications

4.1 Impact of polarization

Notice that in the underinvestment condition (14) only three investors matter: the two pivots, n_L and n_L, and investor 1. In particular, the equilibrium outcome would not change if the optimistic investors become even more optimistic. This result is due to the fact that voting does not allow to express the intensity of preferences. With a majority vote where each person can only cast one vote, the DMG ignores the intensity of preferences over alternative.

Figure 3 describes the underinvestment conditions for a group of $N = 2$ agent. Note that in this case, the left pivot is agent P (pessimist) and the right pivot is agent O (optimist). It follows that for a two-agent group, condition (14) will always be satisfied as long as the two agents have different beliefs about the growth rate of X_t. In the Figure, the value L_{max} represent the highest possible value of the licensing cost for which underinvestment occurs. The vertical segment highlighted in red at the threshold X_O^* represents the value of licensing costs L for which underinvestment occurs. For values of $L/2 > L_{max} = J_P(x, X_P^*)$, agent P will not agree to license even at his own preferred strategy X_P^*. For value of $L/2 < L_{min} = J_P(x, X_O^*)$, agent P will agree to license even if the investment policy is perceived as suboptimal. In both cases we will not have underinvestment.

Figure 4 describes the underinvestment conditions for a group of $N = 4$ agent. Agent 2 and 3 are, respectively, the right and left pivot. The vertical segment $[L_{min}, L_{max}]$ at the level X_3^* represents the values of licensing costs that satisfy the underinvestment condition (14). From the figure we can infer that as the right and left pivot assessment of the drift of X_t diverge, the underinvestment region increases. In fact, as the right pivot becomes more pessimistic, the value $J_2(x, \widehat{X})$ moves towards $J_P(x, \widehat{X})$ and as the left pivot becomes more optimistic, its value $J_3(x, \widehat{X})$ moves towards $J_O(x, \widehat{X})$. In both cases we see that the segment of licensing costs L increases thus suggesting a larger underinvestment region. The opposite is true if the beliefs of the right and left pivot converge to each other. In this case, underinvestment disappears as soon as $J_2(x, X_3^*) > L_{max}$.

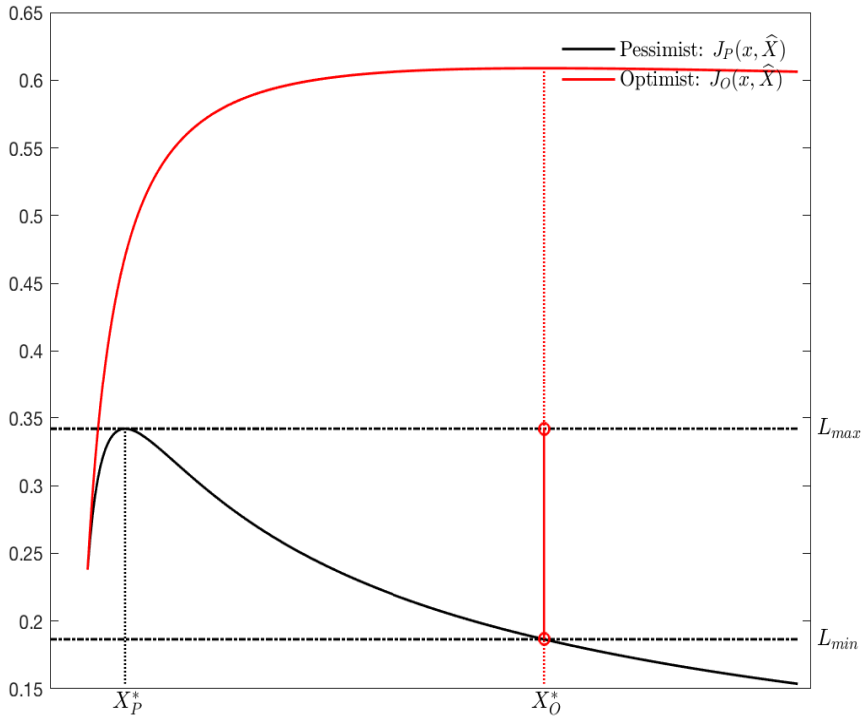

Figure 3: Belief polarization and underinvestment in a two-agent group.
The figure reports the value at time zero of the licensing option of both agents as a function of the investment threshold $\widehat{X}$. The vertical segment $[L_{min}, L_{max}]$ shows the range of licensing costs $L/2$ for which underinvestment occurs. Parameter values: $r = 0.05$, $\sigma = 0.3$, $I = 1$, $\mu_P = 0.01$, $\mu_O = 0.045$, $x = X_P^*/2$ where X_P^* is defined in (1).

The figures also makes it clear that polarization—that is, the divergence between agent 2 and agent 3's beliefs about the expected growth of the project value—is a necessary requirement for underinvestment to occur. To see this, suppose that agent 2's belief μ_2 increases towards μ_3. As μ_2 increases, the distance between L_{min} and L_{max} in the figure shrinks, implying that there is a smaller region of licensing cost L for which condition (14) is satisfied. Similarly, as μ_3 decreases towards μ_2, X_3^* decreases towards X_2^* and the interval $[L_{min}, L_{max}]$ shrinks until underinvestment disappears.

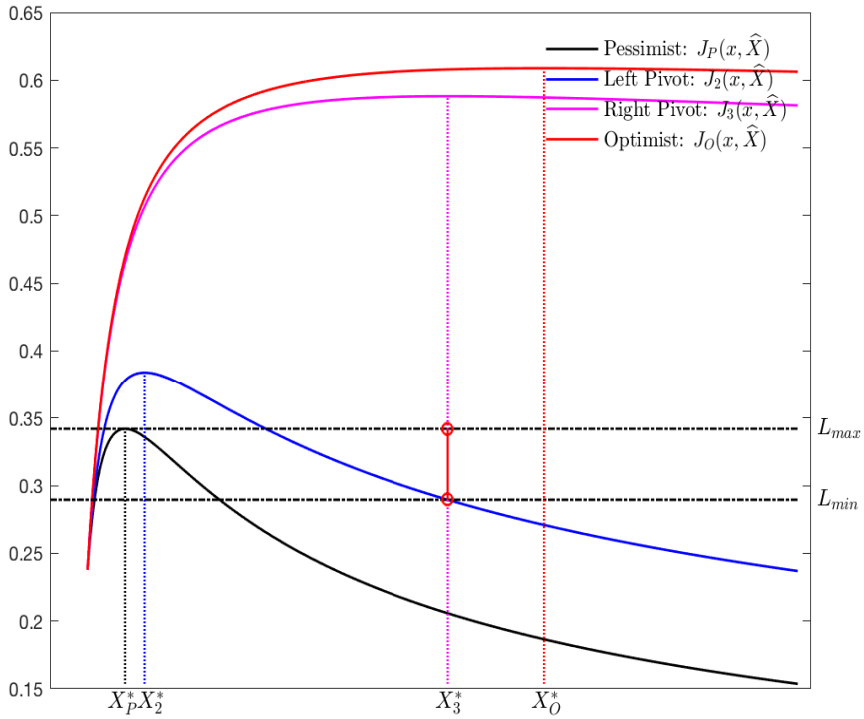

Figure 4: Belief polarization and underinvestment in a four-agent group.

The figure reports the value at time zero of the licensing option of both agents as a function of the investment threshold $\widehat{X}$. The vertical segment $[L_{min}, L_{max}]$ shows the range of licensing costs $L/4$ for which underinvestment occurs. Parameter values: $r = 0.05$, $\sigma = 0.3$, $I = 1$, $\mu_P = 0.01$, $\mu_2 = 0.02$, $\mu_3 = 0.0437$, $\mu_O = 0.045$, $x = X_P^*/2$ where X_P^* is defined in (1).

4.2 Project uncertainty and underinvestment

The licensing and investment decisions analyzed in the previous sections are the workhorse framework of the real option approach to project valuations. A common and well understood insight from that literature is that volatility makes the licensing option more valuable and consequently delays the exercise of the investment option. In this section we investigate the effect of volatility on inefficient underinvestment. The main conclusion is that if polarization is sufficiently strong, then inefficient underinvestment is always possible, independent of the level of volatility. This result is important in that it highlights a unique channel through which underinvestment can occur, that is, the polarization of beliefs.

As in the standard real option setting, high volatility will increase the licensing option value and delay the investment threshold for each player. However, when beliefs are sufficiently polarized, inefficient underinvestment can occur at any level of volatility. While it is true that high volatility leads to a delay of investment, it is important to emphasize that delaying investment is not the same as "inefficient underinvestment".

We illustrate the effect of uncertainty on underinvestment in the context of a four-agent groups analyzed in the previous sections. Figure 5 reports, for different values of the volatility parameter σ the highest possible value of the belief μ_2 for which underinvestment occurs. Formally, the bound $\overline{\mu}_2$ is defined as

$$\overline{\mu}_2 = \sup\{\mu_2 \in [\mu_1, \mu_3] : J_2(X_3^*|\mu_2) \leq J_1(X_1^*)\}.$$

The figures reports the bound for different level of agent 3's belief, μ_3. As the figure shows, for every level of volatility σ there exists a sufficiently lower level of μ_2 for which underinvestment occurs. Given that μ_3 is fixed, a low value of μ_2 implies more disagreement between the two pivots. As volatility increases, the bound $\overline{\mu}_2$ decreases. This means that, when volatitliy is high, more polarization is required for underinvestment to occur. Intuitively, this happens because, all else being equal, high volatiltiy mitigates underinvestment in that it makes the licensing options more valuable, and hence investment more attractive. For underinvestment to occur in this scenario the disagreement betweeen pivots has to be more severe.

We illustrate this point in Figure 6, where we report the region of beliefs (μ_2, μ_3) for which underinvestment occurs, under two separate volatiltiy scenario. The 45-degreee line represents the case of no disagreement. In this case underinvestment will never occur. The blue (red) region represents the set of beliefs (μ_2, μ_3) for which underinvestment occurs in a low (high) volatility scenario. Consistent with the discussion above, underinvestment in a high volatiltiy scenario requires a more polarized set of beliefs. Note finally that, the case of a two-agent group will always exhibit underivestment because the condition $J_1(X_2^*) \leq J_1(X_1^*)$ for any level of volatity. In a four-agent group this will be equivalent to the case of of extreme polarization $\mu_1 = \mu_2$ and $\mu_3 = \mu_4$ or unanimity.

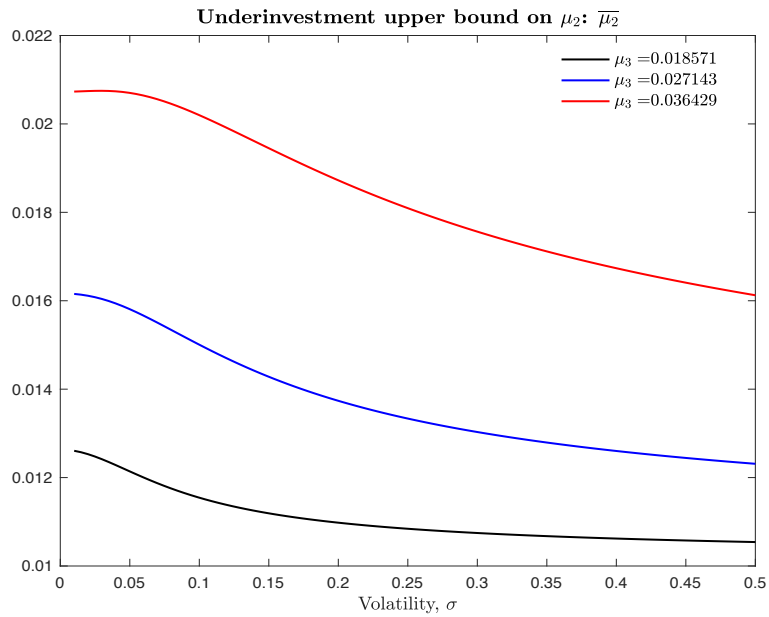

Figure 5: Uncertainty and underinvestment in a two-agent group.

The figure reports, as a function of volatitliy σ, and for three given level of belief μ_3, the upper bound of Agent 2's belief, μ_2 for which underinvestment occurs. Parameter values: $r = 0.05$, $I = 1$, $\mu_P = 0.01$, $\mu_O = 0.045$, $x = X_P^*/1.1$ where X_P^* is defined in (1).

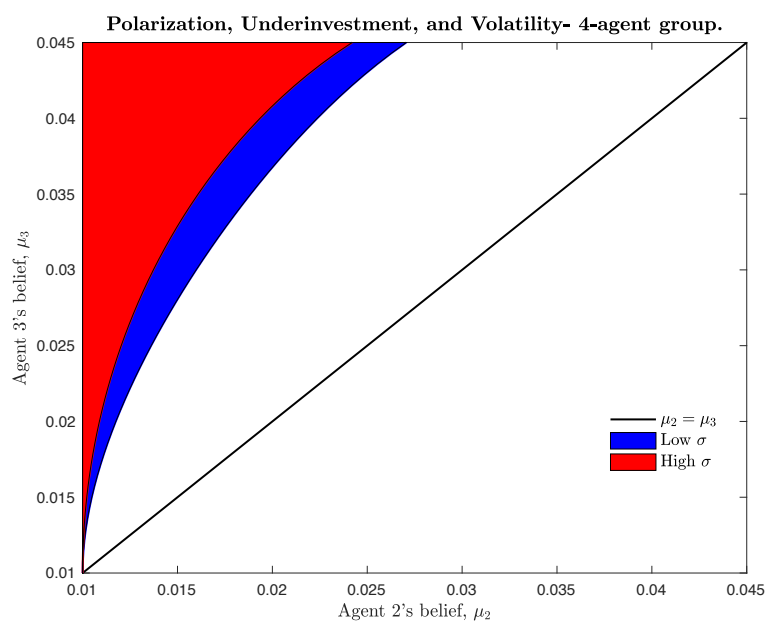

Figure 6: Uncertainty and underinvestment in a four-agent group.

The figure reports the region of beliefs (μ_2, μ_3) for which underinvestment occurs. Parameter values: $r = 0.05$, $I = 1$, $\mu_P = 0.01$, $\mu_O = 0.045$, $x = X_P^*/1.1$ where X_P^* is defined in (1).

5 Other governance rules

In this section, we analyze different governance rules within a DMG. In Section 5.1 we consider groups in which agent have veto power and groups who mediate disagreement using supermajority rules. In Section 5.2 we consider the possibility of bargaining between disagreeing agents within the group.

5.1 More general voting protocols

As noted earlier, there cannot be inefficient underinvestment if the group delegates all the decision power to a single agent—a "dictatorial" governing rule. In contrast, unanimity always leads to inefficient underinvestment, as illustrated in the two-agent group example of Section 2. Dictatorship and unanimity are polar cases of voting protocols within the class of *non-collegial* governance rules, that is, rules in which some members of the DMG have veto power, *the vetoers*. Majority and supermajority are voting protocols within the class of *collegial*, that is rules in which no group member has veto power. In the next two subsections we explore whether the inefficient underinvestment problem can be mitigated by changing the voting rule. The main result from this analysis is that the inefficiency will either be unaffected or worsened by the alternative voting procedures that we explore.

5.1.1 Voting rules

Given a group of $\mathcal{N} = \{1, \ldots, N\}$, a voting rule is defined as a set of *decisive coalitions* $\mathcal{D} \subseteq 2^N/\{\varnothing\}$ (Austen-Smith and Banks, 1999). If a proposal is made and a decisive coalition $\mathcal{C} \in \mathcal{D}$ votes to accept (reject) it, then the DMG accepts (rejects) the proposal. Therefore, defining the voting rules is equivalent to fixing the set $\mathcal{D}$ characterizing the decisive coalition.

The next definition formally characterizes a majority governance rule in which some some group member have veto power.

Definition 4. *A governance rule is called majority with vetoers if and only of there is a set of investors* $\mathcal{V} = \{v_1, v_2, \ldots, v_m\}$ *with* $m \leq N$ *and* $v_1 < v_2 < \ldots < v_m$ *such that*

$$\mathcal{D} = \left\{ \mathcal{C} \subseteq \mathcal{N} :\mid \mathcal{C} \mid > \frac{N}{2} \text{ and } \mathcal{V} \subseteq \mathcal{C} \right\}$$

Majority rule with a single vetoer captures the idea that the DMG gives special veto power to a single member, such as the chair of the board of directors or the founder of the corporation. Unanimity is a polar example of such rule, where every agent has veto power, that is $\mathcal{V} = \mathcal{N}$ and hence the only decisive coalition is $\mathcal{D} = \{\mathcal{N}\}$.

A DMG ruled by a dictator $n_d \in \mathcal{N}$ is defined by the set

$$\mathcal{D} = \{\mathcal{C} \subseteq \mathcal{N} : n_d \in C\}$$

The next definition formally characterizes the supermajority or quota non-collegial rule.

Definition 5. *A governance rule is called supermajority or quota if there exist* q *with* $\frac{N}{2}+1 < q < N$ *such that*

$$\mathcal{D} = \{C \subseteq \mathcal{N} :\mid \mathcal{C} \mid \geq q\}.$$

5.1.2 Majority rule with vetoers

We assume in this section that DMG makes decisions based on a majority rule with vetoers. The following proposition characterizes the equilibrium outcome relative to the equilibrium outcome in the absence of vetoers.

Proposition 3. *Assume that the DMG utilizes the majority with vetoers governance rule and that the number of investors in the DMG is even.*

If the vetoer v_m *is not more optimistic than the right pivot,* $v_m \leq n_R$*, then the equilibrium outcome is not altered and inefficient underinvestment occurs when condition (14) is satisfied.*

If the vetoer v_m *is more optimistic than the right pivot,* $v_m > n_R$*, inefficient underinvestment occurs when*

$$J_{n_L}(x, X^*_{v_m}) < \frac{L}{N} < J_1(x, X^*_1). \tag{15}$$

In presence of vetoers the first investment proposal that will be accepted by the group must include a majority of investors *and* all vetoers. Therefore the investment threshold that the DMG applies should be either the right pivot n_R and/or the vetoer v_m. If the vetoer is (weakly) less optimistic than the right pivot, then the DMG will apply the threshold policy $X^*_{n_R}$. The group will not licence under condition (14) in the exact same way as it occurs in the absence of vetoers: half of the investors will block the licensing. If the vetoer is more optimistic than the right pivot then the group will delay the investment (relative to a DMG with no vetoer) and apply the threshold $X^*_{v_m}$. Similar to the no vetoer case, the underinvestment condition is is given by (15).

It is important to observe than condition (15) is less stringent than condition (14). This is due to the fact that $X^*_{n_R} < X^*_{v_m}$ and hence $J_{n_L}(x, X^*_{v_m}) < J_{n_L}(x, X^*_{n_R})$. This can be seem more easily in Figure 4. Assuming a majority rule with the vetoer $\mathcal{V} = O$, the DMG will then apply the threshold policy X^*_O. The new level of minimal licensing cost is then at the intersection of the dashed red line on the vertical of X^*_O and the blue curve describing $J_{n_L} = J_2$. We see then that introducing the vetoer O to a majority governance rule has the effect of decreasing the lower bound L_{min} thereby increasing the size of the interval of licensing costs that give rise to inefficient underinvestment. In this sense, introducing a veto rule to a simple majority rule can worsen the inefficient underinvestment problem.

The above results also implies that the presence of veto power can generate underinvestment also in a group with an odd number of agents. To see this, consider, for example, the case of a three-agent group. According to our analysis above, in the absence of veto power, the group will behave according to the preference of the median group member, agent 2. If however veto powers rests in the hand of agent 3, then investment will occur at the threshold X^*_3 and therefore the underinvestment condition is $J_2(x, X^*_3) < J_1(x, X^*_1)$. In general underinvestment will occur in a odd-numbered group if veto power is in the hands of agents that are more optimistic than the median agent.

5.1.3 Non collegial rules: supermajority

Proposition 4. *Assume the DMG follows a supermajority rule with q being an integer satisfying $N/2+1 < q < N$. Inefficient underinvestment happens when*

$$J_{N-q+1}(x, X_q^*) < \frac{L}{N} < J_1(x, X_1^*). \tag{16}$$

With a supermajority rule, underinvestment will occur independently of whether the number of group members is odd or even. This is because the investment policy adopted by the DMG is X_q^*. For the supermajority rule, the left pivot becomes the member $n_R = N-q+1$ and the right pivot is the member $n_L = q$. Supermajority creates a natural wedge between the left and the right pivot without requiring an even number of investors. Notice that condition (16) is naturally less stringent than (14) because $X_q^* > X_{n_R}^*$ and $J_{N-q+1}(x, X_q^*) < J_{n_L}(x, X_q^*) < J_{n_L}(x, X_{n_R}^*)$. We thus conclude that the inefficient underinvestment problem is worsened under a supermajority rule.

5.2 Bargaining in presence of disagreement

So far in the paper, we assumed that the DMG voting outcome is a tie as can be the case in a DMG with an even number of investors, the DMG does not license or does not invest. This implies that not licensing and not investing are the (exogenous) the status quo for our voting game.

In this section, we consider without loss of generality the case of a two-agent DMG in which disagreeing investors engage in bargaining. Specifically,

Assumption 5. *When the two DMG members disagree on the investment threshold a bargaining game ensues in which agent O succeeds with probability ρ.*

The above assumption capture the implications of bargaining without modelling the details of the negotiations. The assumption states that the outcome desired by the optimistic voter is obtained with an exogenous independent probability ρ. The parameter ρ represents

the bargaining power of the most optimistic agent who desires a late investment policy. When $\rho = 1$, we obtain the case described in the illustrative example of Section 2.

The following proposition shows that, as long as the parameter $\rho \neq 0$, underinvestment can occur.

Proposition 5. *Assume $x < X_O^*$ and that Assumption 5 holds with $\rho \neq 0$. If*

$$(1-\rho)J_P(x, X_P^*) + \rho J_P(x, X_O^*) < \frac{L}{2} < J_P(x, X_P^*)$$

then inefficient under-investment occurs with probability $1 - \rho$.

When $\rho = 1$ the underinvestment is most severe, as can be seen from Figure 3. As $\rho \to 0$, agent P becomes the dictator and therefore no underinvestment will be possible. Although the underinvestment area is reduced, underinvestment problem is still an issue in this setting.

6 Conclusion

In this paper, we have examined coordination frictions and consequent investment inefficiencies that arise when a group instead of an individual makes corporate decisions. More specifically, we examine the acquisition and subsequent management of a real option by a DMG whose members have heterogeneous beliefs. We show that in such a setting the DMG may reject an investment opportunity on behalf of the corporation even though each member of the DMG sees the opportunity as valuable. This happens when views of group member are polarized. In the dynamic setting we analyze, inefficiencies arises when the composition of the decisive coalition within the DMG changes over the life of the project.

We also characterize and contrast the role of polarization and volatility in investment inefficiency. Volatility is an important determinant of the value and management of a real option. Increases in volatility can lead to a delay in investment as the value of waiting increases. Such delays, however, are optimal and do not constitute inefficiencies. In contrast, polarization of beliefs leads to inefficiencies in that investments are not just delayed but lost altogether. Our result apply generally to groups that mediate conflicts according to either

majority or supermajority voting rules and to groups in which agents have veto power. While descriptive of actual final decision making in groups, in reality votes are cast in the context of dynamic pre-vote interactions. Developing theories that are more descriptive of the political economy of corporate finance will be the subject of future research.

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